Module 2, Lecture 12

**Topic Review:**

* Fundamentals
* NVA
* Power, I & V Measurements
* Resistor & Capacitor Physics
* Thevenin and Norton Equiv
  - Dep. Sources
  - \(\parallel\) & series
* Op Amps
  - NFB
  - Comparators
* Charge Sharing
* Practice Problems
Fundamentals:
Voltages & Currents } Lec 2D
Nodes & Branches } Note 11

Node: any point where 2 or more elements intersect.

e.g., $V_{R_1} = V_2 - V_1$
Node Voltage Analysis

Knowns: Source values, Resistor values
Unknowns: Node voltages

1. Select a reference node
2. Label nodes with known voltages
3. Label remaining nodes
4. Label element voltages & currents, following passive sign convention
5. Write KCL Equations at each unknown node voltage
6. Write Element Current Equations
7. Substitute Element Currents into KCL equations
8. Solve!

Universal algorithm: never fails!
Labeling elements: What if we flip the current directions?

KCL:
\[ I_{R1} = I_{R2} \]
\[ I_{R1} - I_{R2} = 0 \]

Element Equations:
\[ I_{R1} = \frac{V_{R1}}{4k\Omega} = \frac{V_{V2} - V_I}{4k\Omega} \]
\[ I_{R2} = \frac{V_{R2}}{4k\Omega} = \frac{V_I - 0}{4k\Omega} \]

Substituting element equations into KCL:
\[ \frac{V_{V2} - V_I}{4k\Omega} - \frac{V_I - 0}{4k\Omega} = 0 \]

Notice that if we rearrange the equations a bit, they should give the same solution for \( V_I \). \( I_{R1} \) will be equal in magnitude but negative, however, the “real” direction of the current is the same.

**Common Practice:**

Invoke already done analysis results on “common”cite that show up often:

\[ \downarrow \text{Dividers} \]
\[ \text{(Voltage, Current)} \]
Voltage Divider

Note, the resistors MUST be in SERIES to use this equation

\[ V_{R1} = V_s \frac{R_1}{R_1 + R_2} \]
\[ V_{R2} = V_s \frac{R_2}{R_1 + R_2} \]

Current Divider

Note, the resistors MUST be in PARALLEL to use this equation

Also note that the resistor in the numerator is opposite of the one you’d expect. Intuitively, path of least resistance.

\[ I_{R1} = I_s \frac{R_2}{R_1 + R_2} \]
\[ I_{R2} = I_s \frac{R_1}{R_1 + R_2} \]
Power:

\[ P_{el} = V_{el} \cdot I_{el} \]

**Always!**

\[ P_{el} > 0 \rightarrow 
\begin{align*}
1) & \text{Element dissipates power} \\
2) & \text{It follows passive sign convention} \\
3) & \text{It is "passive"}
\end{align*} \]

\[ P_{el} < 0 \rightarrow 
\begin{align*}
1) & \text{Element generates power} \\
2) & \text{It "violates" passive sign convention} \\
3) & \text{It is "active" (i.e., current flows out of the \"+\" in reality)} \\
& \text{e.g., in voltage source + resistor}
\end{align*} \]
* For Resistors:

\[ P_R = I_R V_R = I_R^2 R = \frac{V_R^2}{R} > 0 \]

\[ \implies \text{A resistor always dissipates power!} \]

* Power is always conserved!

\[ P_{el,\text{tot}} = 0 \]
V & I Measurements:

Ideal Measurement Device
Should dissipate 0 power

Ref: Lec 3B - Note 13
Current Measurement

\[ I_{\text{mystery}} = \frac{V_{\text{meas}}}{R} + I_{\text{meas}} \]

\[ I_{\text{meas}} = \frac{V_{\text{meas}}}{R} + I_{\text{meas}} \]

\[ I_{\text{mystery}} = I_{\text{meas}} \quad \text{if} \quad V_{\text{meas}} = 0 \]

\[ P_{\text{meas}} = V_{\text{meas}} \cdot I_{\text{meas}} = 0 \quad \checkmark \quad \text{(measurement circuit does not dissipate any power)} \]

**Disclaimer:**

If approximations are required we will explicitly tell you so in an exam. (i.e.

\[ \frac{R_{\text{mystery}} - R_x}{R_{\text{mystery}} + R_x} \quad \text{if} \quad R_{\text{mystery}} \rightarrow \infty \]
Resistors and Capacitors

\[ R = \rho \cdot \frac{L}{A} \]

material resistivity

\[ C = \varepsilon \cdot \frac{A}{d} \]

material (insulator) permittivity
Superposition

Def of linear: $T(ax + by) = aT(x) + bT(y)$

The passive components we use have linear I-V relations, so we can solve for the effect of each source individually and add the responses to get the same result.

