

Electrical Characteristics of MOS Devices

- **The MOS Capacitor**
 - Voltage components
 - Accumulation, Depletion, Inversion Modes
 - Effect of channel bias and substrate bias
 - Effect of gate oxide charges
 - Threshold-voltage adjustment by implantation
 - Capacitance vs. voltage characteristics
- **MOS Field-Effect Transistor**
 - I-V characteristics
 - Parameter extraction

1) Revisit EE143 Week#2 Reading Assignment

- Introduction to IC Devices, www.icknowledge.com
- Streetman, Chap 3 Energy Band and Charge carriers in Semiconductors.

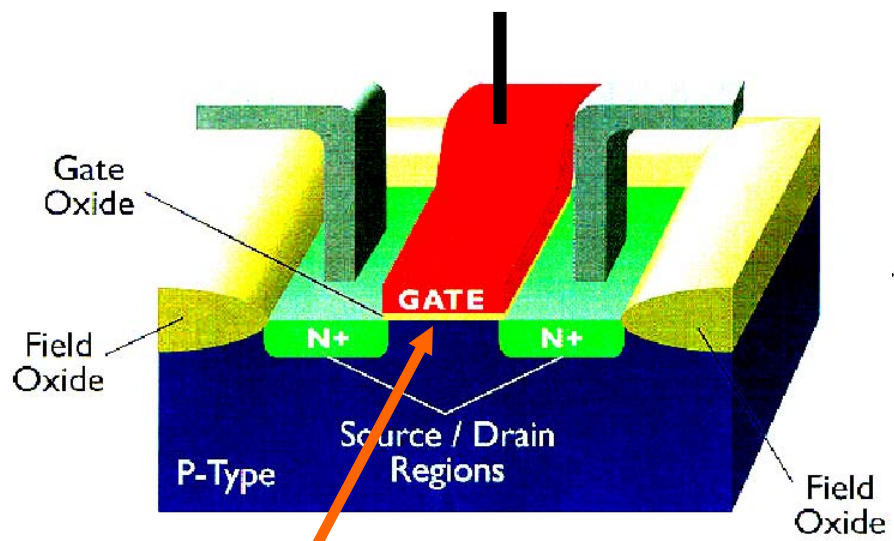
2) Visit the Device Visualization Website

<http://jas.eng.buffalo.edu/>
and run the ***visualization experiments*** of

- 1) Charge carriers and Fermi level,
- 2) pn junctions
- 3) MOS capacitors
- 4) MOSFETs

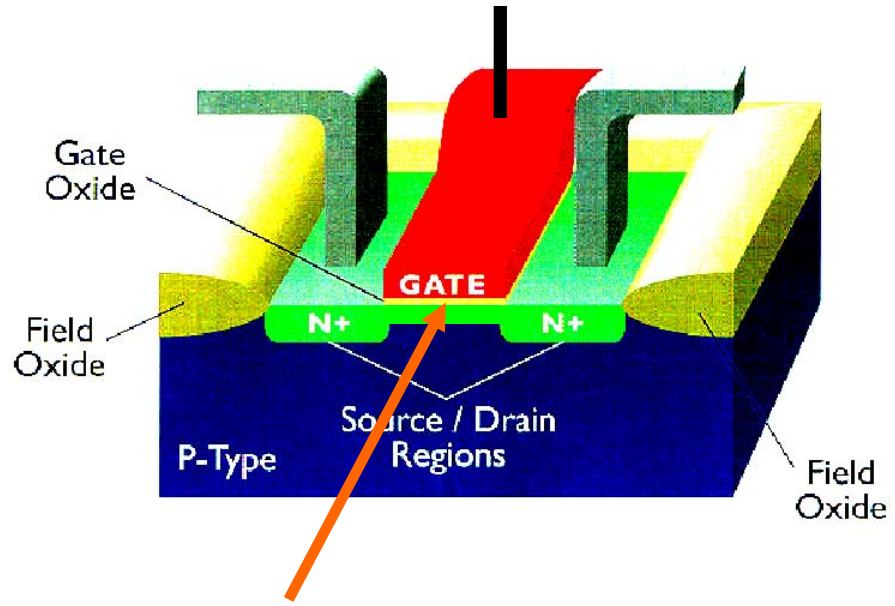
Metal -Oxide-Semiconductor Transistor [n-channel]

$$V_G < V_{\text{threshold}}$$



**Negligible electron concentration underneath Gate region;
Source-Drain is electrically open**

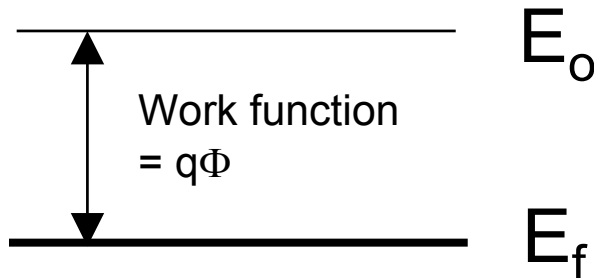
$$V_G > V_{\text{threshold}}$$



**High electron concentration underneath Gate region;
Source-Drain is electrically connected**

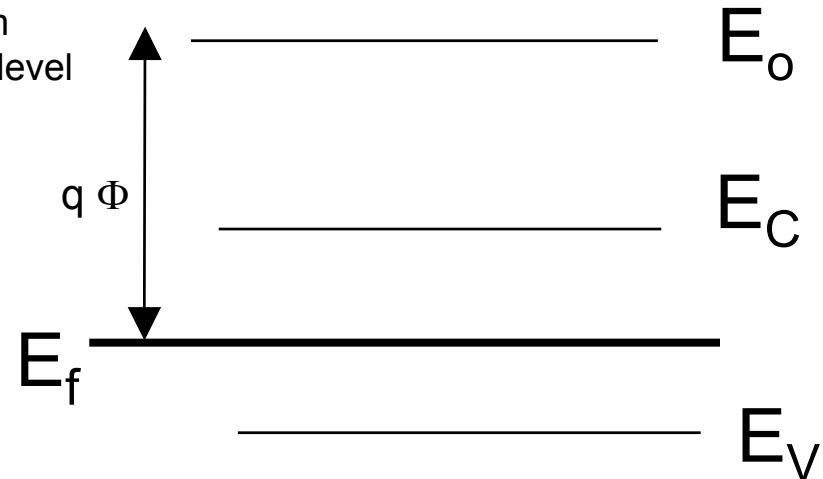
Work Function of Materials

METAL



$q\Phi_M$ is determined
by the metal material

SEMICONDUCTOR



$q\Phi_S$ is determined
by the semiconductor material,
the **dopant type**,
and **doping concentration**

Work Function ($q\Phi_M$) of MOS Gate Materials

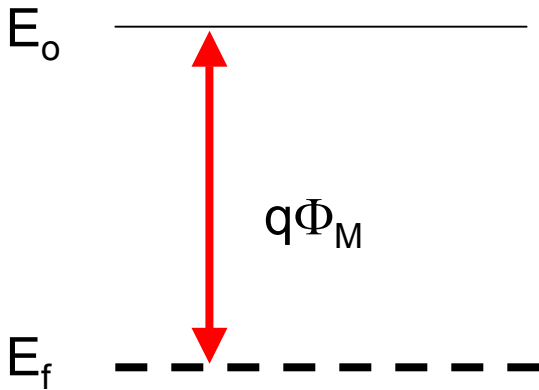
E_o = vacuum energy level

E_f = Fermi level

E_c = bottom of conduction band

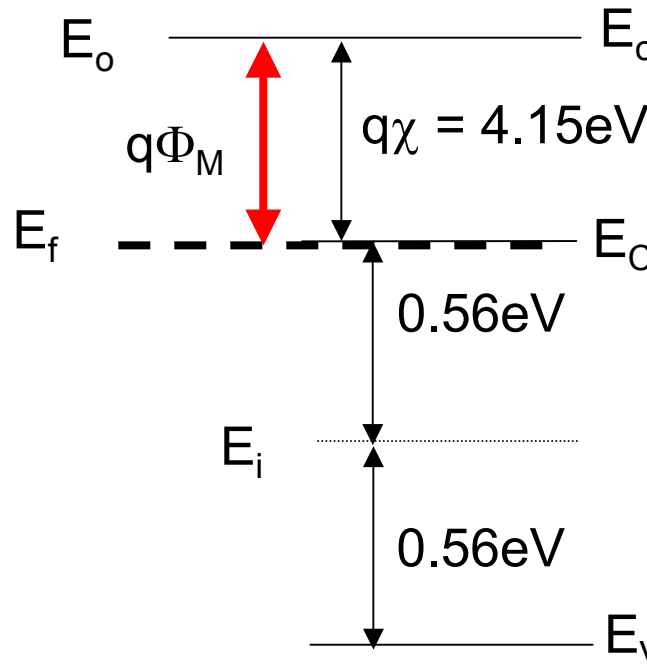
E_v = top of conduction band

$q\chi = 4.15\text{eV}$ (electron affinity)

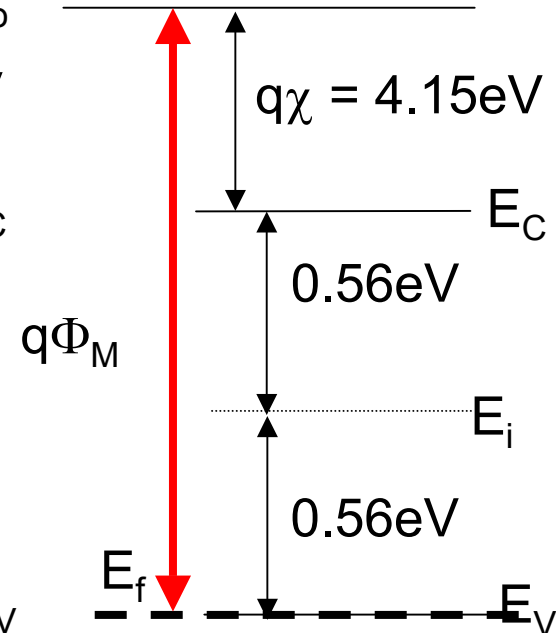


Al = 4.1 eV

TiSi₂ = 4.6 eV



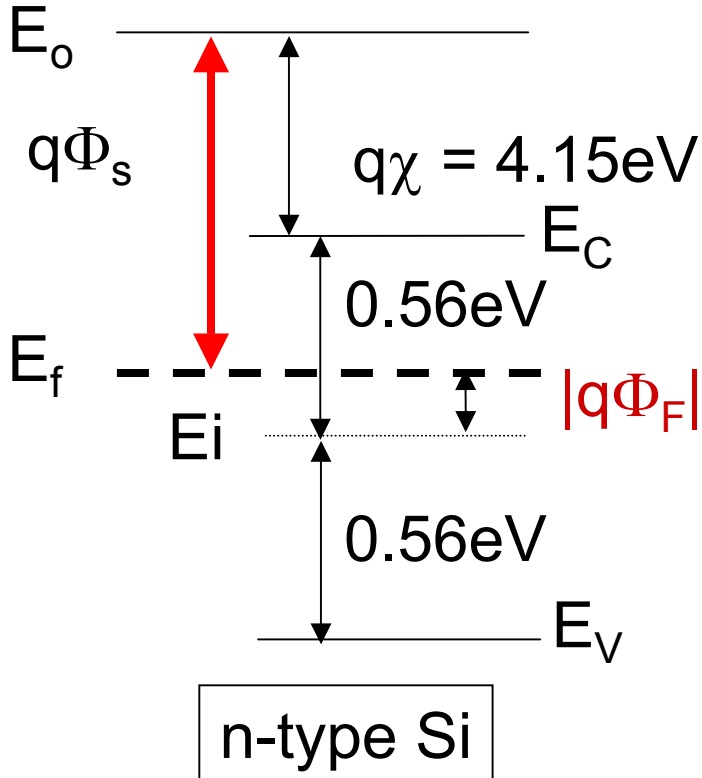
n+ poly-Si



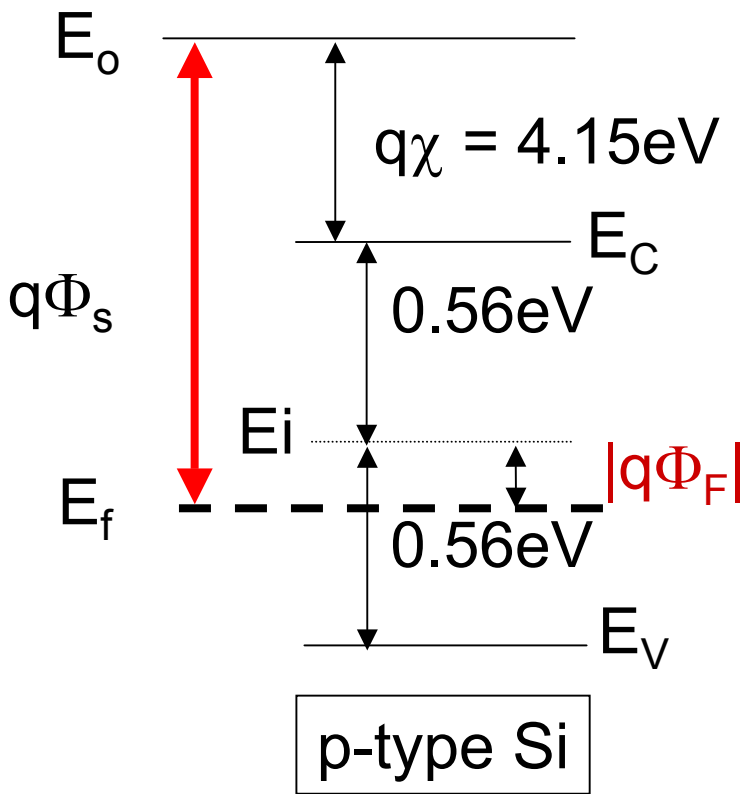
p+ poly-Si

Work Function of doped Si substrate

* Depends on substrate concentration N_B $|\Phi_F| = \frac{kT}{q} \ln\left(\frac{N_B}{n_i}\right)$

