1. Passive Sign Convention and Power v 2.0

Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let \( V_s = 5 \text{V}, I_s = 0.5 \text{A} \) and \( R_1 = 5 \Omega \).

**Answer:** We'll start by solving the circuit for the unknown node potentials and currents.

The KCL equations for nodes \( u_1, u_2 \) in this circuit are:

\[
I_{V_i} + I_{R_1} = 0
\]
\[
-I_{R_1} + I_s = 0
\]

The Element equations for the voltage source and the resistor are:

\[
u_1 - 0 = V_1 = V_s
\]
\[
V_{R_1} = u_1 - u_2 = I_{R_1} R_1
\]

Finally, the KVL equation around the one loop formed by the circuit is:

\[
V_1 - V_{R_1} - V_{I_s} = 0
\]
Solving the above equations with $V_s = 5 \, \text{V}$, $I_s = 0.5 \, \text{A}$ and $R_1 = 5 \, \Omega$:

\[
\begin{align*}
I_{R_1} &= 0.5 \, \text{A} \\
I_v &= -0.5 \, \text{A} \\
V_{R_1} &= 2.5 \, \text{V} \\
V_i &= 2.5 \, \text{V}
\end{align*}
\]

From above, we can solve for the power dissipated across the resistor:

\[
P_{R_1} = IV = I_{R_1} V_{R_1} = 0.5 \, \text{A} \cdot 2.5 \, \text{V} = 1.25 \, \text{W}
\]

Next we can solve for the power dissipated across the voltage source:

\[
P_V = IV = I_v V_1 = -0.5 \, \text{A} \cdot 5 \, \text{V} = -2.5 \, \text{W}
\]

Finally, we can solve for the power dissipated across the current source:

\[
P_i = IV = I_s V_s = 0.5 \, \text{A} \cdot 2.5 \, \text{V} = 1.25 \, \text{W}
\]

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is actually supplying power to the circuit.

**Note:** In this case the current source is also dissipating power but it could be also supplying if the numbers were picked differently. For example, if $I_s = 2 \, \text{A}$ the same equations would give $P_{R_1} = 20 \, \text{W}$, $P_V = -10 \, \text{W}$, $P_i = -10 \, \text{W}$. Also, numbers could have been selected such that the voltage source dissipated and the current source supplied power to the circuit.

2. **Resist the Touch**

In this question, we will be re-examining the 2-dimensional resistive touchscreen which we previously discussed in lecture and will soon also see in lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length $L$ and width $W$ and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position $(1,1)$.

Figure 1: Model and components of a general touchscreen
The top layer has \( N \) vertical strips denoted by \( x_1, x_2, \ldots, x_N \). These vertical strips all have cross sectional area \( A \), and resistivity \( \rho_x \).

The bottom layer has \( N \) horizontal strips denoted by \( y_1, y_2, \ldots, y_N \). These horizontal strips all have cross sectional area \( A \) as well, and resistivity \( \rho_y \).

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).

(a) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance \( R_x \) for the vertical strips and \( R_y \) for the horizontal strips, as a function of the screen dimensions \( W \) and \( L \), the respective resistivities, and the cross-sectional area \( A \).

**Answer:** The equation for resistance is \( R = \frac{\rho l}{A} \).

Therefore, \( R_x = \frac{\rho_x L}{A} \).

For the bottom, \( R_y = \frac{\rho_y W}{A} \).

(b) Consider a \( 2 \times 2 \) example for the touchscreen circuit as in shown in Figure 2.

![Figure 2: 2 × 2 Case of the Resistive Touchscreen](image)

Given that \( V_s = 3 \text{V} \), \( R_x = 2000\Omega \), and \( R_y = 2000\Omega \), draw the equivalent circuit for when the point \((2, 2)\) is pressed and solve for the voltage at terminal \( V_{O2} \) with respect to ground.

**Answer:**

Since all of the resistive strips are equally spaced, the resistor above point \((2, 2)\) on strip \( x_2 \) becomes \( \frac{2}{3}R_x \) and the resistor below point \((2, 2)\) on strip \( x_2 \) becomes \( \frac{1}{3}R_x \).

The bottom layer resistors, although they must be drawn in the equivalent circuit, do not affect the voltage at \( V_{O2} \) as there are open circuits, leading to no currents through the bottom layer resistors, and therefore no voltage drops over either bottom layer resistor.
Observing that the top layer resistive strips form a voltage divider, we can determine $V_{O2}$ using the voltage divider equation.

Therefore, $V_{O2} = V_{(2,2)} = \frac{1}{3} \frac{R_x}{\frac{2}{3} R_x + \frac{1}{3} R_x} = \frac{1}{3} V_s = 1V$.

(c) Suppose a touch occurs at coordinates $(i, j)$ in Figure 1(a). Find an expression for $V_{O2}$ as a function of $V_s$, $N$, $i$, and $j$. The upper left corner is the coordinate $(1, 1)$ and the upper right coordinate is $(N, 1)$.

Answer:

$$V_{O2} = \frac{N+1-j}{N+1} R_x V_s$$

$$= \frac{N+1-j}{N+1} V_s$$

3. Practice: Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals $A$ and $B$ using the resistor combination rules for series and parallel resistors.

Answer:

$$5k\Omega \parallel ((6k\Omega \parallel 6k\Omega \parallel 6k\Omega) + 8k\Omega) = 5k\Omega \parallel (2k\Omega + 8k\Omega) = 5k\Omega \parallel 10k\Omega = 3.33k\Omega$$