Problem 1: Perturbed nonlinear systems.
Suppose that some physical system obeys the differential equation
\[ \dot{x} = p(x,t), \quad x(t_0) = x_0, \quad \forall t \geq t_0 \]
where \( p(\cdot, \cdot) \) obeys the conditions of the fundamental theorem. Suppose that as a result of some perturbation the equation becomes
\[ \dot{z} = p(z,t) + f(t), \quad z(t_0) = x_0 + \delta x_0, \quad \forall t \geq t_0 \]
Given that for \( t \in [t_0, t_0 + T] \), \( ||f(t)|| \leq \epsilon_1 \) and \( ||\delta x_0|| \leq \epsilon_0 \), find a bound on \( ||x(t) - z(t)|| \) valid on \( [t_0, t_0 + T] \).

Problem 2: Linear systems.
Given the following discrete-time state update equation
\[ x[k+1] = ax[k], \quad x[0] = x_0, \quad a, x \in \mathbb{R} \]
with the output read out map \( y[k] = x[k] \), derive the state transition function. Is this a linear system?

Problem 3: Dynamical systems, time invariance.
Suppose that the output of a system is represented by
\[ y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} u(\tau) d\tau \]
Is the system time invariant? You may select the input space \( U \) to be the set of bounded, piecewise continuous, real-valued functions defined on \( (-\infty, \infty) \).