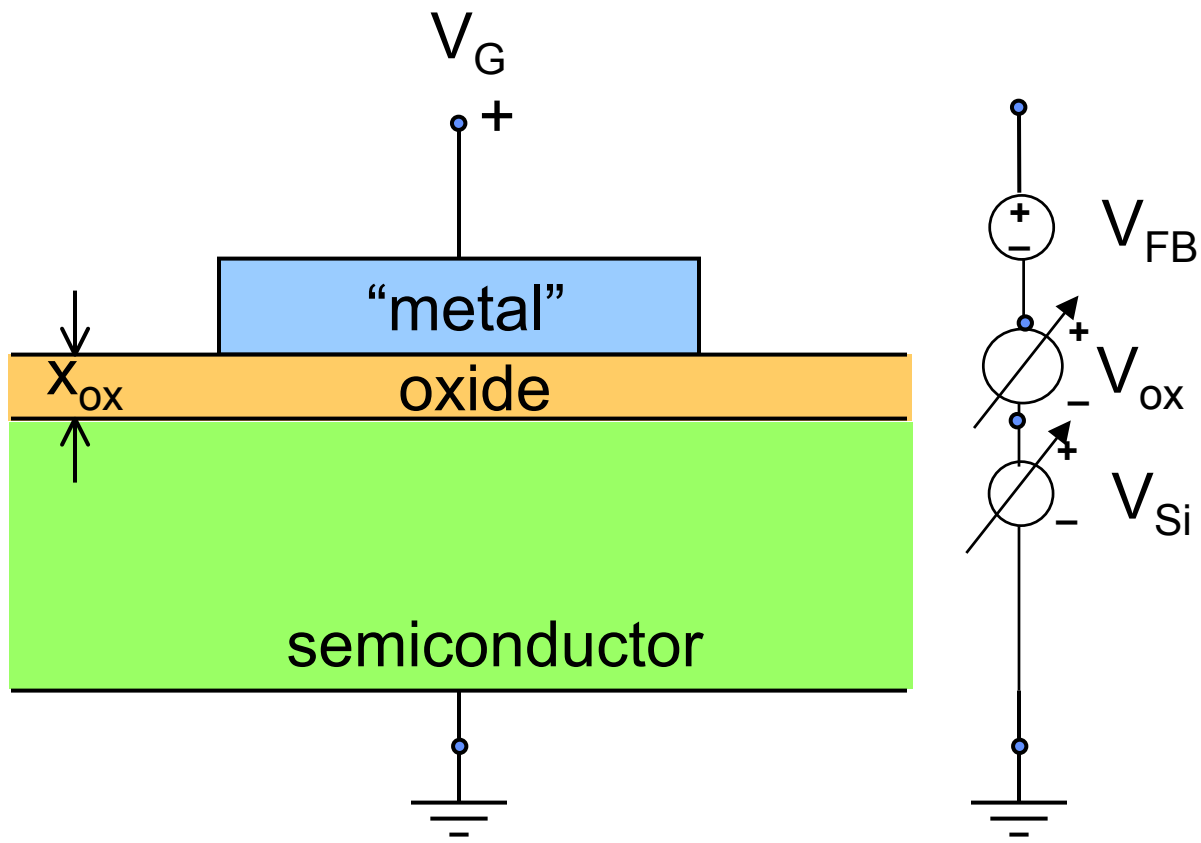


$$\Phi_s \text{ (volts)} = 4.15 + 0.56 - |\Phi_F|$$



$$\Phi_s \text{ (volts)} = 4.15 + 0.56 + |\Phi_F|$$

The MOS Capacitor



$$V_G = V_{FB} + V_{ox} + V_{Si}$$

$$C_{ox} = \frac{\epsilon_{ox}}{x_{ox}} \quad [\text{F/cm}^2]$$

Oxide capacitance/unit area

Flat Band Voltage

- V_{FB} is the “built-in” voltage of the MOS:

$$V_{FB} \equiv \Phi_M - \Phi_S$$

- Gate work function Φ_M :
Al: 4.1 V; n+ poly-Si: 4.15 V; p+ poly-Si: 5.27 V
- Semiconductor work function Φ_S :

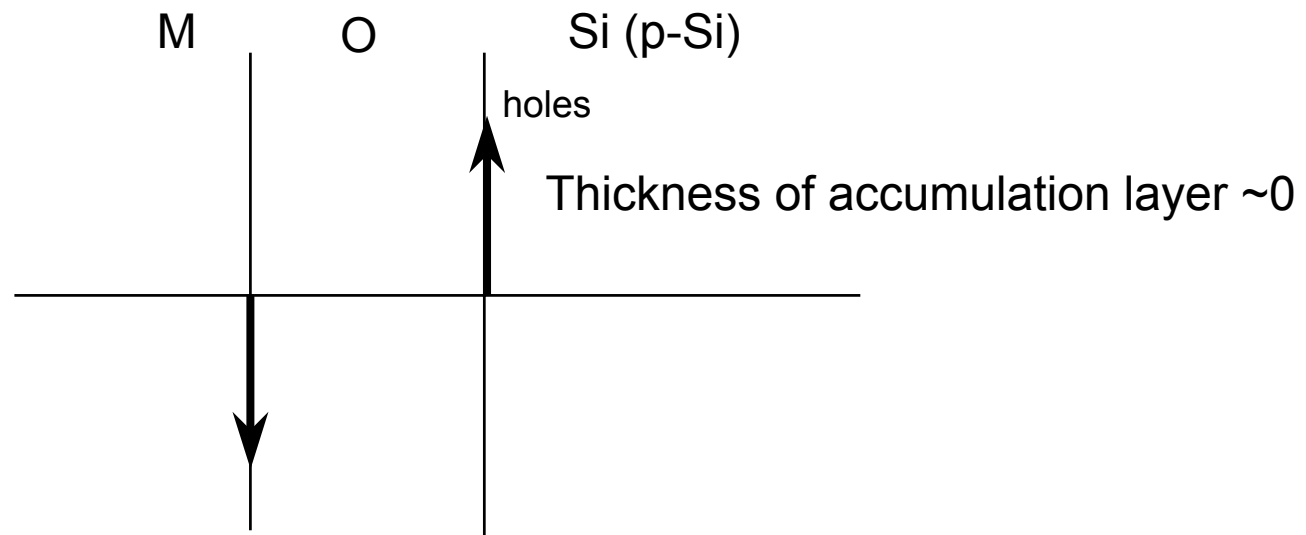
$$\Phi_s \text{ (volts)} = 4.15 + 0.56 - |\Phi_F| \text{ for n-Si}$$

$$\Phi_s \text{ (volts)} = 4.15 + 0.56 + |\Phi_F| \text{ for p-Si}$$

- V_{ox} = voltage drop across oxide (depends on V_G)
- V_{Si} = voltage drop in the silicon (depends on V_G)

MOS Operation Modes

A) Accumulation: $V_G < V_{FB}$ for p-type substrate



$$V_{Si} \approx 0, \text{ so } V_{ox} = V_G - V_{FB}$$

$$Q_{Si}' = \text{charge/unit area in Si}$$

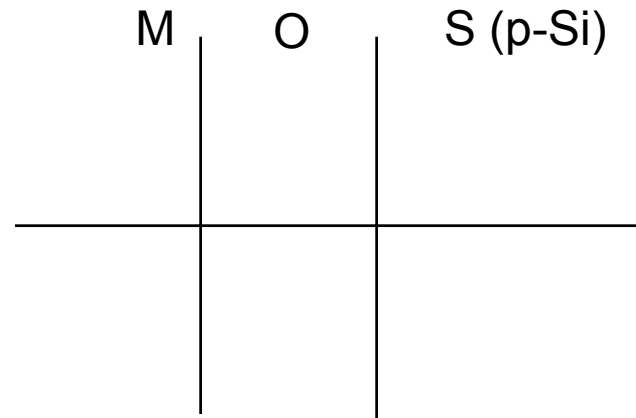
$$= C_{ox} (V_G - V_{FB})$$

MOS Operation Modes

- **B) Flatband:** $V_G = V_{FB}$

No charge in Si (and hence no charge in metal gate)

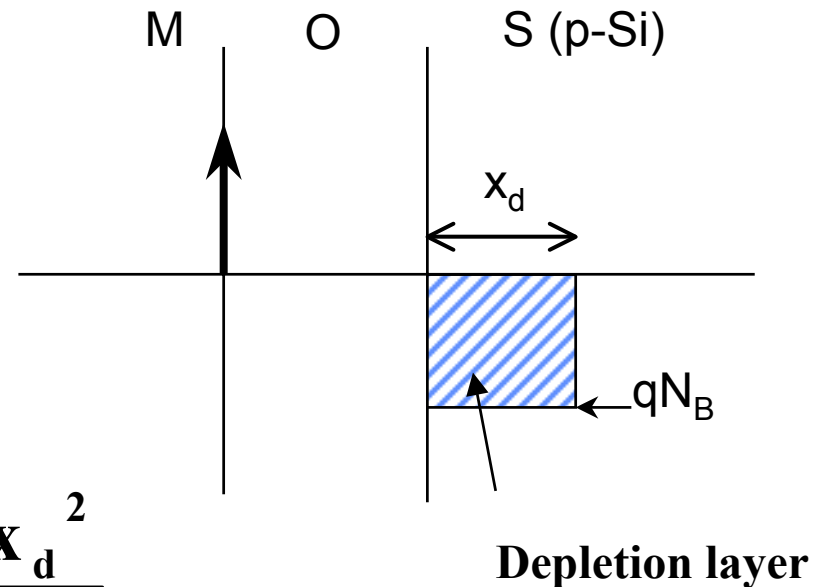
- $V_{Si} = V_{ox} = 0$



MOS Operation Modes (cont.)

C) Depletion: $V_G > V_{FB}$

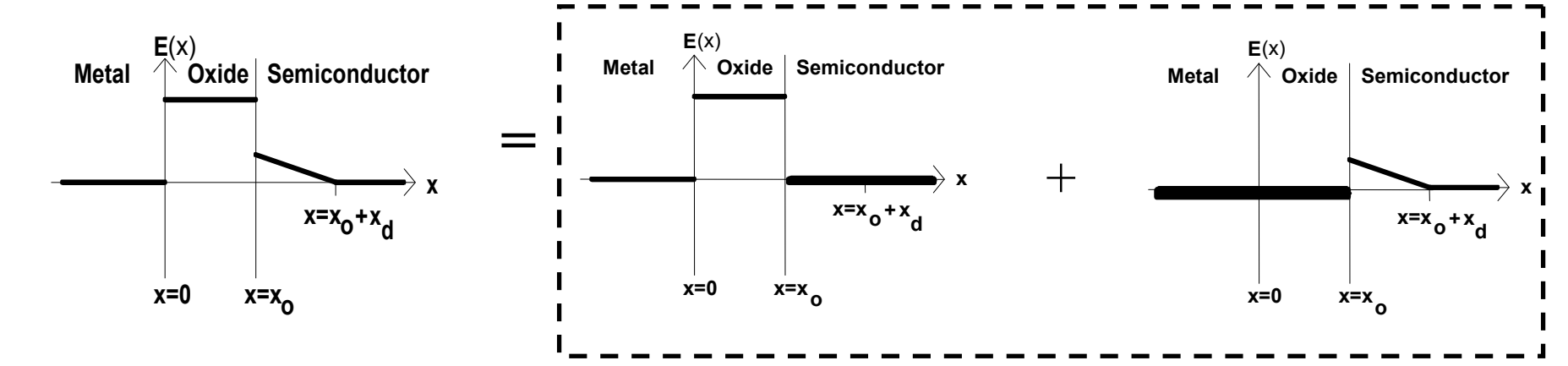
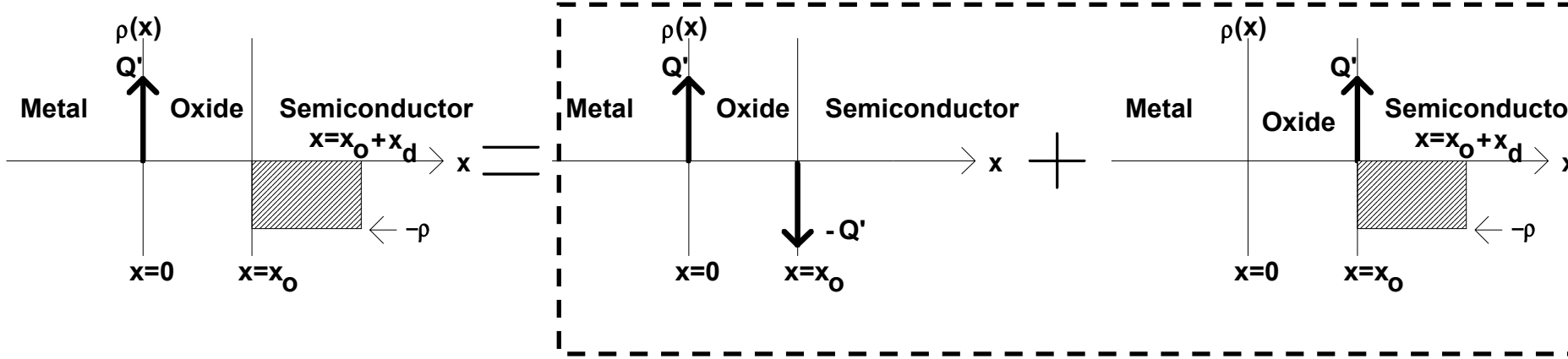
$$x_d = \sqrt{\frac{2\epsilon_{Si} V_{Si}}{qN_B}}$$



$$V_G = V_{FB} + \underbrace{\frac{qN_B x_d}{C_{ox}}}_{V_{ox}} + \underbrace{\frac{qN_B x_d^2}{2\epsilon_s}}_{V_{Si}}$$

(can solve for x_d)

Depletion Mode : Charge and Electric Field Distributions by Superposition Principle of Electrostatics



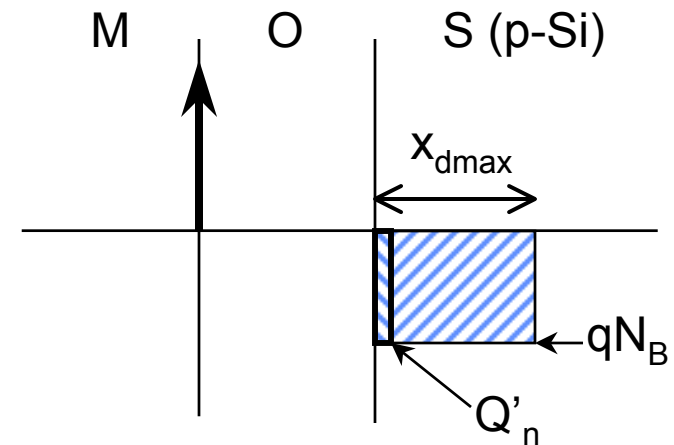
MOS Operation Modes (cont.)