When to use? When you have **multiple sources** and **considering all at the same time is confusing**

Note: you can solve any circuit in this class without superposition with just normal nodal analysis.
Series?
Two ways to tell if components are in series:

1. same EXACT current. Not same value of current, but same exact current through elements by KCL. If the current can split off then they aren’t series!
2. If two and only two elements are connected to a single node (no other elements are connected to that node), the elements are in series.

Parallel?
Parallel: they share the same NODES on either side. Not just the same voltage, but same NODES

\[ C_{AB} = C_1 + C_2 + C_3 \]
Parallel:
\[ R_{AB} = \frac{R_1 R_2}{R_1 + R_2} \]

Series:
\[ R_{AB} = R_1 + R_2 \]

Capacitors:
\[ C_{AB} = C_1 + C_2 \]
\[ C_{AB} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} \]

For the circuit with \( R_3, R_4, R_5 \):
\[ R_{AB} = \frac{R_1}{R_5} \]
Note: $R_{eq}$ depends on where I connect my terminals:

$$R_{AB}' = R_4 \parallel (R_1 + R_2 + R_3)$$

$\neq R_{AB}$ from above

**Capacitor Example**

$$C_{AB} = C_1 \parallel (C_2 + C_3)$$
Thevenin and Norton Equiv:

1. $V_{th} = I_{no} \cdot R_{no}$
2. $I_{no} = \frac{V_{th}}{R_{th} + R_{no}}$
3. $R_{th} = R_{no}$

for resistive only cells

$I-V$ plot through the origin
Finding Thevenin - Norton Equiv.

1) Find $V_{th}$ by open-circuiting the terminals A-B and finding $V_{oc}=V_{th}$
   How: NVA or Superposition

2) Find $I_{no}$ by short-circuiting the terminals A-B (i.e., connect them with a wire) and find $I_{sc}=I_{no}$
   How: NVA or Superposition

3) Find $R_{th}$:
   How: 3 options → pick the easiest!
   a) $R_{th} = \frac{V_{th}}{I_{no}}$  → fails with res. only ckt

E.g.

Resistive-only ckt

No source → cannot apply method a)

since $V_{th}=I_{no}=0$
b) Use \( \parallel \) and series combination with independent source zed-out to fastest.

\[ \text{fails when dep. sources are not zeroed out (see ex. below)} \]

\[ \text{But takes the longest} \]

\[ \text{Procedure:} \]

With Indep. Sources zed-out

Apply \( V_{\text{test}} \rightarrow \text{measure} \ I_{\text{test}} \)

or

Apply \( I_{\text{test}} \rightarrow \text{measure} \ V_{\text{test}} \)

\[ R_{\text{flu}} = \frac{V_{\text{test}}}{I_{\text{test}}} \]
Dependent Sources:
Model Some Complicated
Circuit Element in our "Black Box" ckt (e.g. transistors, opamps)

\[ V_c = A \cdot I_c \]

Should not be shut off when finding \( R_{th} \).

Independent Sources:
Inputs to our ckt.

\[ V_s \]
\[ I_s \]
Find $V_{th}, R_{th}$ looking into terminals C, D:

Disclaimer: $g$ has units of $\frac{1}{\Omega}$

(c) We modify the circuit as shown below:

\[ V_{AB} = V_A = \frac{R_3}{R_3 + R_2} \cdot V_s \quad (1) \]

\[ V_{out} = g(V_A - V_{out}) \frac{R_1 || R_0}{1 + gR_1 || R_0} \]

\[ = \frac{g \cdot R_1 || R_0}{1 + gR_1 || R_0} \cdot V_A \]

\[ (1) = \frac{g \cdot R_1 || R_0}{1 + gR_1 || R_0} \cdot \frac{R_3}{R_3 + R_2} \cdot V_s \]

\[ V_{out} = V_{oc} = V_{th}. \]
Step 2: Find $R_{th}$

a) Zero out independent sources

b) Apply $V_{test}$ and measure $I_{test}$ using NVA. (I need to do this here since the dep. source is not zeroed out)

