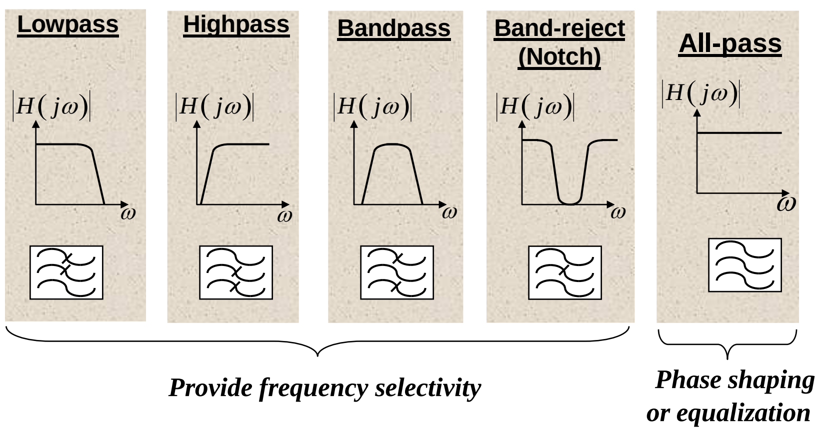
EE247 - Lecture 2
Filters

- Filters:
  - Nomenclature
  - Specifications
    - Quality factor
    - Magnitude/phase response versus frequency characteristics
    - Group delay
  - Filter types
    - Butterworth
    - Chebyshev I & II
    - Elliptic
    - Bessel
  - Group delay comparison example
  - Biquads

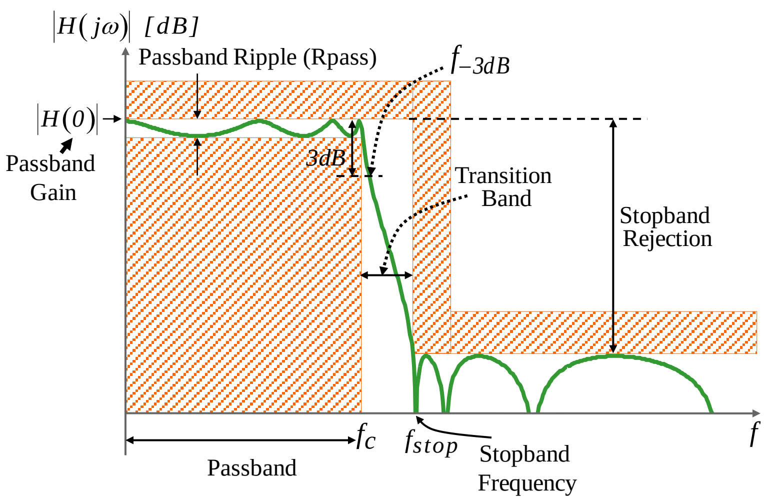
Nomenclature
Filter Types wrt Frequency Range Selectivity



Filter Specifications

- Magnitude response versus frequency characteristics:
  - Passband ripple (Rpass)
  - Cutoff frequency or *-3dB* frequency
  - Stopband rejection
  - Passband gain
- Phase characteristics:
  - Group delay
- SNR (Dynamic range)
- SNDR (Signal to Noise+Distortion ratio)
- Linearity measures: IM3 (intermodulation distortion), HD3 (harmonic distortion), IIP3 or OIP3 (Input-referred or output-referred third order intercept point)
- Area/pole & Power/pole

Filter Magnitude versus Frequency Characteristics
Example: Lowpass



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Quality Factor (Q)

- The term quality factor (Q) has different definitions in different contexts:
  - Component quality factor (inductor & capacitor Q)
  - Pole quality factor
  - Bandpass filter quality factor
- Next 3 slides clarifies each

Component Quality Factor (Q)

- For any component with a transfer function:

$$H(j\omega) = \frac{1}{R(\omega) + jX(\omega)}$$

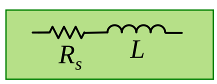
- Quality factor is defined as:

$$Q = \frac{X(\omega)}{R(\omega)} \rightarrow \frac{Energy\ Stored}{Average\ Power\ Dissipation}\ per\ unit\ time$$