D) Threshold of Inversion: $V_G = V_T$

$$n_{\text{surface}} = N_B \quad (\text{for p-type substrate})$$

$$\Rightarrow V_{Si} = 2|\Phi_F|$$

This is a **definition**
for onset of
strong inversion



$$V_G = V_T = V_{FB} + \frac{\sqrt{2\epsilon_s (2|\Phi_F|) q N_B}}{C_{ox}} + 2|\Phi_F|$$

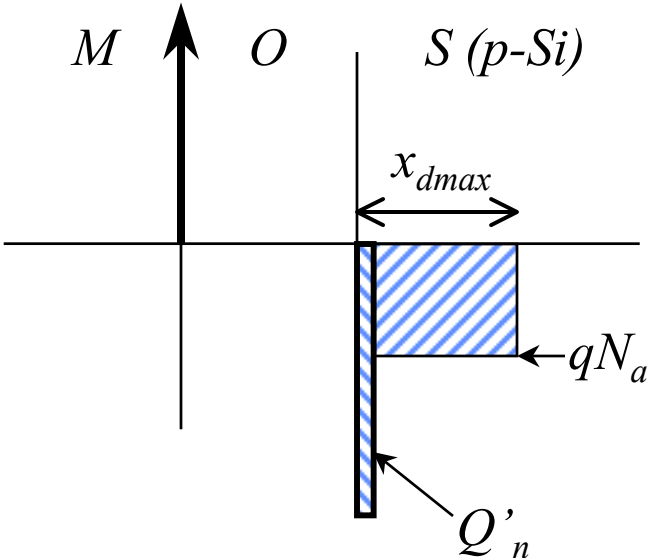
MOS Operation Modes (cont.)

E) Strong Inversion: $V_G > V_T$

x_{dmax} is approximately unchanged when $V_G > V_T$

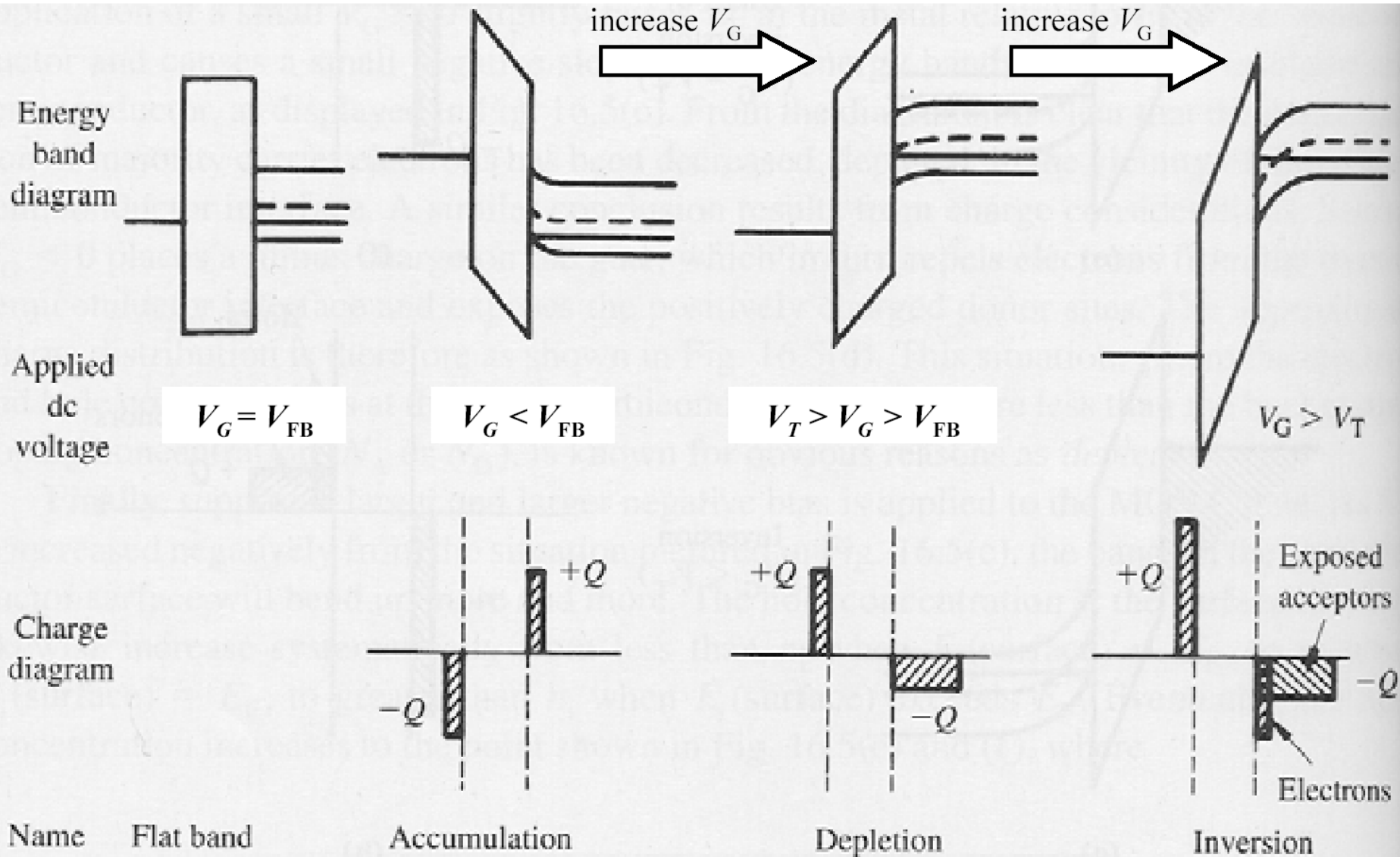
$$x_{dmax} = \sqrt{\frac{4\epsilon_{Si} |\Phi_F|}{qN_B}}$$

$$V_{ox} = \frac{qN_B x_{dmax} + |Q'_n|}{C_{ox}}$$
$$Q'_n \approx -C_{ox} (V_G - V_T)$$



electrons

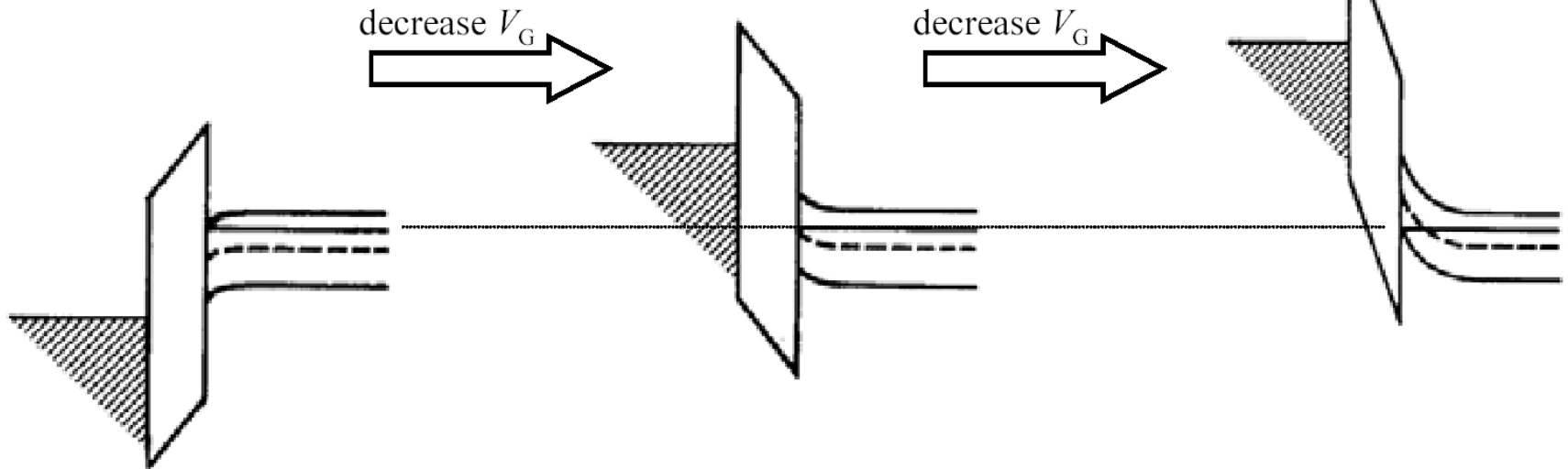
Biasing Conditions for p-type Si



MOS Band Diagrams (n-type Si)

Decrease V_G (toward more negative values)

