1. Passive Sign Convention and Power

(a) Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\, \text{V}$ and let $R_1 = 5\, \Omega$.

(b) Suppose we change the label of the currents in the circuit to be as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\, \text{V}$ and let $R_1 = 5\, \Omega$.

2. Modular Circuits

In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations. (Note that the so-called analog signal processing – where these kinds of mathematical operations are performed on continuously-valued voltages by analog circuits – is extremely common in real-world applications; without this capability, essentially none of our radios or sensors would actually work.) Specifically, let’s assume that we want to implement the block diagram shown below:

In other words, we want to implement a circuit with two outputs $V_x$ and $V_y$, where $V_x = \frac{1}{2} V_{\text{in}}$ and $V_y = \frac{1}{3} V_x$.

(a) Design two voltage divider circuits that each independently would implement the two multiplications shown in the block diagram above (i.e., multiply by $\frac{1}{2}$ and multiply by $\frac{1}{3}$). Note that you do not need to include the input voltage sources in your design – you can simply define the input to each block as being at the appropriate potential (e.g., $V_{\text{in}}$ or $V_x$).
(b) Assuming that \( V_{\text{in}} \) is created by an ideal voltage source, implement the original block diagram as a circuit by directly replacing each block with the designs you came up with in part (a).

(c) For the circuit from part (b), find \( V_x \) and \( V_y \). Do you get the desired relationship between \( V_y \) and \( V_x \)? How about between \( V_x \) and \( V_{\text{in}} \)? Be sure to explain why or why not each block retains its desired functionality.

3. Practice: Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don’t last a long time. For example, a Google Pixel phone, under typical usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality the voltage drops gradually, but let’s keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel’s battery has a battery capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will a Pixel’s full battery last under typical usage conditions?

(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has \( 1.602 \times 10^{-19} \) C of charge.)

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a Ws.

(d) Suppose PG&E charges $0.12 per kWh. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?

(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor \( R_{\text{bat}} \). We now wish to charge the battery by plugging into a wall plug. The wall plug can be modeled as a 5V voltage source and 200mΩ resistor, as pictured in Figure 1. What is the power dissipated across \( R_{\text{bat}} \) for \( R_{\text{bat}} = 1 \) mΩ, 1 Ω, and 10kΩ? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of \( R_{\text{bat}} \)?

![Figure 1: Model of wall plug, wire, and battery.](image)