Component Quality Factor (Q)
Inductor & Capacitor Quality Factor

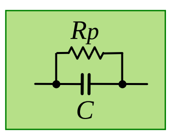
- Inductor Q :
  - ❖R_s → series parasitic resistance

$$Y_L = \frac{1}{R_s + j\omega L} \qquad Q_L = \frac{\omega L}{R_s}$$



- Capacitor Q :
  - ❖R_p → parallel parasitic resistance

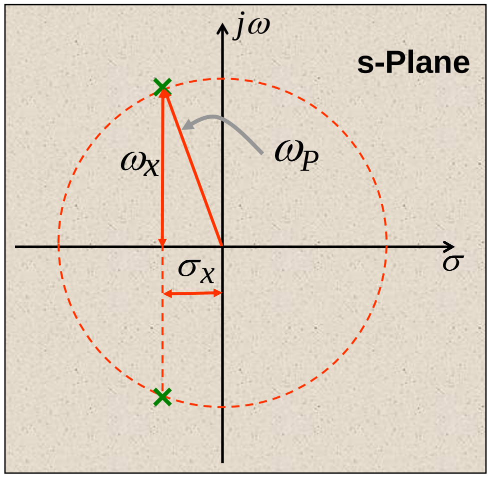
$$Z_C = \frac{1}{\frac{1}{Rp} + j\omega C} \qquad Q_C = \omega C Rp$$



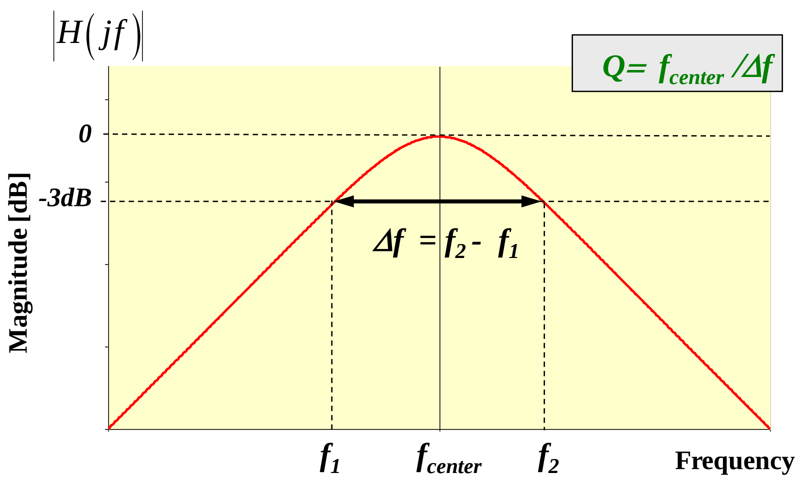
Pole Quality Factor

- Typically filter singularities include pairs of complex conjugate poles.
- Quality factor of complex conjugate poles are defined as:

$$Q_{Pole} = \frac{\omega_p}{2\sigma_x}$$



Bandpass Filter Quality Factor (Q)



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What is Group Delay?

- Consider a continuous-time filter with s-domain transfer function G(s):

$$G(j\omega) \equiv |G(j\omega)|\, e^{j\theta(\omega)}$$

- Let us apply a signal to the filter input composed of sum of two sine waves at slightly different frequencies ($\Delta\omega \ll \omega$):

$$v_{IN}(t) = A_1 \sin(\omega t) + A_2 \sin[(\omega+\Delta\omega)\, t]$$

- The filter output is:

$$v_{OUT}(t) = A_1\, |G(j\omega)|\, \sin[\omega t+\theta(\omega)] + A_2\, |G[\, j(\omega+\Delta\omega)]|\, \sin[(\omega+\Delta\omega)t+ \theta(\omega+\Delta\omega)]$$

What is Group Delay?

$$v_{OUT}(t) = A_1 \left|G(j\omega)\right| \sin\left\{\omega\left[t + \frac{\theta(\omega)}{\omega}\right]\right\} +$$

$$+ A_2 \left|G[j(\omega+\Delta\omega)]\right| \sin\left\{(\omega+\Delta\omega)\left[t + \frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega}\right]\right\}$$

Since $\frac{\Delta\omega}{\omega} \ll 1$ then $\left[\frac{\Delta\omega}{\omega}\right]^2 \rightarrow 0$

$$\frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \cong \left[\theta(\omega) + \frac{d\theta(\omega)}{d\omega}\Delta\omega\right]\left[\frac{1}{\omega}\left(1 - \frac{\Delta\omega}{\omega}\right)\right]$$

$$\cong \frac{\theta(\omega)}{\omega} + \left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega}\right)\frac{\Delta\omega}{\omega}$$

What is Group Delay?
Signal Magnitude and Phase Impairment

$$v_{OUT}(t) = A_1 \left|G(j\omega)\right| \sin\left\{\omega\left[t + \frac{\theta(\omega)}{\omega}\right]\right\} +$$

$$+ A_2 \left|G[j(\omega+\Delta\omega)]\right| \sin\left\{(\omega+\Delta\omega)\left[t + \frac{\theta(\omega)}{\omega} + \underbrace{\left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega}\right)\frac{\Delta\omega}{\omega}}_{\delta}\right]\right\}$$