-> move the gate energy-bands up, relative to the Si



- Accumulation

- $V_G > V_{FB}$
- Electrons accumulate at surface

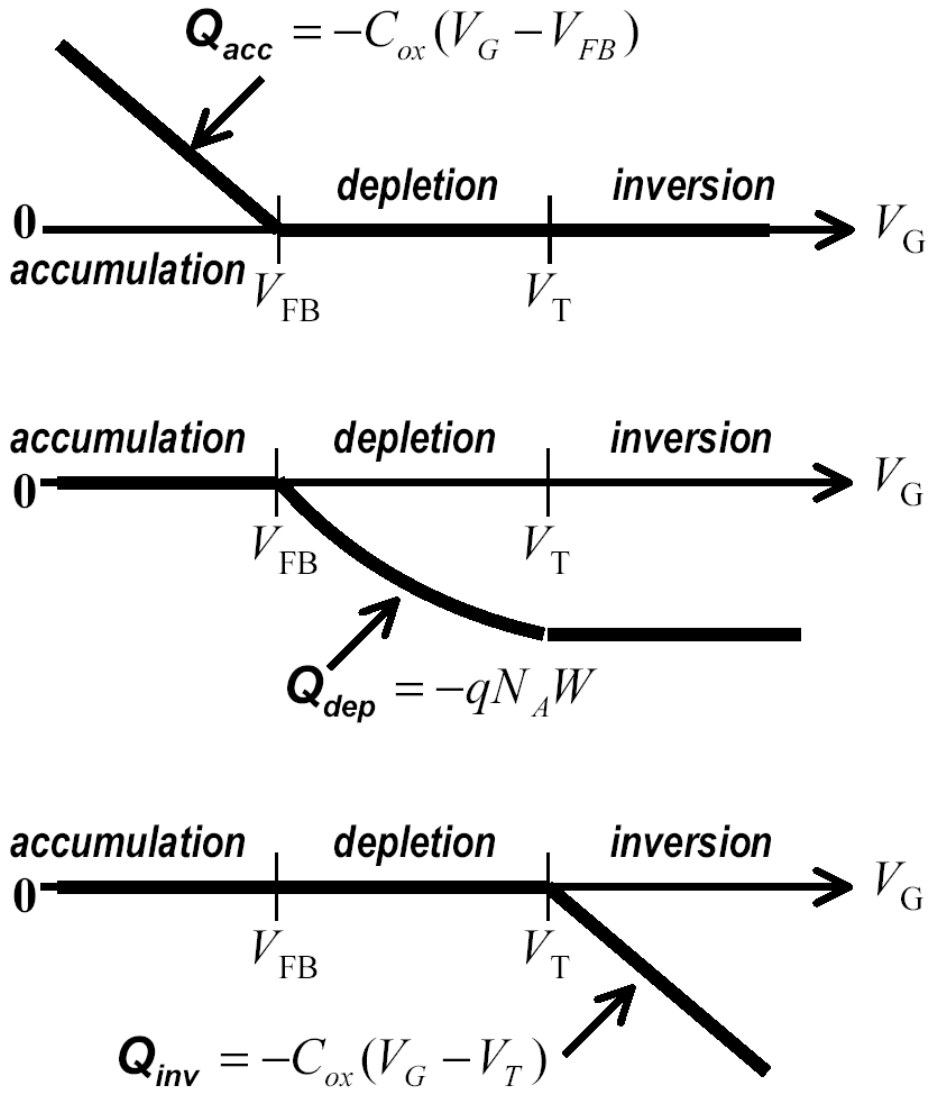
- Depletion

- $V_G < V_{FB}$
- Electrons repelled from surface

- Inversion

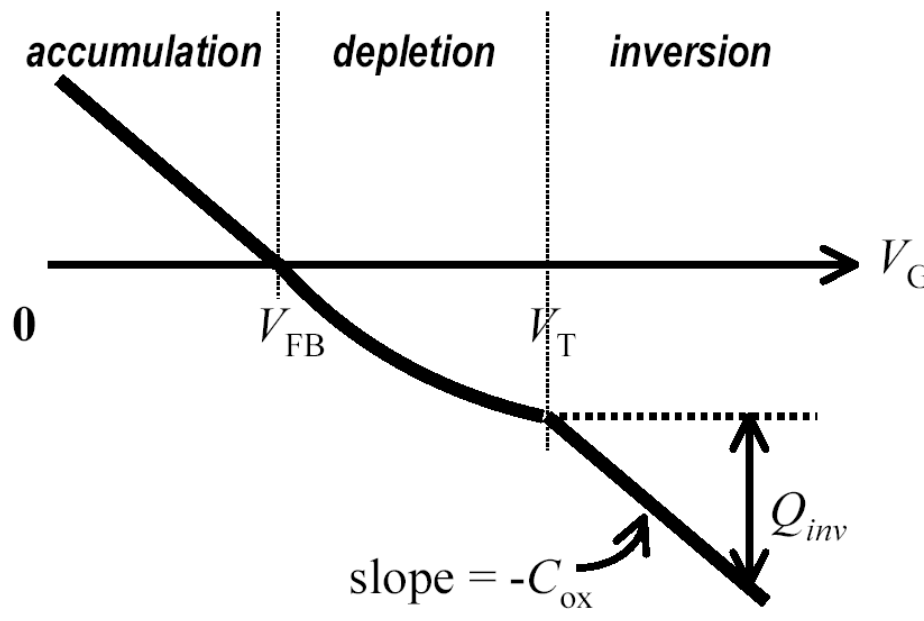
- $V_G < V_T$
- Surface becomes p-type

Total Charge Density in Si, Q_s



p-Si

$$Q_s = Q_{acc} + Q_{dep} + Q_{inv}$$

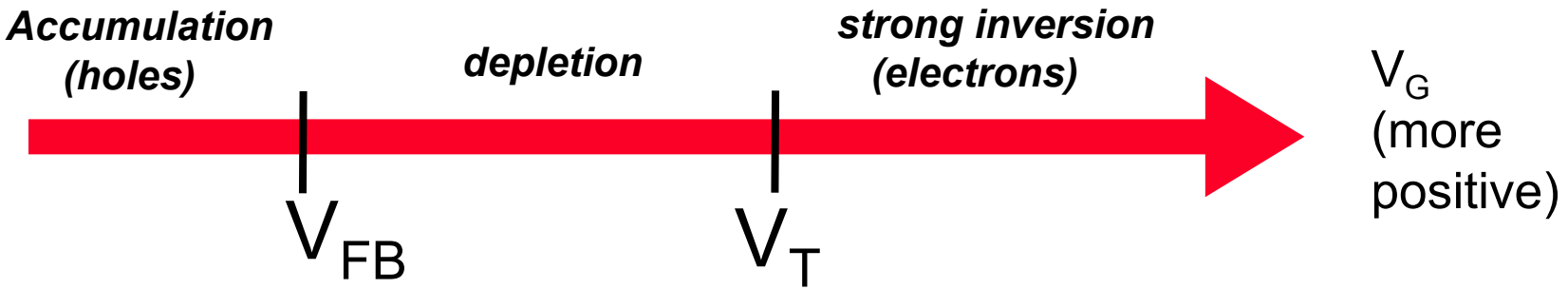


Suggested Exercise

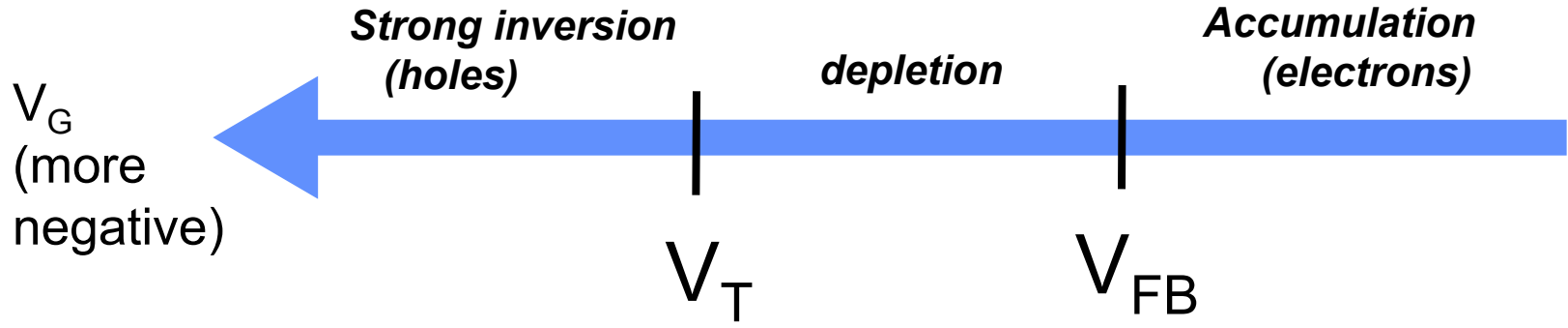
Most derivations for MOS shown in lecture notes are done with p-type substrate (NMOS) as example.

Repeat the derivations yourself for n-type substrate (PMOS) to test your understanding of MOS.

p-Si substrate (NMOS)



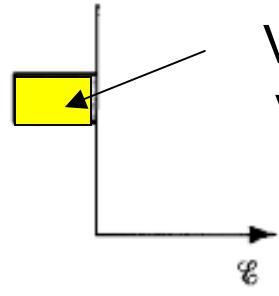
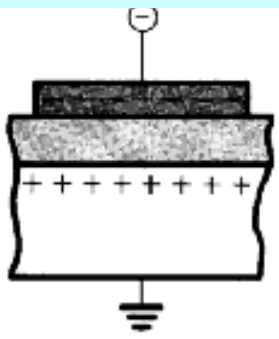
n-Si substrate (PMOS)



Voltage drop = area under E-field curve

- Key:**
 + Holes
 - Electrons
 ⊖ Acceptor ions

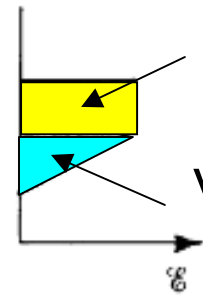
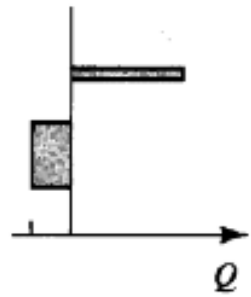
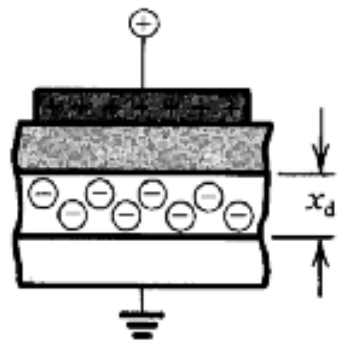
Accumulation



$$V_{ox} = Q_a / C_{ox}$$

$$V_{Si} \sim 0$$

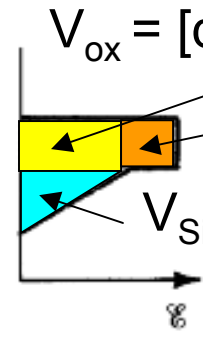
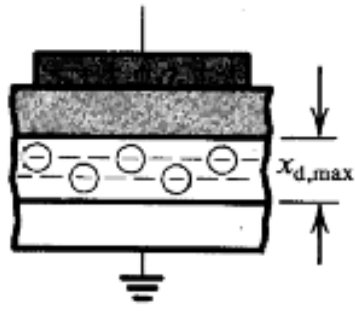
Depletion



$$V_{ox} = qN_a x_d / C_{ox}$$

$$V_{Si} = qN_a x_d^2 / (2\epsilon_s)$$

Inversion



$$V_{ox} = [qN_a x_{d,max} + Q_n] / C_{ox}$$

$$V_{Si} = qN_a x_{d,max}^2 / (2\epsilon_s)$$

$$= 2|\Phi_F|$$

* For simplicity, dielectric constants assumed to be same for oxide and Si in E-field sketches

Appendix

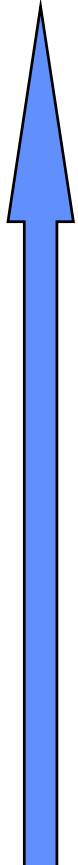
-Electron Energy Band

- Fermi Level

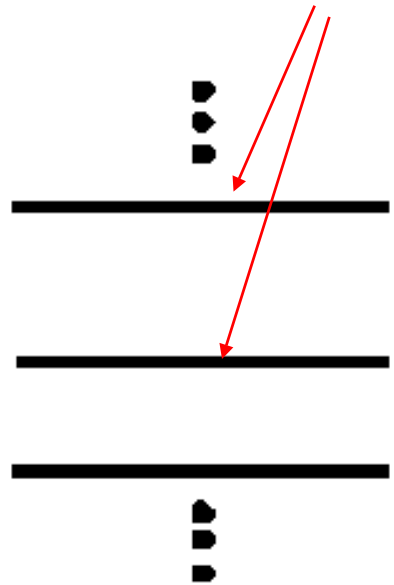
-Electrostatics of device charges

Conduction Band and Valence Band

Electron Potential Energy

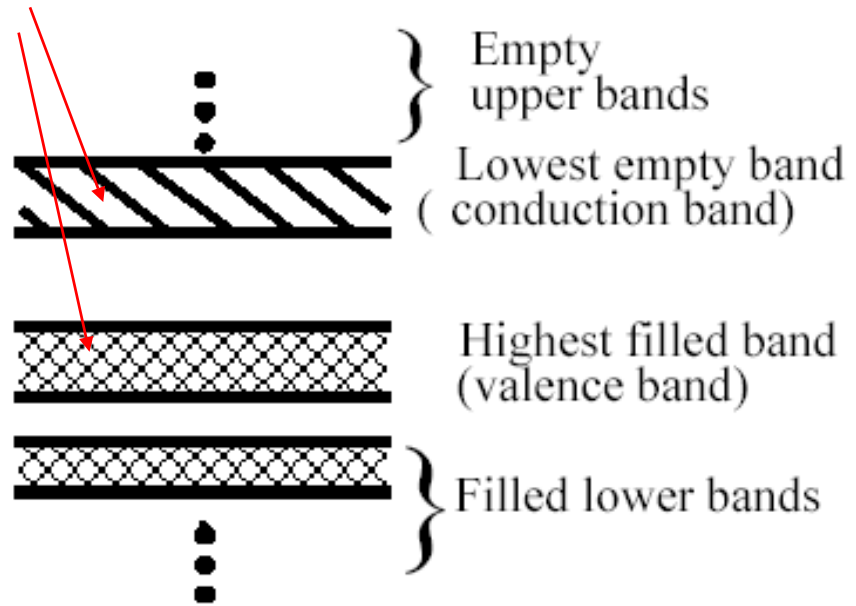


Available states at discrete energy levels



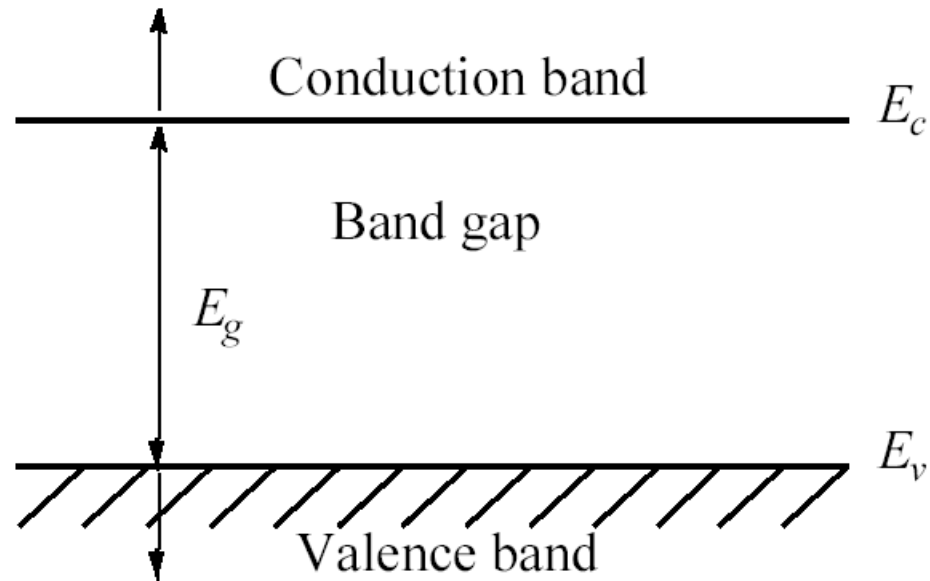
Isolated atoms

Available states as continuous energy levels inside energy bands



Atoms in a solid

The Simplified Electron Energy Band Diagram

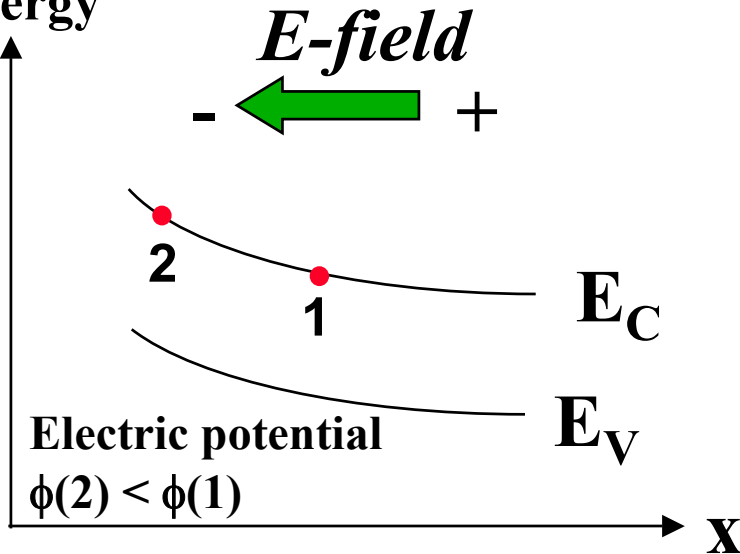


- *Energy band diagram* shows the bottom edge of conduction band, E_c , and top edge of valence band, E_v .
- E_c and E_v are separated by the *band gap energy*, E_g .