KCL (on node $C$):

$\text{entering } I_{test} + g_m (V_{out}) - I_1 = 0$

$\text{exitting } I_{test} + g_m (-V_{out}) - V_{out}/R_{L/R_0} = 0$

$\Rightarrow -(g_m + \frac{1}{R_{L/R_0}}) V_{test} = -I_{test}$
\[
\frac{V_{\text{test}}}{I_{\text{test}}} = \frac{1}{g_m + \frac{1}{R_c // R_0}}
\]

\[
R_{ft} = \frac{1}{g_m + \frac{1}{R_c // R_0}}
\]
Looking only at this:

\[ V_{th} = V_{AB} = \frac{R_3}{R_2 + R_3} V_s \]

\[ R_{th} : (\text{zero } V_s) \]

Find \( V_{th} \), \( R_{th} \) looking at A-B.

\[ R_{AB} = R_2 || R_3 \]

\( R_2, R_3 \) are coll. in parallel since \( V_s \to \text{wire} \)

(P1 is shorted so neglect - we did wrong in lec.)
Op Amps

Model:

Symbol:

Caveat: $V_{pp}$ and $V_{ss}$ supply power to the op amp (often not drawn for convenience)
Uses:

a) As a comparator

Is this a linear function?

No!
b) As an amplifier and a means of cascading blocks

\[ f() \rightarrow g() \]

Cascading Cut Blocks

Before connection:
\[ V_{\text{in}} = V_{\text{in}} \]

After connection:
\[ V_{\text{in}} = V_{\text{in}} = \frac{R_{\text{th},g} V_{\text{in}} + R_{\text{th},f} V_{\text{out}}}{R_{\text{th},f} + R_{\text{th},g}} \]

In general:
\[ V_{\text{in}} = V_{\text{in}} \]

Except when:
\[ R_{\text{th},f} = 0 \text{ (wire)} \]
\[ R_{\text{th},g} = \infty \text{ (open-circuit)} \]

Ideal Isolation

From the perspective of block \( f \): see an open-circuit \( R_{\text{th},g} = \infty \)

From the perspective of block \( g \): see a voltage source \( R_{\text{th},f} = 0 \)
Motivation:

Audio System - "JGAboombox"

Digital to Analog Converter (DAC)
Converts a digital-binary value to
an analog voltage

\[ 3.3V = V_{dd} \]

\[ V_{out, DAC} = V_{ds} \]

\[ V_{out, DAC} \in [0, 3.3V] \]

Thévenin Equivalent

\[ R_{th, DAC} = 100 \Omega \]

\[ V_{th, DAC} \in [0, 3.3V] \]

Voltage Divider

\[ V_{speaker} = \frac{R_{speaker}}{R_{speaker} + R_{th, DAC}} \cdot V_{th, DAC} \]

\[ = \frac{8\Omega}{8\Omega + 100\Omega} \cdot V_{th, DAC} \approx 0.1 \cdot V_{th, DAC} \]

[Loading Effect?]

Caution: Direct connection with \( 10V \) and \( 0V \) into a \( 8\Omega \) speaker is not recommended.
Want Linear Amplification

\[ V_{out} = Av \cdot V_{in} \]

- Goal: \( Av \) needs to be reasonable and well controlled.

- Problem: huge uncertain gain \( \Rightarrow \) non-linear \( V_{in} - V_{out} \)

- Solution: Use NFB!
\[ V_{\text{out}} = A_v \cdot V_{\text{in}} \]

\[ A_v = \frac{A}{1 + A f} \quad \xrightarrow{\text{feedback}} \quad \frac{1}{1 + A f} \]

\[ \Rightarrow \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{f} \quad \text{not that easy to calculate.} \]
That’s why we invoked the...