- $\tau_{PD} \equiv -\theta(\omega)/\omega$ is called the "phase delay" and has units of time
- If the delay term δ is zero → the filter's output at frequency $\omega+\Delta\omega$ and the output at frequency ω are each delayed in time by $-\theta(\omega)/\omega$
- If the term δ is non-zero → the filter's output at frequency $\omega+\Delta\omega$ is time-shifted differently than the filter's output at frequency ω

  → "Phase distortion"

What is Group Delay?
Signal Magnitude and Phase Impairment

- Phase distortion is avoided only if:

$$\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} = 0$$

- Clearly, if $\theta(\omega)=k\omega$, k a constant, → no phase distortion
- This type of filter phase response is called "linear phase"
  →Phase shift varies linearly with frequency
- $\tau_{GR} \equiv -d\theta(\omega)/d\omega$ is called the "group delay" and also has units of time. For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = -k$
  → $\tau_{GR} = \tau_{PD}$ implies linear phase
- Note: Filters with $\theta(\omega)=k\omega+c$ are also called linear phase filters, but they're not free of phase distortion

What is Group Delay?
Signal Magnitude and Phase Impairment

- If $\tau_{GR} = \tau_{PD}$ → No phase distortion

$$v_{OUT}(t) = A_1 \left|G(j\omega)\right| \sin\left[\omega\left(t - \tau_{GR}\right)\right] + $$
$$+ A_2 \left|G[j(\omega+\Delta\omega)]\right| \sin\left[(\omega+\Delta\omega)\left(t - \tau_{GR}\right)\right]$$

- If also $\left|G(j\omega)\right| = \left|G[j(\omega+\Delta\omega)]\right|$ for all input frequencies within the signal-band, v_{OUT} is a scaled, time-shifted replica of the input, with no "signal magnitude distortion"

- In most cases neither of these conditions are exactly realizable

Summary
Group Delay

- Phase delay is defined as:

$$\tau_{PD} \equiv -\theta(\omega)/\omega \quad \text{[time]}$$

- Group delay is defined as :

$$\tau_{GR} \equiv -d\theta(\omega)/d\omega \quad \text{[time]}$$

- If $\theta(\omega)=k\omega$, k a constant, → no phase distortion

- For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = -k$

Filters

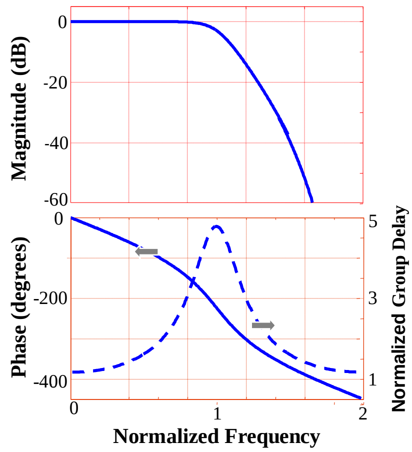
- Filters:
  – Nomenclature
  – Specifications
    - Magnitude/phase response versus frequency characteristics
    - Quality factor
    - Group delay
  – Filter types (examples considered all lowpass, the highpass and bandpass versions similar characteristics)
    - Butterworth
    - Chebyshev I & II
    - Elliptic
    - Bessel
  – Group delay comparison example
  – Biquads

Filter Types wrt Frequency Response
Lowpass Butterworth Filter

- Maximally flat amplitude within the filter passband

$$\left.\frac{d^N |H(j\omega)|}{d\omega}\right|_{\omega=0} = 0$$

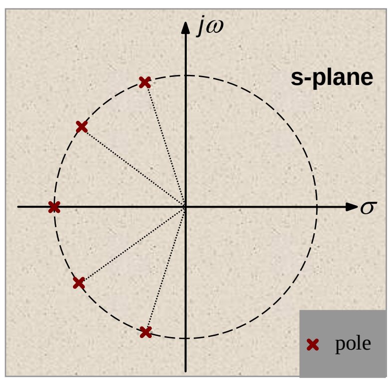
- Moderate phase distortion



Example: 5th Order Butterworth filter

Lowpass Butterworth Filter

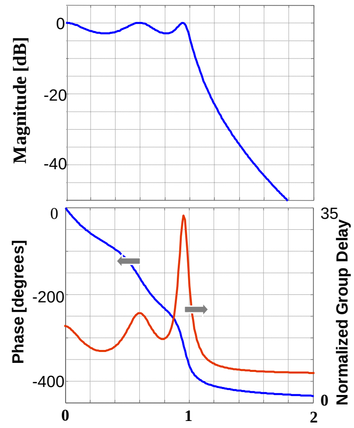
- All poles
- Number of poles equal to filter order
- Poles located on the unit circle with equal angles



