complex circuit
\[ \Rightarrow \]
equivalent simpler circuit
\[ \text{Thevenin Equivalent} \quad \text{Norton Equivalent} \]

* 2 circuits are equivalent if they have the same I-V relationship

\[ \text{sensor circuit} \quad V_{\text{test}} \quad I_{\text{test}} \]

When \( V_{\text{test}} = 0 \)
→ no voltage drop
→ short circuit current \( I_{\text{test}} = I_{\text{sc}} \)

\[ \text{sensor circuit} \quad I_{\text{sc}} \quad V_{\text{test}} = 0 \]

\[ \text{sensor circuit} \quad I_{\text{sc}} \quad V_{\text{test}} = 0 \]

Finding Thevenin and Norton Equivalents

1) \( V_{\text{th}} = V_{\text{sc}} \) - NVA or Superposition
2) \( I_{\text{no}} = I_{\text{sc}} \) - NVA or Superposition

3) \[ R_{\text{th}}, R_{\text{no}} \]
   a) \( R_{\text{th}} = R_{\text{no}} = \frac{V_{\text{sc}}}{I_{\text{sc}}} = \frac{V_{\text{no}}}{I_{\text{no}}} \) (only works if \( V_{\text{th}} \neq 0, I_{\text{no}} \neq 0 \))
   b) turn off independent sources, find \( R_{\text{eq}} \)
      (only works if all sources turn off) *quickest
   c) turn off independent sources, excite output
      w/ test source \( R_{\text{th}} = R_{\text{no}} = \frac{V_{\text{test}}}{I_{\text{test}}} \) *slowest
      (always works)
(b) (6 points) Find and draw the Norton equivalent circuit between terminals a and b in the circuit below.

Clearly label the Norton equivalent current, \( I_{N_{eq}} \), and the Norton equivalent resistance \( R_{N_{eq}} \).

1. Find \( I_{N_{eq}} = I_{sc} \)
   
   \[ u_1 = u_2 \quad V_{a-b} = u_1 - u_2 = i_R R = 0 \]
   
   \[ \Rightarrow i_R = 0 \]

   KCL at \( u_1 \):
   
   \[ g_{V o} - \frac{1}{R} + I_{sc} = 0 \]
   
   \[ I_{sc} = -g_{V o} V_o \]

2. Find \( R_{N_{eq}} \) and which methods can we use? a) b) c)

\[ V_{a-b} = 0 \quad R_{eq} = R = R_{N_{eq}} \]

9. Thévenin Equivalence (12 points)

(a) Find the Thévenin resistance \( R_{th} \) of the circuit shown below, with respect to terminals A and B.
Assume that \( R_1 = 4R, R_2 = R \) and \( R_3 = 9R \).

\[ R_{th} = R_2 || R_3 \]

(b) Now a load resistor, \( R_L = 9R \), is connected across terminals A and B as shown in the circuit below. Find the supply voltage, \( V_s \), such that 1 mW is dissipated across the load resistor. Let \( R = 3k\Omega \).

\[ V_{th} = u_1 - u_2 = i_R R_1 = 0 \]

\[ i_R = 0 \]

Want to find Thévenin equiv \( \Rightarrow \) find \( V_{th} \)

\[ V_{th} = \frac{R_3}{R_2 + R_3} V_s \]

\[ P_L = I_{th} V_{th} = \frac{V_{th}^2}{R_L} \quad \Rightarrow \text{solve for } V_s \]
(c) We modify the circuit as shown below:

![Circuit Diagram]

Find a symbolic expression for \( V_{\text{out}} \) as a function of \( V_g \).

Let's also find \( R_{\text{th}} \) with C and D.

Which method can we use? Not B.

Let's use method C.

\[
V_{\text{out}} = g \left( V_g - V_{\text{out}} \right)
\]

\[
V_{\text{out}} = i \left( R_{\text{L}} || R_0 \right)
\]

\[
V_{\text{out}} + g V_{\text{out}} (R_{\text{L}} || R_0) = g V_a (R_{\text{L}} || R_0)
\]

\[
V_{\text{out}} (1 + g (R_{\text{L}} || R_0)) = g V_a (R_{\text{L}} || R_0)
\]

\[
V_{\text{out}} = g \left( \frac{R_{\text{L}} || R_0}{1 + g (R_{\text{L}} || R_0)} \right) V_a
\]

\[
V_{\text{out}} = \frac{R_3}{R_2 + R_3} g \left( \frac{R_{\text{L}} || R_0}{1 + g (R_{\text{L}} || R_0)} \right) V_s
\]

KCL at A: \( I_{\text{test}} + g \left( V_g - V_{\text{out}} \right) - i_R = 0 \)

\[ I_{\text{test}} = i_R + g V_{\text{out}} = i_R + g V_{\text{test}} \]

\[ R_{\text{th}} = \frac{V_{\text{test}} - g V_{\text{test}}}{\left( \frac{1}{R_{\text{L}} || R_0} + g \right) V_{\text{test}}} \]

\[ R_{\text{th}} = \frac{1}{\frac{1}{R_{\text{L}} || R_0} + g} \]

* Didn't get to this problem but wrote out the solutions

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5. Thévenin and Norton Equivalence (10 points)

\[
I_{\text{test}} = \frac{V_{\text{test}}}{R_{\text{th}}}
\]

\[
R_{\text{th}} = \frac{V_{\text{test}}}{I_{\text{test}}} = R_{\text{eq}}
\]

\[
R_{\text{th}} = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}
\]

(a) (4 points) Redraw the circuit with all sources nulled, then calculate \( R_{\text{th}} \) between terminals a and b.

Let's use method C. * couldn't use method B.

\[
R_{\text{th}} = \frac{R_2 + R_3 + R_4}{R_1}
\]

\[
R_{\text{th}} = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}
\]
(b) 6 points) Find the Thévenin voltage between the terminals a and b. Hint: superposition may be useful.

\[ V_{oc1} = \frac{R_1}{R_1 + R_2 + R_3} V_S \]

*Recall current divider:

\[ I_{R_2} = \frac{R_1}{R_1 + R_2} I_S \]

\[ I_{R_1} = \frac{R_3}{R_1 + R_2 + R_3} I_S \]

\[ V_{oc2} = i_{R_1} R_1 = \frac{R_3 R_1}{R_1 + R_2 + R_3} I_S \]

\[ V_{oc} = V_{oc1} + V_{oc2} = \frac{R_1}{R_1 + R_2 + R_3} V_S + \frac{R_3 R_1}{R_1 + R_2 + R_3} I_S = V_{thn} \]