Energy Band Diagram with E -field

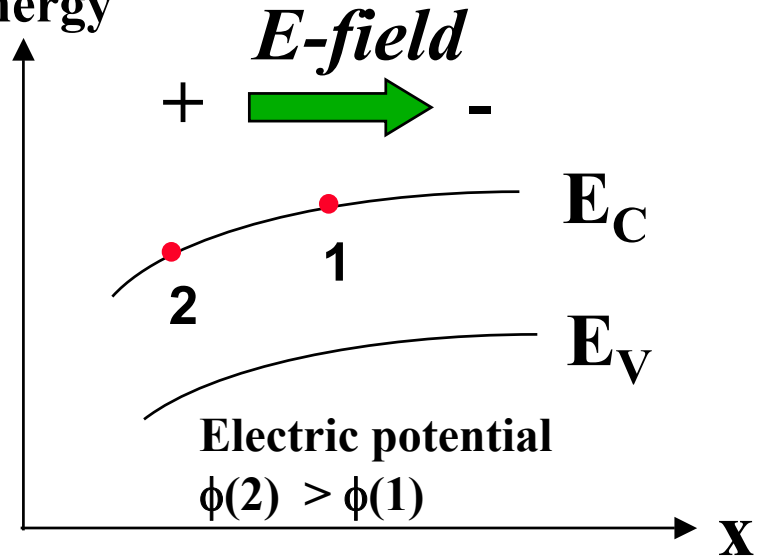
Electron

Energy



Electron

Energy



Electron concentration n

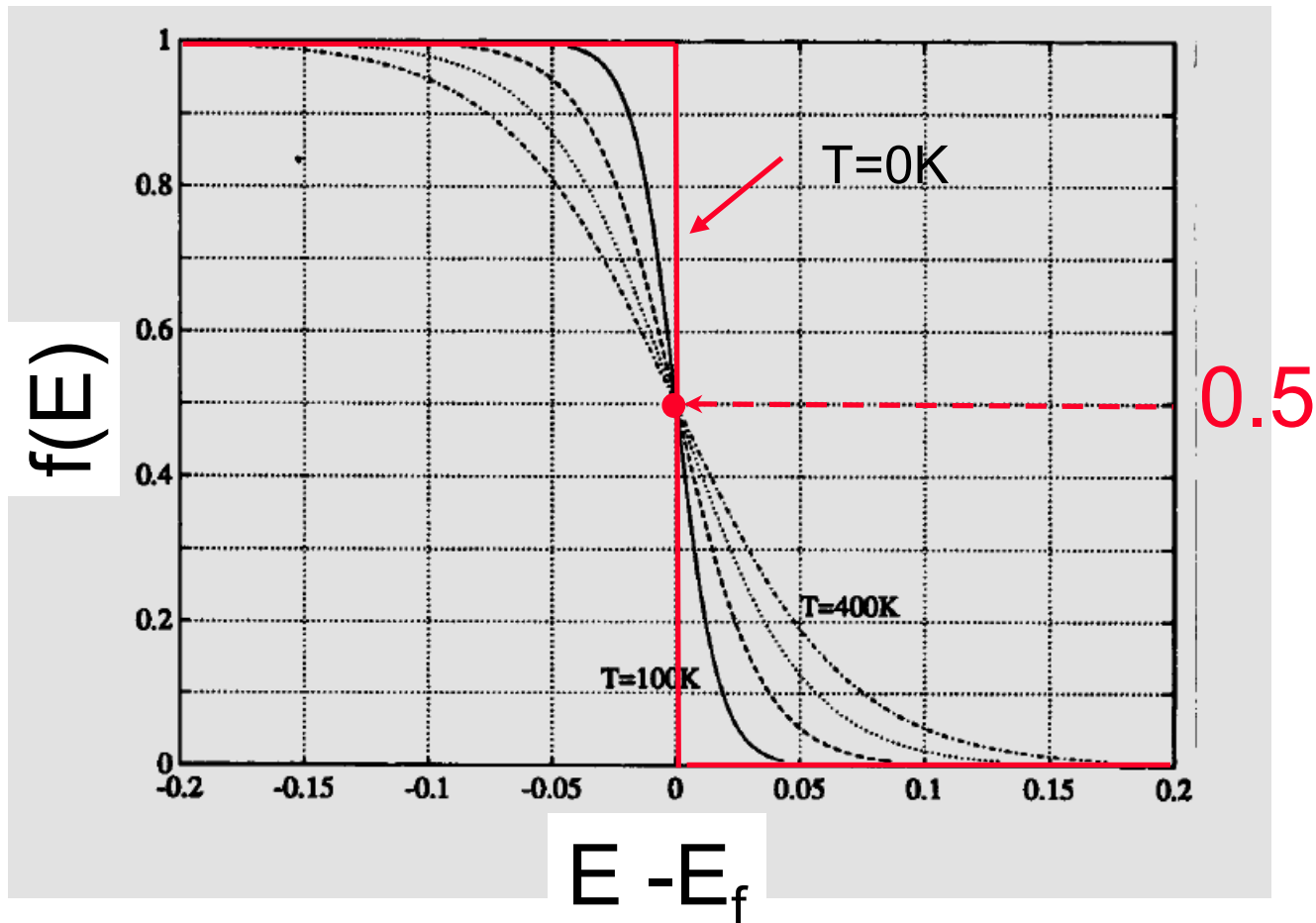
$$\frac{n(2)}{n(1)} = \frac{e^{q\phi(2)/kT}}{e^{q\phi(1)/kT}} = e^{q[\phi(2)-\phi(1)]/kT}$$

The Fermi-Dirac Distribution (Fermi Function)

Probability of available states at energy E being occupied

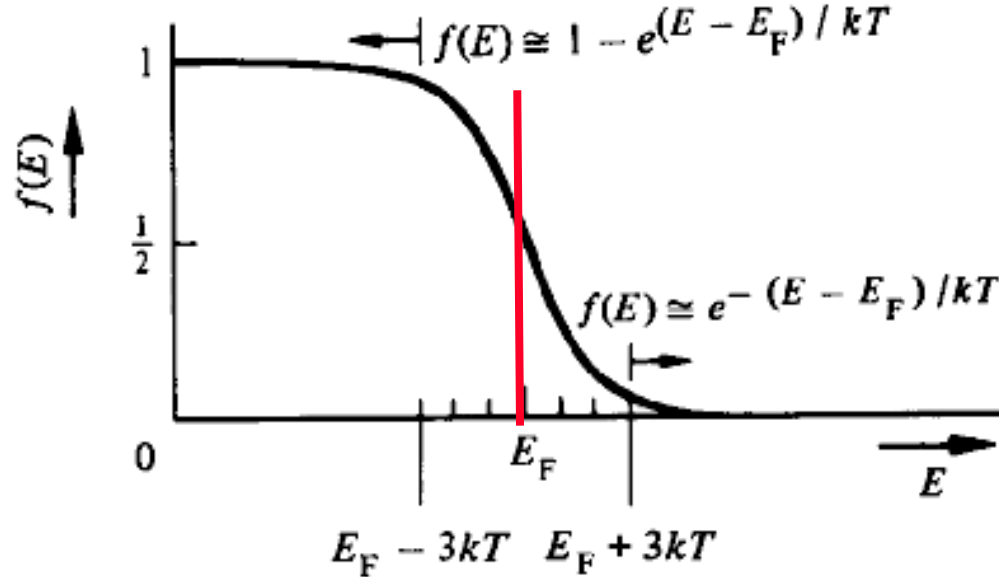
$$f(E) = 1 / [1 + \exp (E - E_f) / kT]$$

where E_f is the Fermi energy and $k = \text{Boltzmann constant} = 8.617 * 10^{-5} \text{ eV/K}$



Properties of the Fermi-Dirac Distribution

Probability
of electron state
at energy E
will be occupied



(b) $T > 0$ K

(1) $f(E) \cong \exp [-(E - E_f) / kT]$ for $(E - E_f) > 3kT$

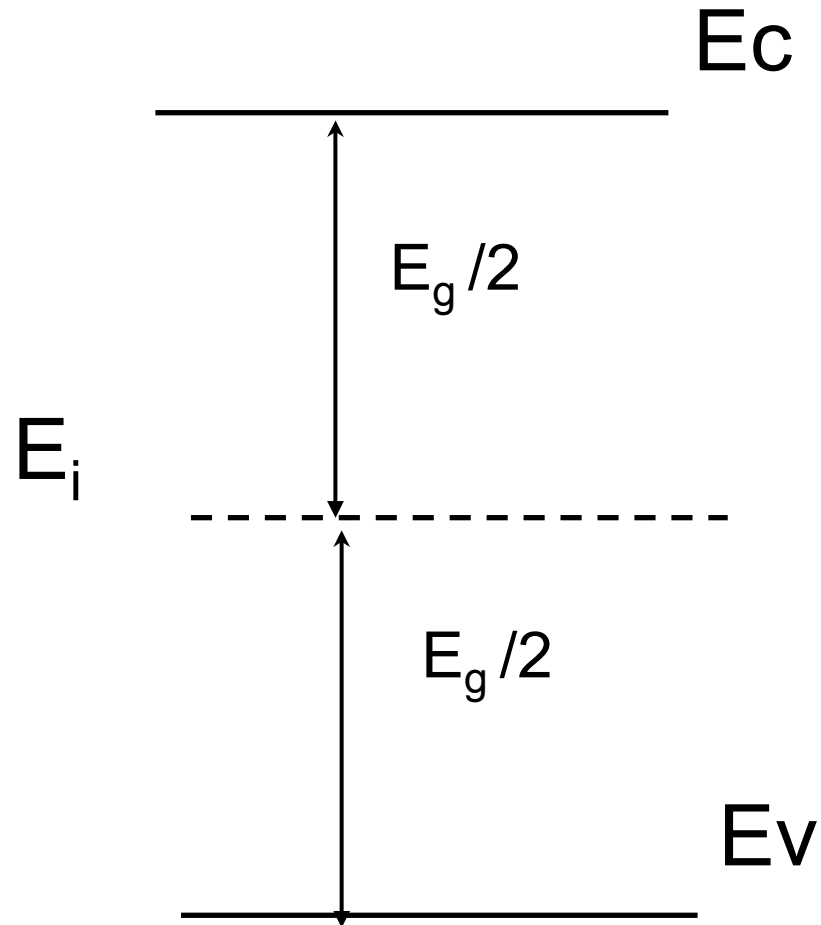
• This approximation is called Boltzmann approximation