Example: 5th Order Butterworth Filter

Filter Types
Chebyshev I Lowpass Filter

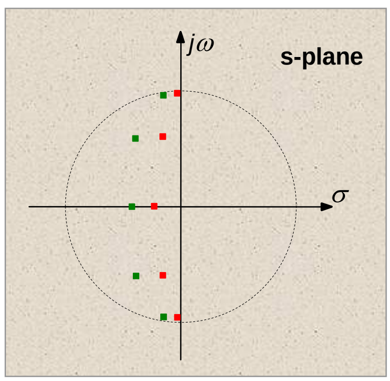
- Chebyshev I filter
  - Ripple in the passband
  - Sharper transition band compared to Butterworth (for the same number of poles)
  - Poorer group delay compared to Butterworth
  - More ripple in passband → poorer phase response



Example: 5th Order Chebyshev filter

Chebyshev I Lowpass Filter Characteristics

- All poles
- Poles located on an ellipse inside the unit circle
- Allowing more ripple in the passband:
  - ⇨Narrower transition band
  - ⇨Sharper cut-off
  - ⇨Higher pole Q
  - ⇨Poorer phase response

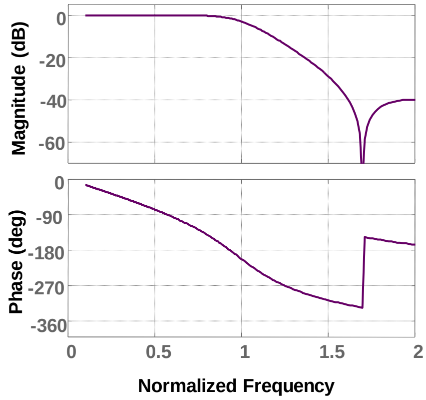


▪ Chebyshev I LPF 3dB passband ripple
▪ Chebyshev I LPF 0.1dB passband ripple

Example: 5th Order Chebyshev I Filter

Filter Types
Chebyshev II Lowpass

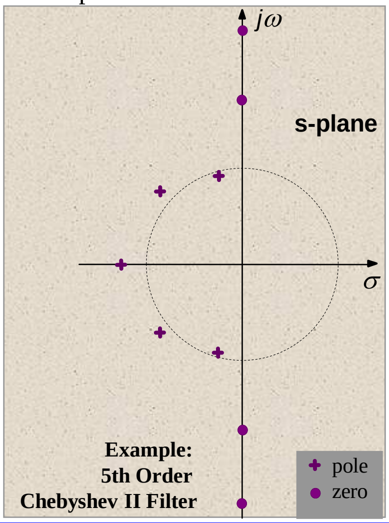
- Chebyshev II filter
  - No ripple in passband
  - Nulls or notches in stopband
  - Sharper transition band compared to Butterworth
  - Passband phase more linear compared to Chebyshev I



Example: 5th Order Chebyshev II filter

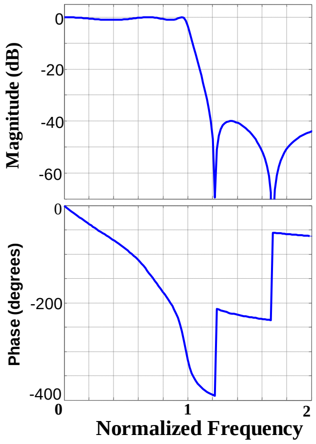
Filter Types
Chebyshev II Lowpass

- Poles & finite zeros
  - No. of poles *n* (*n* –> filter order)
  - No. of finite zeros: *n-1*
- Poles located both inside & outside of the unit circle
- Complex conjugate zeros located on $j\omega$ axis
- Zeros create nulls in stopband



Filter Types
Elliptic Lowpass Filter

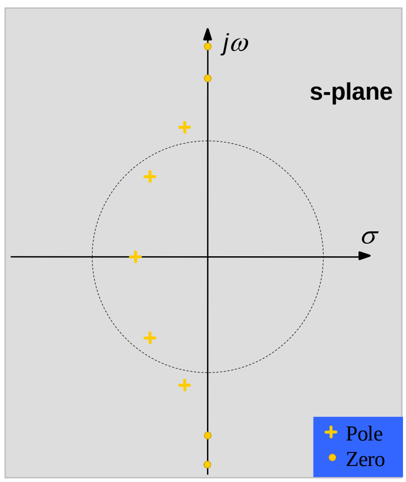
- Elliptic filter
  - Ripple in passband
  - Nulls in the stopband
  - Sharper transition band compared to Butterworth & both Chebyshevs
  - Poorest phase response