**GOLDEN RULES!**

#1 \( i^+ = i^- = 0 \) (Always)

#2 \( u^+ = u^- \) when \( A \to \infty \)

and op-amp is in NFB.
Using golden rules we derived the following:

<table>
<thead>
<tr>
<th>Circuit Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Divider</td>
<td>$V_{R2} = V_S \left( \frac{R_S}{R_1 + R_2} \right)$</td>
</tr>
<tr>
<td>Voltage Summer</td>
<td>$V_{out} = V_1 \left( \frac{R_1}{R_1 + R_2} \right) + V_2 \left( \frac{R_1}{R_1 + R_2} \right)$</td>
</tr>
<tr>
<td>Unity Gain Buffer</td>
<td>$V_{out} = \frac{V_{in}}{V_{in}} \frac{1}{1}$</td>
</tr>
<tr>
<td>Inverting Amplifier</td>
<td>$V_{out} = V_{in} \left( -\frac{R_f}{R_i} \right) + V_{REF} \left( \frac{R_f}{R_i} + 1 \right)$</td>
</tr>
<tr>
<td>Non-inverting Amplifier</td>
<td>$V_{out} = V_{in} \left( 1 + \frac{R_{op}}{R_{omin}} \right) - V_{REF} \left( \frac{R_{op}}{R_{omin}} \right)$</td>
</tr>
<tr>
<td>Transresistance Amplifier</td>
<td>$V_{out} = i_{in} (-R) + V_{REF}$</td>
</tr>
</tbody>
</table>

Keep them handy - learn to pattern match and recognize them
Example #1:  Want this:

\[ V_{in} \rightarrow \frac{R_2}{R_1 + R_2} \rightarrow V_{uid} \rightarrow \text{Av} = 10 \rightarrow V_{out} \rightarrow V_{out} = 10 \cdot V_{uid} \]

Implement:

\[ V_{uid} \quad \text{Av} = 1 + \frac{R_2}{R_{tot}} \]

Verify:

Before connection:

\[ V_{uid} = \frac{R_2}{R_1 + R_2} \cdot V_{in} \]

After connection:

\[ V_{uid} = V_{uid,2} = \frac{R_2}{R_1 + R_2} \cdot V_{in} \cdot \text{Av} = \frac{R_2}{R_1 + R_2} \cdot (1 + \frac{R_2}{R_{tot}}) \cdot V_{in} \cdot 10 \]

\[ V_{out} = Av \cdot V_{uid,2} \]
Practice Handout

\[ V_{0,s1} = -\frac{R_2}{R_1} \cdot V_{s1} \]

\[ V_{s2} \]

\[ R_2 \]

\[ R_1 \]
\[ V_{0,3} = \left(1 + \frac{R_2}{R_1}\right) \cdot V_{S2} \]

\[ U^- = U^+ = 0 \]

\[ \Rightarrow V_{R_1} = ? \]

I \text{cl on } U^- : \quad = 0 - U^-

\[ I_{R_3} = I_{R_2} = I_S \]

Ohm's law: \quad \Rightarrow V_{R_2} = I_S \cdot R_2

\[ 0 - V_{0,IS} = I_S \cdot R_2 \]
\[ V_o = V_{o51} + V_{o52} + V_{oIS} \]

\[ = - \frac{R_2}{R_1} V_{s1} + \left( 1 + \frac{R_2}{R_1} \right) V_{R2} = I_s R_2 \]

(sol also on Class Website)
(a) Determine the voltage at $O$ for the following configuration.

\begin{figure}[h]
\centering
\begin{circuitikz}
\draw (0,0) node[ground]{} -- (0,1) node[ground]{} -- (1,1) node[ground]{} -- (1,0) node[ground]{} -- (0,0);
\draw (0,0) to [v=$v_1$, v^<=$+$] (0,1);
\draw (1,0) to [v=$v_2$, v^<=$+$] (1,1);
\draw (0,0) to [25 k\Omega] (0,1);
\draw (1,0) to [25 k\Omega] (1,1);
\node at (0.5,1.5) {25 k\Omega};
\node at (0.5,-0.5) {25 k\Omega};
\node at (0,0.5) {A};
\node at (1,0.5) {C};
\node at (0,-0.5) {B};
\node at (1,-0.5) {D};
\node at (0.5,2) {O};
\end{circuitikz}
\end{figure}

Practice: show that

$v_o = v_1 + v_2$

(sols on Piazza)