Note:
At 300K,
 $kT = 0.026\text{eV}$

(2) Probability of available states at energy E **NOT** being occupied

$$1 - f(E) = 1 / [1 + \exp (E_f - E) / kT]$$

Fermi Energy (E_i) of Intrinsic Semiconductor



How to find n , p when N_a and N_d are known

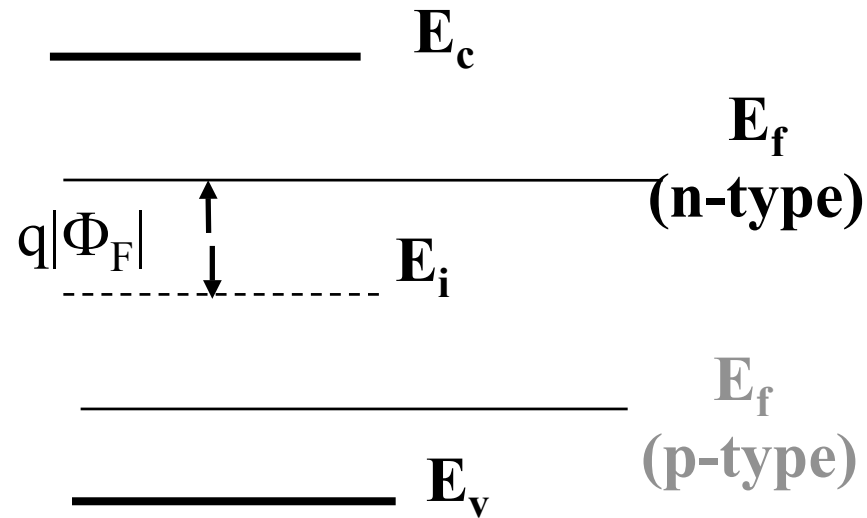
$$n - p = N_d - N_a \quad (1)$$

$$pn = n_i^2 \quad (2)$$

(i) If $N_d - N_a > 10 n_i$: $n \equiv N_d - N_a$

(ii) If $N_a - N_d > 10 n_i$: $p \equiv N_a - N_d$

How to find E_f when n (or p) is known



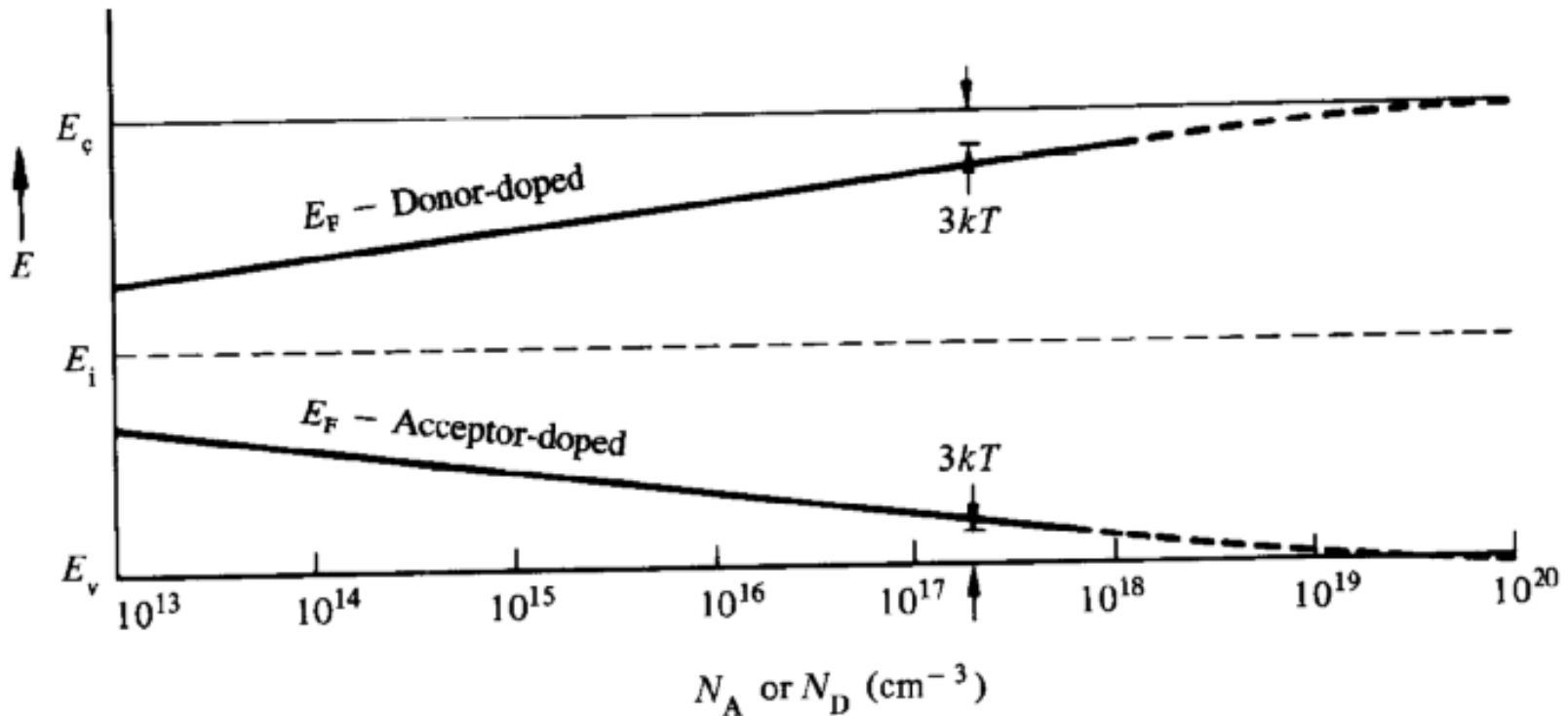
$$n = n_i \exp [(E_f - E_i)/kT]$$

$$\text{Let } q\Phi_F \equiv E_f - E_i$$

$$\therefore n = n_i \exp [q\Phi_F / kT]$$

Dependence of Fermi Level with Doping Concentration

$E_i \equiv (E_C + E_V)/2$ Middle of energy gap

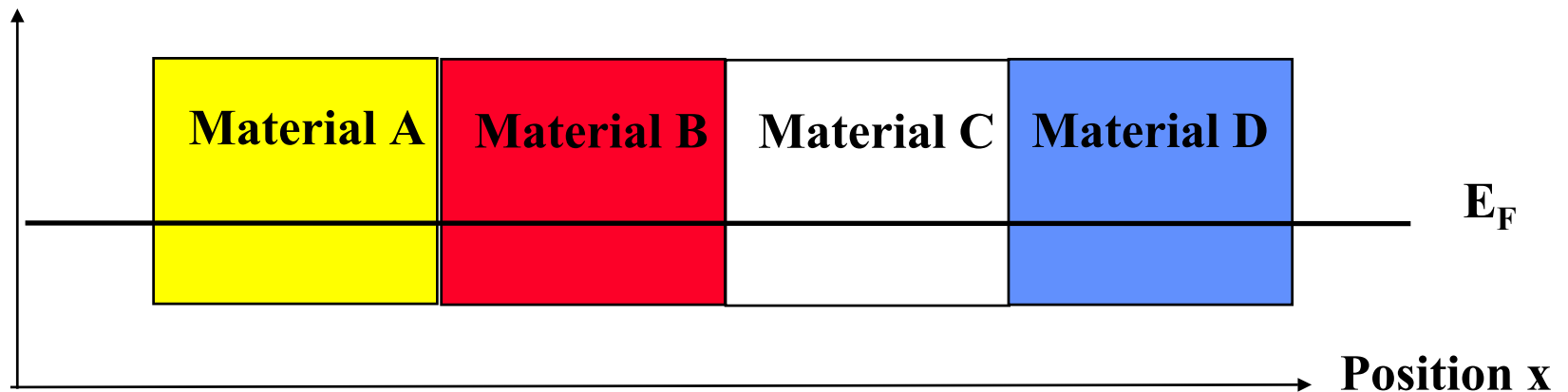


When Si is undoped, $E_f = E_i$; also $n = p = n_i$

The Fermi Energy at thermal equilibrium

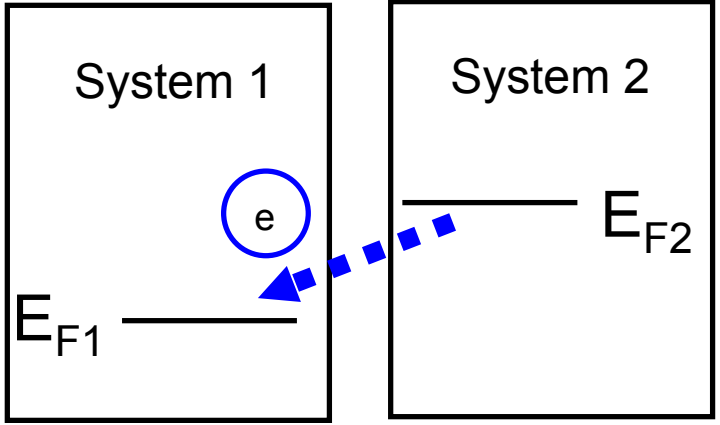
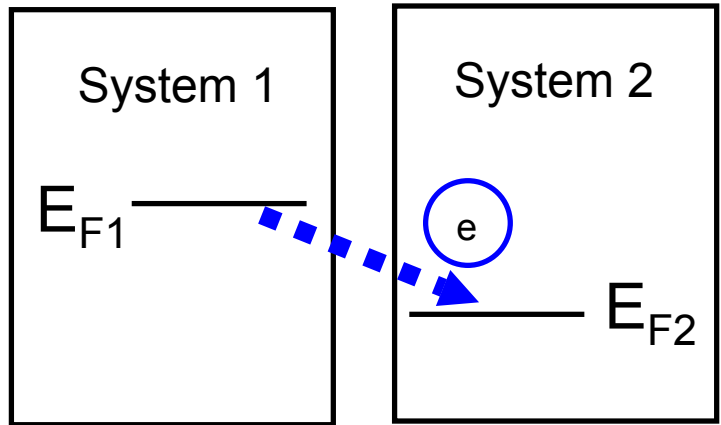
**At thermal equilibrium (i.e., no external perturbation),
The Fermi Energy must be constant for all positions**

Electron energy

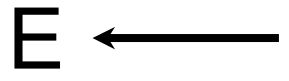
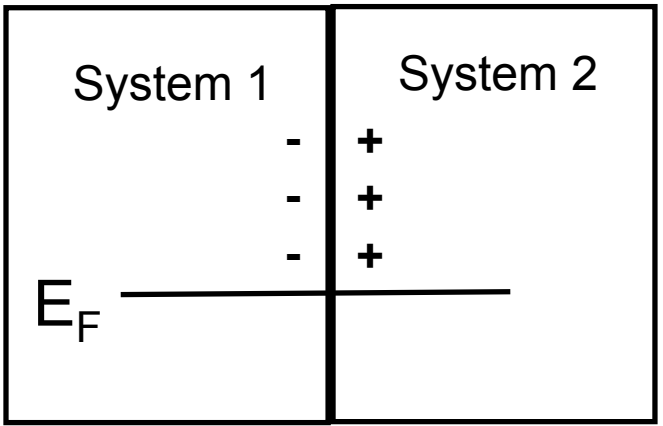
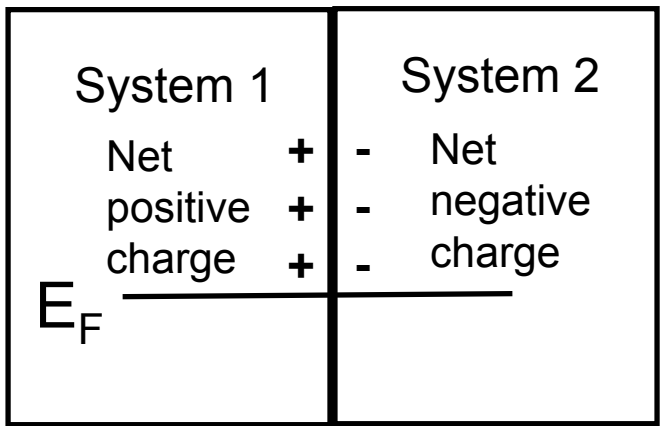


Electron Transfer during contact formation

Before contact formation

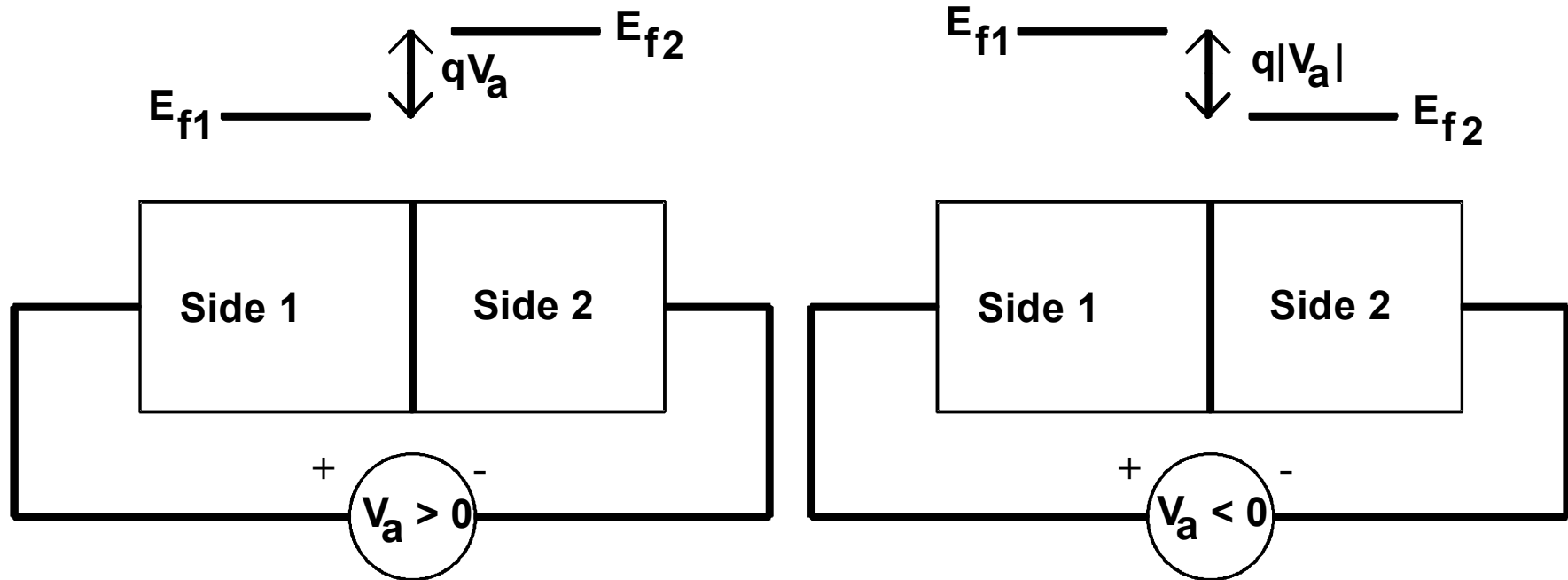


After contact formation



Applied Bias and Fermi Level

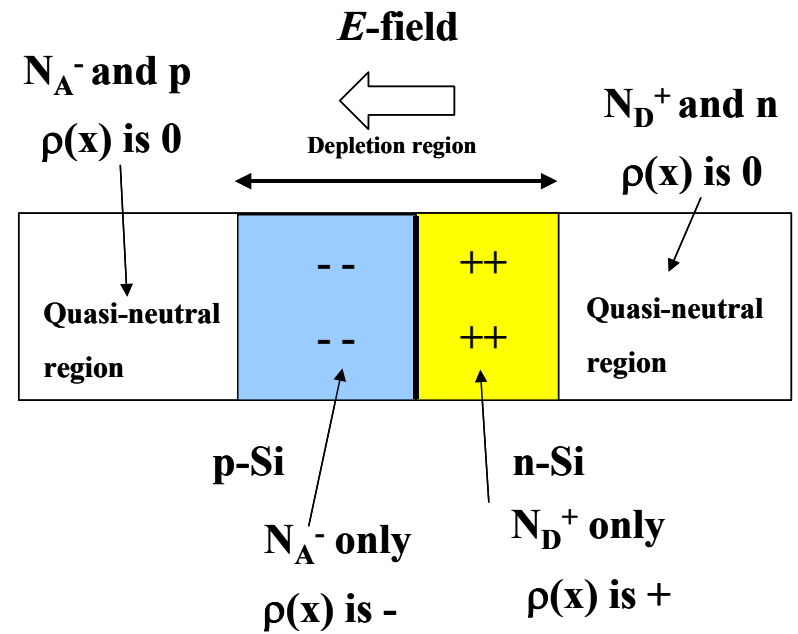
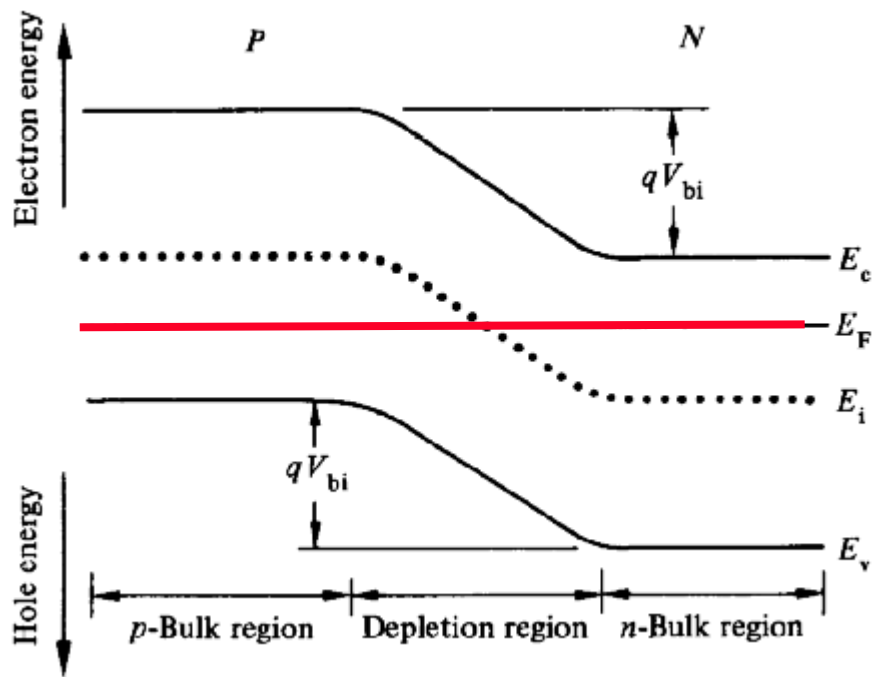
Fermi level of the side which has a relatively higher electric potential will have a relatively lower electron energy (Potential Energy = $-q \cdot$ electric potential.) Only difference of the E 's at both sides are important, not the absolute position of the Fermi levels.