Example: 5th Order Elliptic filter

Filter Types
Elliptic Lowpass Filter

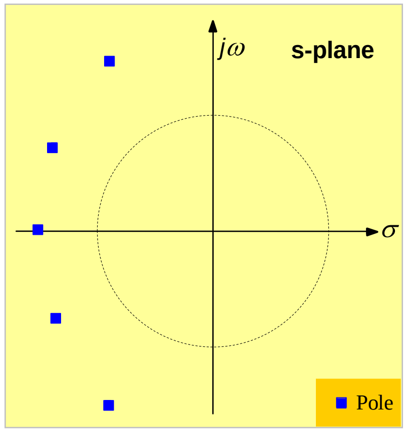
- Poles & finite zeros
  - No. of poles: *n*
  - No. of finite zeros: *n-1*
- Zeros located on $j\omega$ axis
- Sharp cut-off
  - ⇨Narrower transition band
  - ⇨Pole Q higher compared to the previous filter types



Example: 5th Order Elliptic Filter

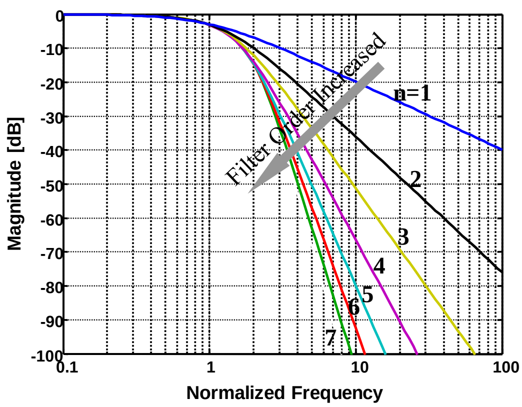
Filter Types
Bessel Lowpass Filter

- Bessel
  - –All poles
  - –Poles outside unit circle
  - –Relatively low Q poles
  - **–Maximally flat group delay**
  - –Poor out-of-band attenuation



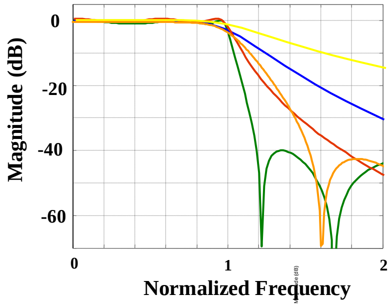
Example: 5th Order Bessel filter

Magnitude Response Behavior as a Function of Filter Order Example: Bessel Filter



n→ Filter order

Filter Types
Comparison of Various Type LPF Magnitude Response

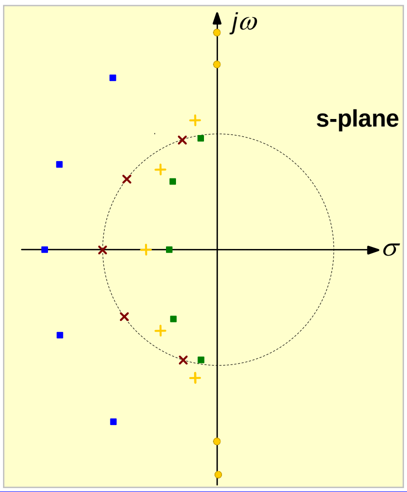


All 5th order filters with same corner freq.

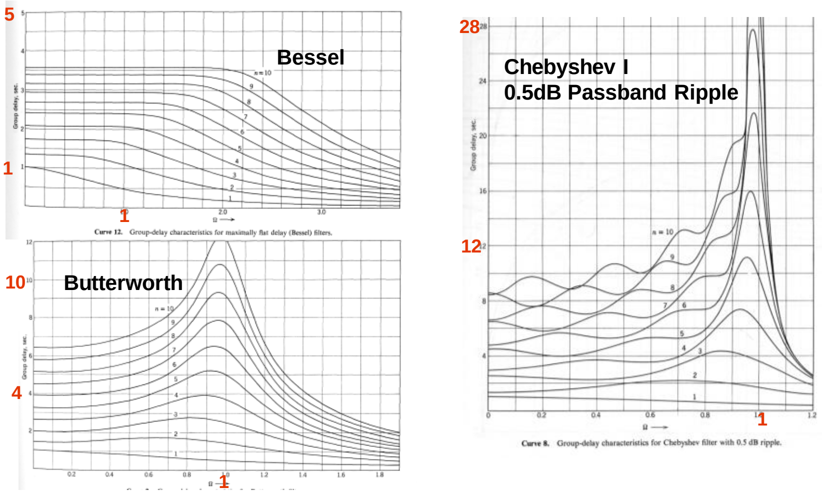
Bessel
Butterworth
Chebyshev I
Chebyshev II
Elliptic

Filter Types
Comparison of Various LPF Singularities

- Poles Bessel
- Poles Butterworth
- Poles Elliptic
- Zeros Elliptic
- Poles Chebyshev I 0.1dB



Comparison of Various LPF Groupdelay



Ref: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.

