1. Identifying a Subspace: Proof

Is the set
\[ V = \{ \vec{v} \mid \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \} \]
a subspace of \( \mathbb{R}^3 \)? Why/why not?

2. Exploring Column Spaces and Null Spaces

- The column space is the span of the column vectors of the matrix.
- The null space is the set of input vectors that output the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of \( A \)? What is its dimension?

ii. What is the null space of \( A \)? What is its dimension?

iii. Are the column spaces of the row reduced matrix \( A \) and the original matrix \( A \) the same?

iv. Do the columns of \( A \) form a basis for \( \mathbb{R}^2 \)? Why or why not?

(a) \[
\begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
0 & 1 \\
0 & 1
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 2 \\
-1 & 1
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
-2 & 4 \\
3 & -6
\end{bmatrix}
\]

(e) \[
\begin{bmatrix}
1 & -1 & -2 & -4 \\
1 & 1 & 3 & -3
\end{bmatrix}
\]

3. Mechanical Determinants

(a) Compute the determinant of \[
\begin{bmatrix}
2 & 0 \\
0 & 3
\end{bmatrix}
\].

(b) Compute the determinant of \[
\begin{bmatrix}
2 & 1 \\
0 & 3
\end{bmatrix}
\].
Reference Definitions: Matrices and Linear (In)Dependence

The following statements are equivalent for an $n \times n$ matrix $A$, meaning, if one is true then all are true:

(a) $A$ is invertible
(b) $\iff$ The equation $A\vec{x} = \vec{b}$ has a unique solution for any $\vec{b}$
(c) $\iff$ $A$ has linearly independent columns
(d) $\iff$ $A$ has a trivial nullspace
(e) $\iff$ the determinant of $A \neq 0$.

In class have shown/proven that:

(a) $A$ is invertible $\implies$ the equation $A\vec{x} = \vec{b}$ has a unique solution for any $\vec{b}$.
(b) $A$ is invertible $\implies$ $A$ has linearly independent columns
(c) $A$ is invertible $\implies$ $A$ has a trivial nullspace.

We have not yet shown/proven the implications in the other direction.