Potential difference across depletion region
 $= V_{bi} - V_a$

PN junctions

Thermal Equilibrium



Complete Depletion Approximation used for charges inside depletion region

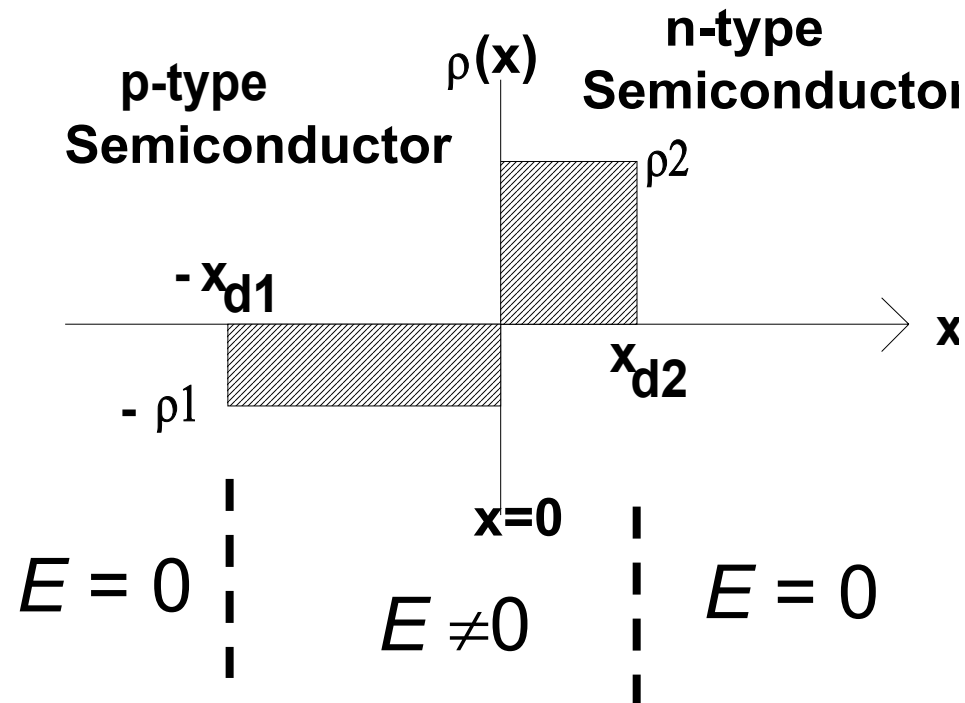
$$\rho(x) \approx N_D^+(x) - N_A^-(x)$$

<http://jas.eng.buffalo.edu/education/pn/pnformation2/pnformation2.html>

Electrostatics of Device Charges

1) Summation of all charges = 0 $\rho_2 \cdot x_{d2} = \rho_1 \cdot x_{d1}$

2) E -field = 0 outside depletion regions



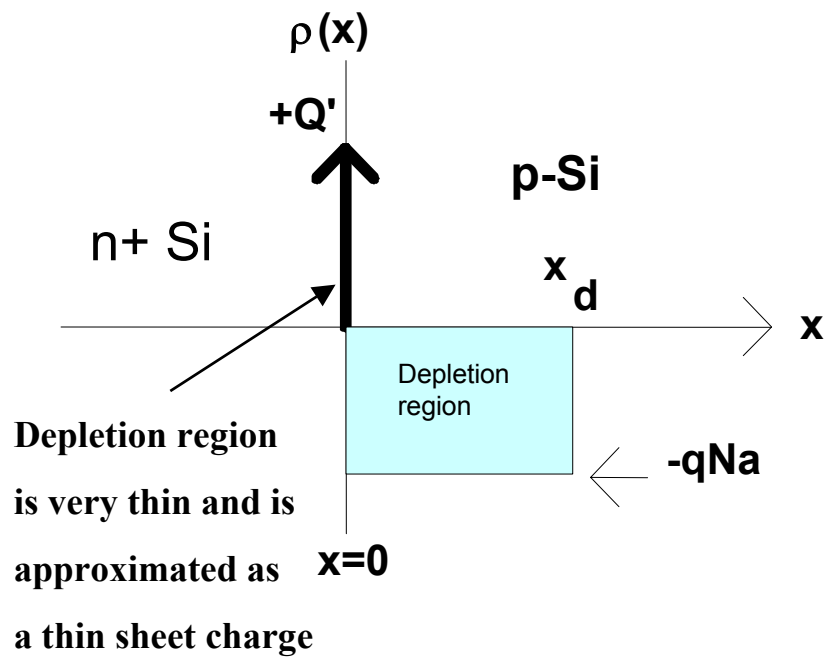
3) Relationship between E -field and charge density $\rho(x)$

$$d [\epsilon E(x)] / dx = \rho(x) \quad \text{“Gauss Law”}$$

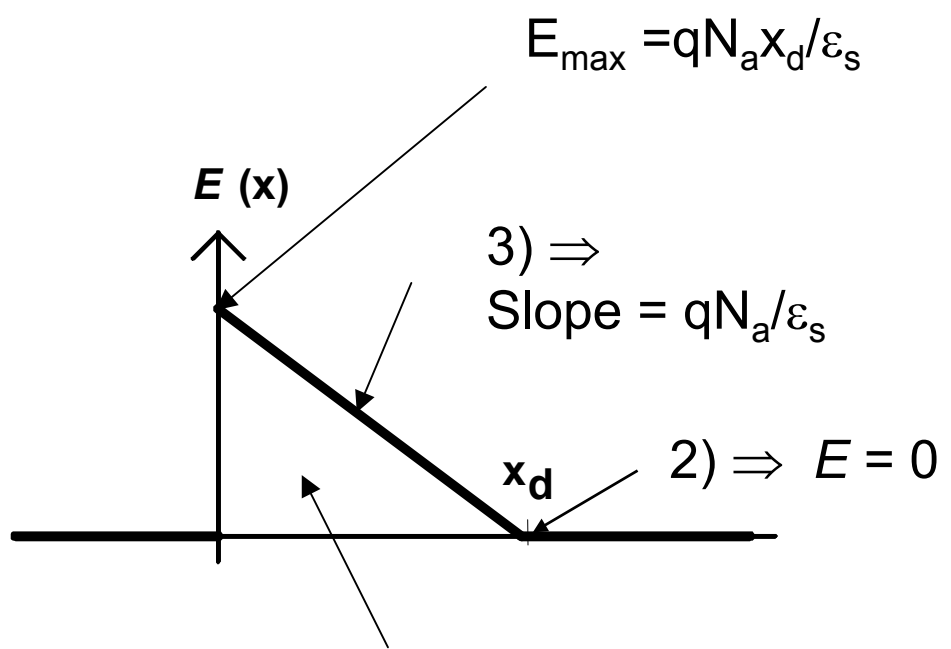
4) Relationship between E -field and potential ϕ

$$E(x) = - d\phi(x)/dx$$

Example Analysis : n+/ p-Si junction



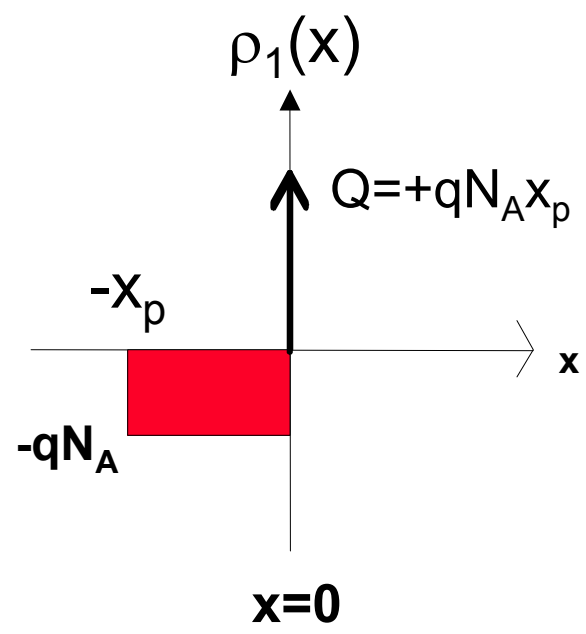
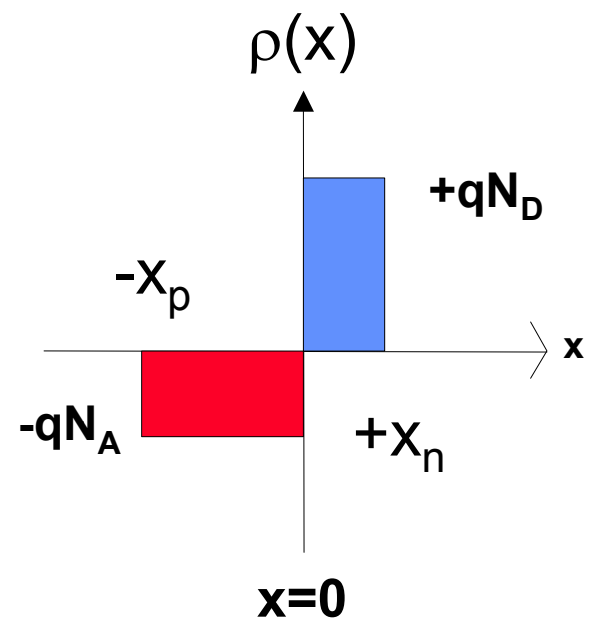
1) $\Rightarrow Q' = qN_a x_d$



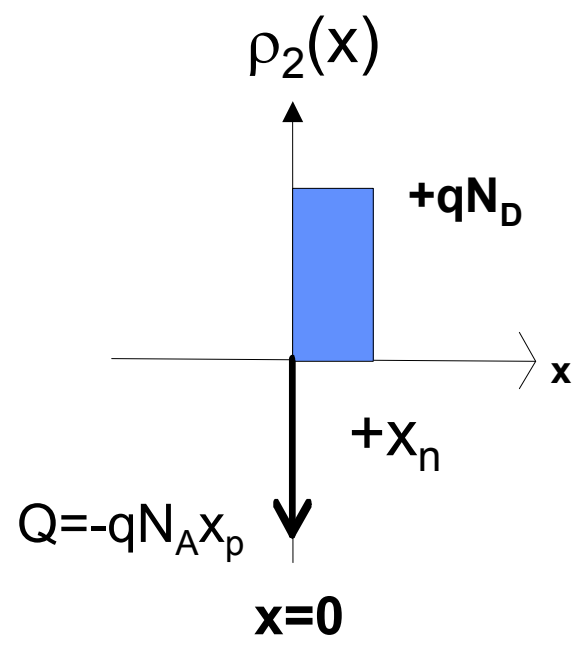
4) \Rightarrow Area under E-field curve = voltage across depletion region = $qN_a x_d^2 / 2\epsilon_s$

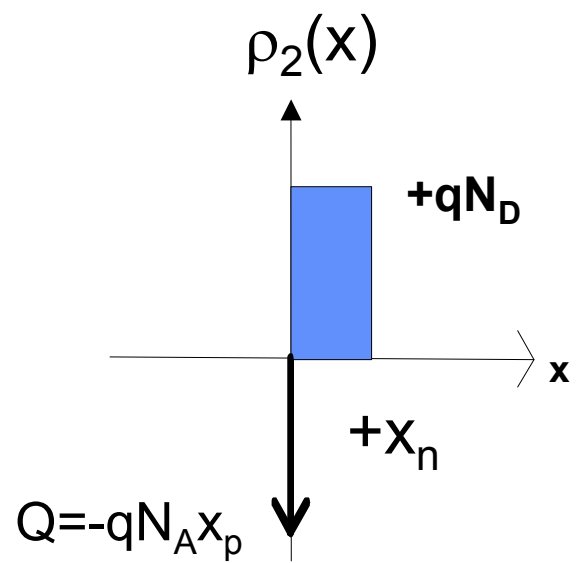
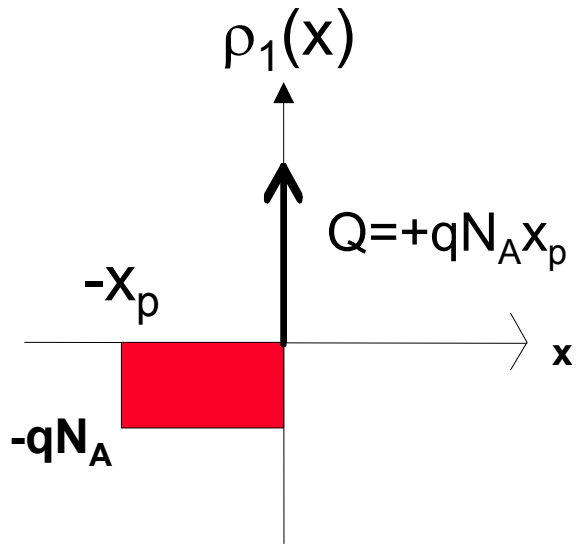
Superposition Principle