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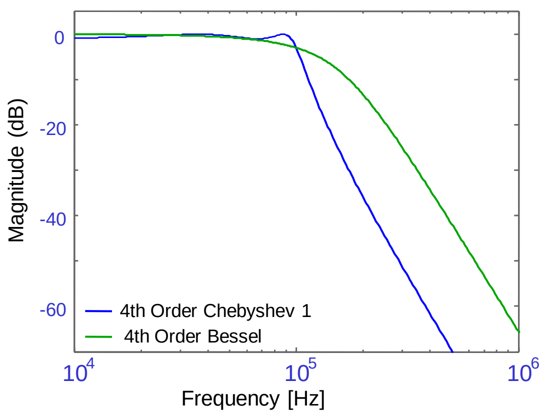
Group Delay Comparison Example

- Lowpass filter with 100kHz corner frequency
- Chebyshev I versus Bessel
  - Both filters 4th order- same ***-3dB*** point
  - Passband ripple of ***1dB*** allowed for Chebyshev I

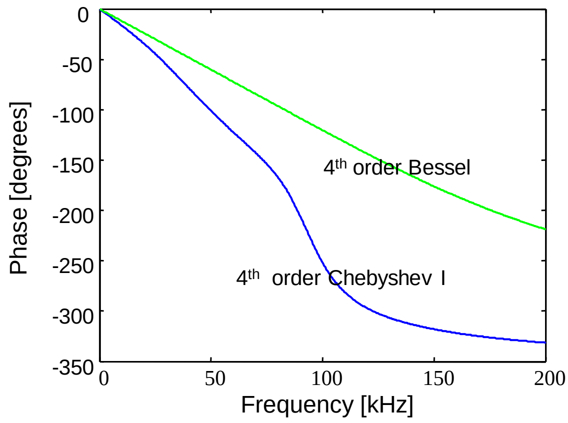
Magnitude Response

4th Order Chebyshev I versus Bessel



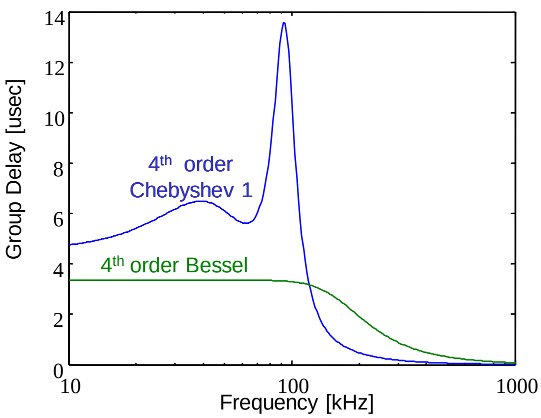
Phase Response

4th Order Chebyshev I versus Bessel

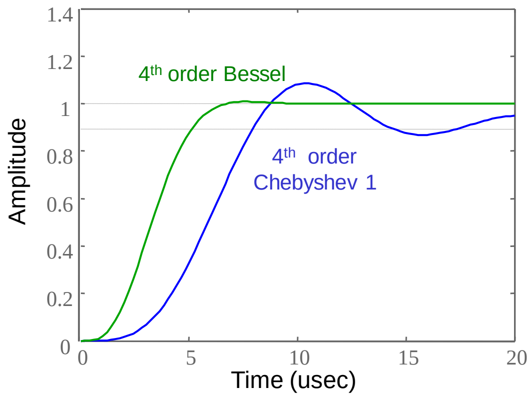


Group Delay

4th Order Chebyshev I versus Bessel



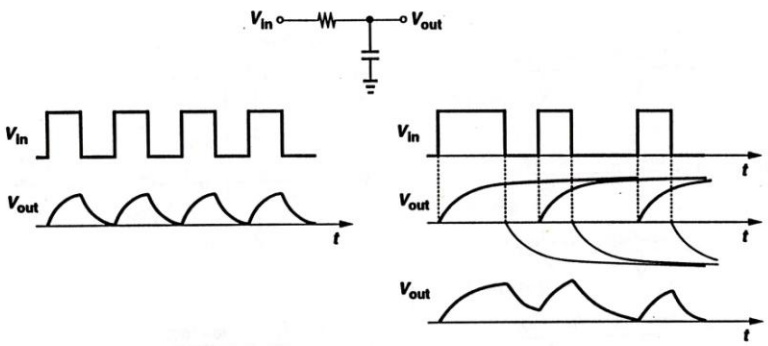
Step Response
4^{th} Order Chebyshev I versus Bessel



