EECS 16A DIS 4B

* Don’t forget, there’s a checkoff today.

Today’s topics

1. Capacitor Review (voltage, current, charge, energy)
2. Time dependent behavior of a changing capacitor
3. If time: Capacitor Equivalence derivations + practice
   → Appeared in lecture yesterday.

\[ \text{Energy (stored in the capacitor)} \]
\[ E = \frac{1}{2} CV^2 \]
\[ = \frac{1}{2} \frac{Q^2}{C} \]
\[ = \frac{1}{2} AV \left[ \frac{C}{\epsilon} \right] \]
\[ \rightarrow \text{[J]} \]

\[ i = \frac{dQ}{dt} \]
\[ Q = CV \]

Assuming a constant \( C \)

\[ i = \frac{dQ}{dt} = \frac{d}{dt} (CV) = C \frac{dV}{dt} \]

\[ i = \frac{dV}{dt} \]

\[ C = \text{Capacitance} \] [Farads]
What is the charge + energy?

\[ Q_1 = C V_1 \]

\[ [F][V] = [C] \text{ coulombs, unit of charge} \]

\[ V_5 = 1 \text{ V} \]

\[ V_1 = V_5 \sqrt{\text{V}} \]

\[ Q_1 = (1 \mu F)(1 \text{ V}) = \boxed{1 \mu C} \]

Q: How do SI prefixes combine?

\[ 10^3 \text{ milli}[A] \times \text{ milli}[B] = 10^{-3} \times 10^{-3} \text{ [AB]} = 10^{-6} \text{ [AB]} = \frac{\text{micro}[AB]}{\mu} \]

\[ \text{milli}[A] \times \text{ [B]} = 10^{-3} \text{ [AB]} = \text{ milli}[AB] \]

\[ E = \frac{1}{2} C V^2 = \frac{1}{2} (1 \mu F)(1 \text{ V})^2 = \boxed{\frac{1}{2} \mu \text{J}} \]

\[ [F][V]^2 \text{ [V]} = \boxed{\frac{[J]}{[C][V]}} \]
After capacitors have had time to change:

\[ V_s = V_1 + V_2 \] (true after charging)

\[ V_s = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \]

\[ Q = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \]

\[ V_s = \frac{C_1 C_2}{C_1 + C_2} \cdot V_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \]

\[ 1 \text{V} = \frac{3}{4} \text{MC} \]

\[ Q_1 = Q_2 = Q \]

Method 1:

1. Method 1: Look at both caps changing

2. Method 2: Use equivalence

Using method 2:

\[ q \]

\[ E_1 = \frac{1}{2} \frac{Q_1^2}{C_1} = \frac{1}{2} \cdot \frac{Q_1^2}{C_1} \]

\[ E_2 = \frac{1}{2} \cdot \frac{Q_2^2}{C_2} = \frac{1}{2} \cdot \frac{(\frac{3}{4}MC)^2}{3 \text{MC}} = \frac{9}{32} \text{MJ} \]

\[ \text{energy stored in each cap.} \]
How capacitors behave (with time) when charged by a constant current source.

\[ I_s \uparrow \quad V_C = \frac{Q}{C_1} \quad V_{out}(t) \]

\[ I_C = I_s \quad \text{(KCL @ top)} \]

\[ C_1 \frac{dV_C}{dt} = I_s \quad V_C(t) = V_{out}(t) \]

\[ \frac{dV_{out}}{dt} = \frac{I_s}{C_1} \quad V_{out}(t) = \frac{I_s}{C_1} (t - \tau) \]

\[ V_{out}(t) = V_{out}(0) + \frac{I_s}{C_1} t \]

\[ V_{out}(0) = 0 \quad \Rightarrow \quad V_{out}(t) = \frac{I_s}{C_1} t \]

Constant current leads to constantly increasing (linearly) voltage.
\[ C_{eq} = c_1 + c_2 \]  
(parallel capacitors)

\[ V_{at}(t) = \frac{I_s}{C_{eq}} t = \left[ \frac{I_s}{c_1 + c_2} \right] t \]  
Using cap. equivalence
3. Deriving Parallel equivalent capacitance