If $\rho_1(x) \Rightarrow E_1(x)$ and $V_1(x)$
 $\rho_2(x) \Rightarrow E_2(x)$ and $V_2(x)$
 then
 $\rho_1(x) + \rho_2(x) \Rightarrow E_1(x) + E_2(x)$ and
 $V_1(x) + V_2(x)$



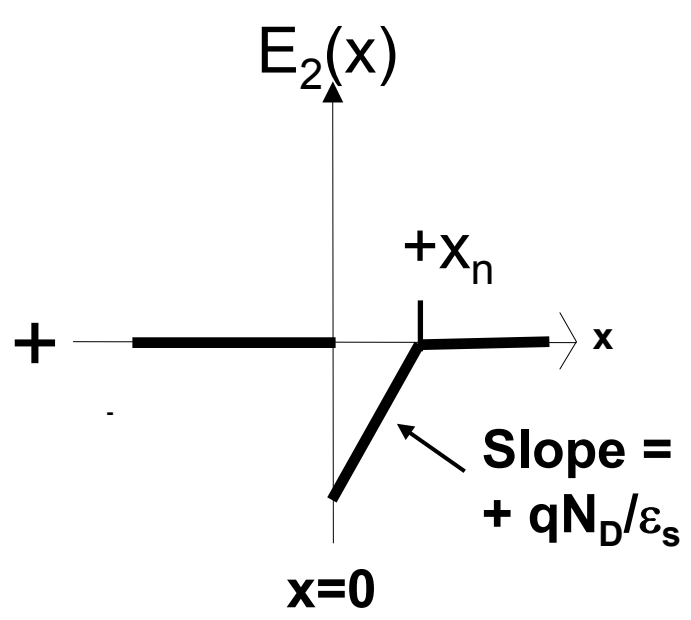
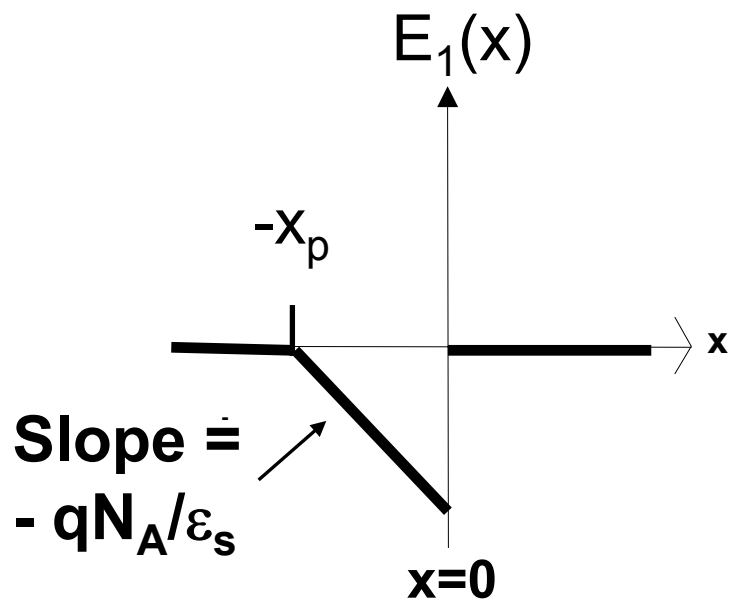
+





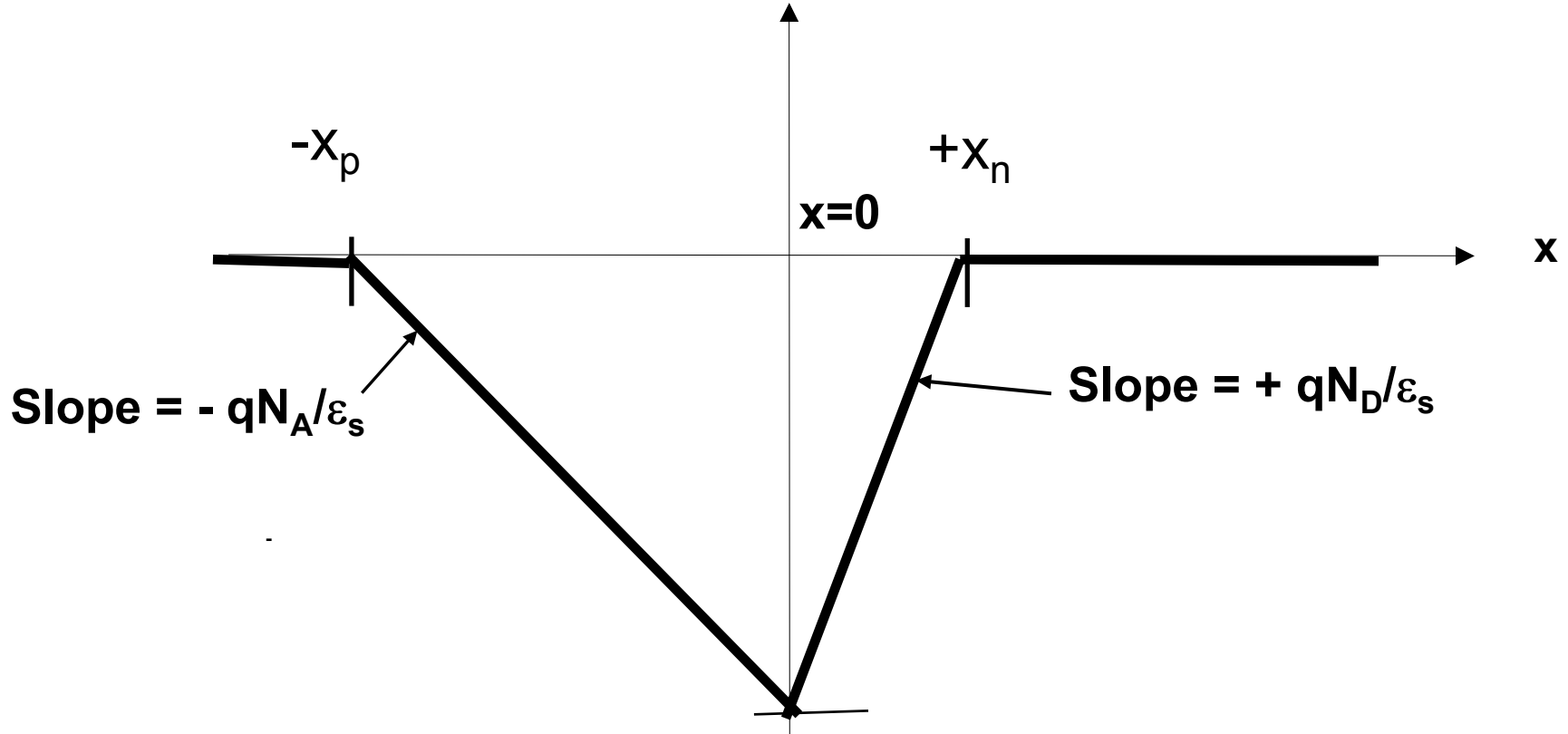
$x=0$

$x=0$



Sketch of $E(x)$

$$E(x) = E_1(x) + E_2(x)$$



$$\begin{aligned} E_{\max} &= -qN_A x_p / \epsilon_s \\ &= -qN_D x_n / \epsilon_s \end{aligned}$$

Why $x_{dmax} \sim$ constant beyond onset of strong inversion ?

$$V_G = V_{FB} + V_{OX} + V_{Si}$$

Higher than V_T

Picks up all the changes in V_G

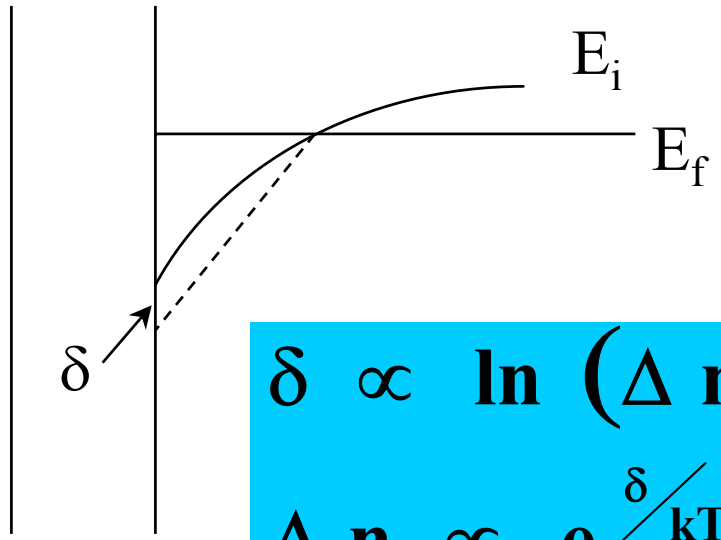
Approximation assumes **V_{Si} does not change much**

Justification:

If surface electron density changes by Δn

$$\Delta V_{OX} \propto \frac{\Delta n}{C_{OX}}$$

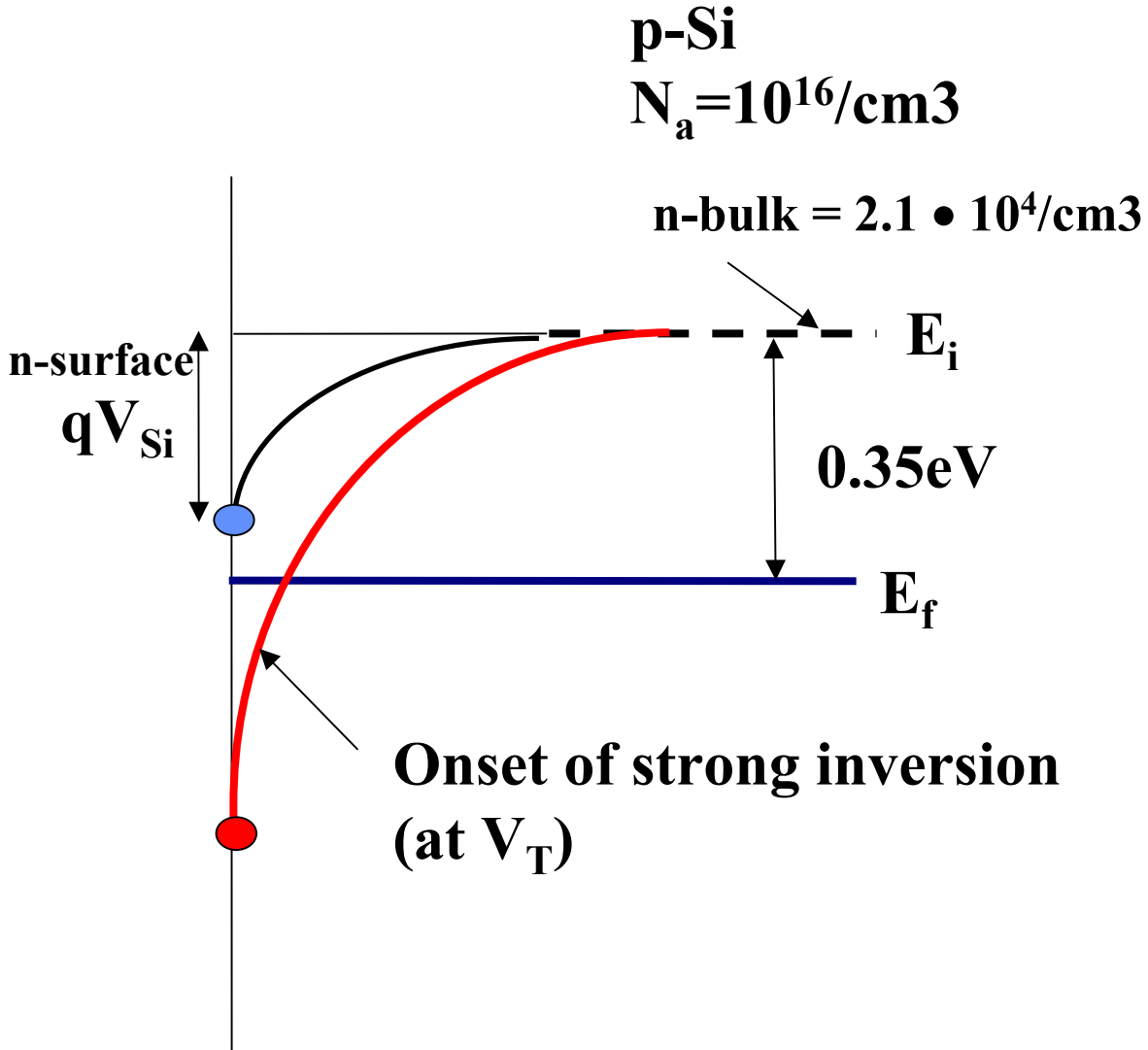
but the change of V_{Si} changes only by $kT/q [\ln(\Delta n)]$ – **small!**



$$\delta \propto \ln(\Delta n)$$

$$\Delta n \propto e^{\delta/kT}$$

$$n\text{-surface} = n\text{-bulk} \cdot e^{qV_{Si}/kT}$$



V_{Si}	$n\text{-surface}$
0	2.10E+04
0.1	9.84E+05
0.2	4.61E+07
0.3	2.16E+09
0.4	1.01E+11
0.5	4.73E+12
0.6	2.21E+14
0.7	1.04E+16
0.8	4.85E+17
0.9	2.27E+19