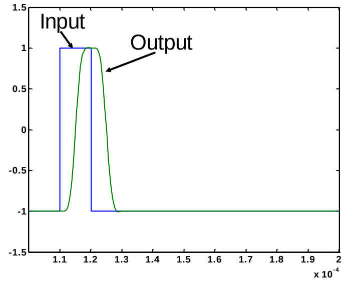
Intersymbol Interference (ISI)

ISI→ Broadening of pulses resulting in interference between successive transmitted pulses

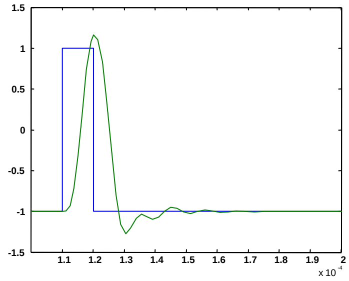
Example: Simple RC filter



Pulse Impairment
Bessel versus Chebyshev



4th order Bessel



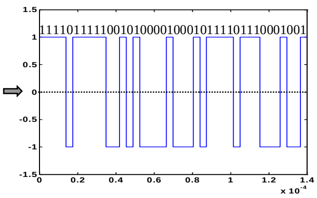
4th order Chebyshev I

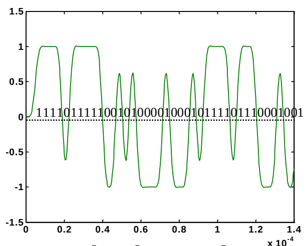
Note that in the case of the Chebyshev filter not only the pulse has broadened <u>but it also has a long tail</u>

→ More ISI for Chebyshev compared to Bessel

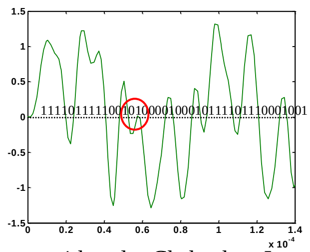
Response to Pseudo-Random Data
Chebyshev versus Bessel

Input Signal:
Symbol rate 1/130kHz ⇨





4th order Bessel



4th order Chebyshev I

Summary
Filter Types

– Filter types with high signal attenuation per pole ⇨ poor phase response

– For a given signal attenuation, requirement of preserving constant groupdelay → Higher order filter
  - In the case of passive filters ⇨ higher component count
  - For integrated active filters ⇨ higher chip area & power dissipation

– In cases where filter is followed by ADC and DSP
  - In some cases possible to digitally correct for phase impairments incurred by the analog circuitry by using digital phase equalizers & thus possible to reduce the required analog filter order

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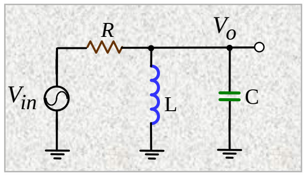
  - ⇒ – Biquads

RLC Filters

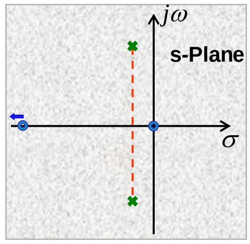
• Bandpass filter (2nd order):



$$\frac{V_o}{V_{in}} = \frac{\frac{s}{RC}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\omega_0 = 1 \Big/ \sqrt{LC}$$

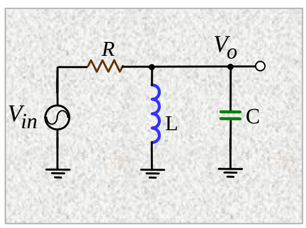
$$Q = \omega_0 RC = \frac{R}{L\omega_0}$$



Singularities: Pair of complex conjugate poles
Zeros @ $f=0$ & $f=inf$.

RLC Filters
Example

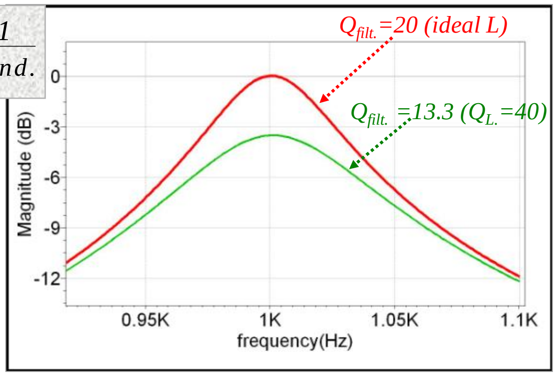
• Design a bandpass filter with:
  - Center frequency of 1kHz
  - Filter quality factor of 20



• First assume the inductor is ideal
• Next consider the case where the inductor has series R resulting in a finite inductor Q of 40
• What is the effect of finite inductor Q on the overall filter Q?