(b) Deriving series

(c) Factoring using equivalence

\[ Q = CV \]

\[ i = C \frac{dV}{dt} \]

in lecture this was used

\[ V_{eq} = V_1 = V_2 \text{ (parallel)} \]

\[ Q_{eq} = \frac{Q_1 + Q_2}{Q_1} \text{ or } Q_{eq} \]

\[ V_{eq} \sim Q_{eq} ? \]

\[ Q_{eq} = C_1 V_{eq} + C_2 V_{eq} \]

\[ Q_1 = C_1 V_1 = C_1 V_{eq} \]

\[ Q_2 = C_2 V_1 = C_2 V_{eq} \]

\[ Q_{eq} = C_{eq} V_{eq} = (C_1 + C_2) V_{eq} \]

\[ C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \text{ (in parallel)} \]
Series eq. C: Assumption: both start unchanged both start with the same amount of charge

\[ Q_{\text{eq}} = Q_1 = Q_2 \]

( series, same current dump's same change )

\[ V_{\text{eq}} = V_1 + V_2 \]

\[ V_{\text{eq}} = \frac{Q_{\text{eq}}}{C_1} + \frac{Q_{\text{eq}}}{C_2} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right) Q_{\text{eq}} \]

\[ Q_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \frac{V_{\text{eq}}}{V_{\text{eq}}} = \frac{C_1 C_2}{C_1 + C_2} V_{\text{eq}} \]

\[ C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} \]

( series eq., )

\[ C_1 \| C_2 \]

\[ \frac{A}{1B} \frac{A}{1+B} \]

operation, not shape of circuit
Extra: Not covered, but here for your use/learning/checking

3. Practice using series and parallel equivalence to reduce to a single equivalent

1. Check series \& parallel for pairs
   - $C_1 \cap C_2$? No, neither (bc of $C_3$, not series, bc of $C_4$ not parallel)
   - $C_4 \cap C_2$? also Neither
   - $C_2 \cap C_3$? Yes! Parallel

2. Calculate value
   \[ C_{eq} = C_2 + C_3 \]

3. Redraw with substitution: The equivalent should be connected to the same pair of nodes ($\bullet$, $\bullet$)

4. Iterate!
   1. $C_1 \cap C_2 + C_3$ series
   2. $C_{eq} = C_1 \parallel (C_2 + C_3)$ (parallel operator doesn't distribute)
   3. $C_4 \parallel C_1 \parallel (C_2 + C_3)$
   4. $C_{eq} = \frac{1}{C_4 + C_1 \parallel (C_2 + C_3)}$
Extra: not covered during discussion but here for your use/learning.

Q: Why is equivalence useful?
   So far, we have learned of 4 kinds of equivalence:
   - Norton
   - Thevenin
   - Resistor
   - Capacitor
   Equivalence is actually useful to **analysis** (finding voltages + currents)

A: Equivalence is actually useful to **analysis** (finding voltages + currents)


A: It converts hard problems into easier problems with the same behavior.

Q: How to use it then? Here is a rough procedure:

A: 1. Reduce some subpart of a circuit to its equivalent (redraw the circuit)
   2. Calculate a voltage/current in the easier circuit
   3. Go back to un-reduced circuit/rewind the simplification you made
   4. Use new quantity you know to iterate, apply value
Examples:

**Deriving Voltage divider using equivalence**

1. **Reduce**
   - \( V_S \)
   - \( I \)
   - \( R_1 \)
   - \( R_2 \)

2. **Calculate**
   \[ I = \frac{V_S}{R_1 + R_2} \]

3. **Rewind**

4. **Apply**
   - \( V_{R_1} = R_1 I = \frac{V_S}{R_1 + R_2} \)  
   - \( V_{R_2} = R_2 I = \frac{V_S}{R_1 + R_2} \)

\( \Rightarrow \)

Tada! Very succinct!
Examples:

Objective: find \( V_{af} \) in terms of \( V_{in} \)

1. Reduce: \( 2R_2 \) \& \( R_2 \) in series

2. Calculate!
   Use voltage division to find \( V_i \)
   \[
   V_i = \frac{R_i}{R_i + R_1} \cdot \frac{R_1}{3R_2} \cdot V_{in}
   \]

3. Rewind! \( \times 2 \) → 4. Apply: Use voltage divider again

Final answer:

\[
V_{af} = \frac{1}{3} \cdot \frac{R_1}{R_1 + R_1} \cdot \frac{3R_2}{V_{in}}
\]

Note: compute \( R_1/3R_2 \), compare to previous notes
Exercise: Try using Thevenin/Norton on previous example.

Hint: This circuit has a Norton equivalent.

Thing to look for in future:

Example:

Calculate $Q$ on $C_1, C_2, C_3, C_4$.