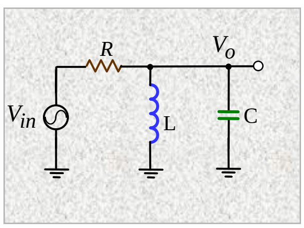
RLC Filters
Effect of Finite Component Q

$$\frac{1}{Q_{filt}} \approx \frac{1}{Q_{filt}^{ideal}} + \frac{1}{Q_{ind.}}$$



⇨Need to have component Q much higher compared to desired filter Q

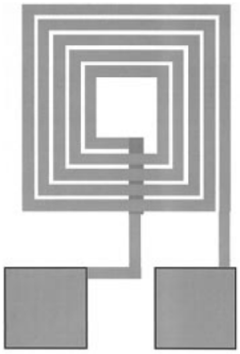
RLC Filters



Question:

Can RLC filters be integrated on-chip?

Monolithic Spiral Inductors



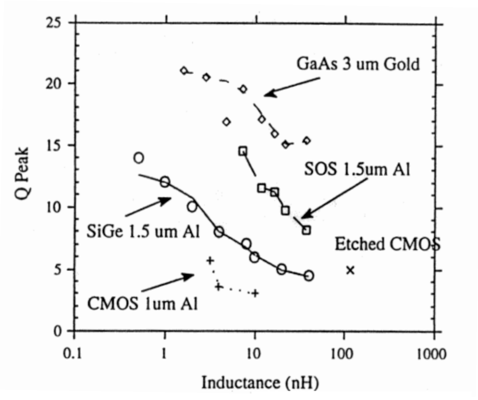
Top View

Monolithic Inductors
Feasible Quality Factor & Value

Typically, on-chip inductors built as spiral structures out of metal/s layers

$Q_L = (\omega L/R)$

Q_L measured at frequencies of operation ($>1GHz$)



➪ Feasible monolithic inductor in CMOS tech. <10nH with Q <7

❖Ref: "Radio Frequency Filters", Lawrence Larson; Mead workshop presentation 1999

Integrated Filters

- Implementation of RLC filters in CMOS technologies requires on-chip inductors
  - Integrated L<10nH with Q<10
  - Combined with max. cap. 20pF
  - → *LC filters in the monolithic form feasible: freq>350MHz*
  - → *(Learn more in EE242 & RF circuit courses)*

- Analog/Digital interface circuitry require fully integrated filters with critical frequencies << 350MHz

- Hence:

⇨ Need to build active filters without using inductors

Filters

2nd Order Transfer Functions (Biquads)

- Biquadratic (2nd order) transfer function:

$$H(s)=\frac{1}{1+\frac{s}{\omega_P Q_P}+\frac{s^2}{\omega_P^2}}$$

$$|H(j\omega)|=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_P^2}\right)^2+\left(\frac{\omega}{\omega_P Q_P}\right)^2}} \Rightarrow \begin{cases} |H(j\omega)|_{\omega=0}=1 \\ |H(j\omega)|_{\omega\to\infty}=0 \\ |H(j\omega)|_{\omega=\omega_P}=Q_P \end{cases}$$

$$\text{Biquad poles @: } s=-\frac{\omega_P}{2Q_P}\left(1\pm\sqrt{1-4Q_P^2}\right)$$

Note: for $Q_P \le \frac{1}{2}$ *poles are real, complex otherwise*

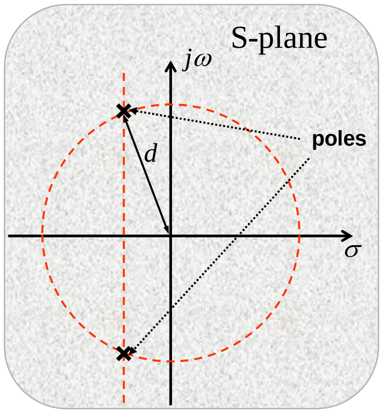
Biquad Complex Poles

$Q_P > \frac{1}{2} \quad \rightarrow$ Complex conjugate poles:

$$s = -\frac{\omega_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

Distance from origin in s-plane:

$$d^2 = \left(\frac{\omega_P}{2Q_P}\right)^2 \left(1 + 4Q_P^2 - 1\right)$$
$$= \omega_P^2$$



s-Plane