Lecture #7

Quiz #1 Results
(undergrad. scores only)

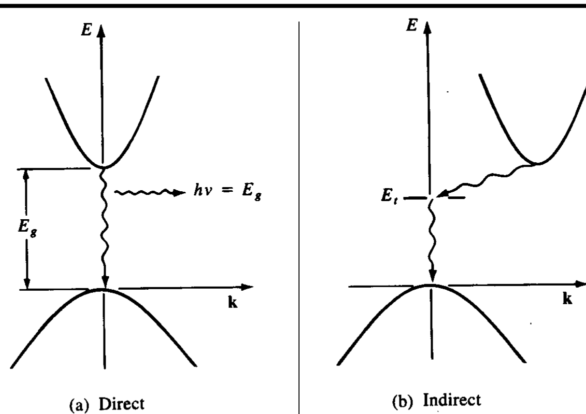
N = 73; mean = 21.6; σ = 2.1; high = 25; low = 14

OUTLINE

– Continuity equations
– Minority carrier diffusion equations
– Quasi-Fermi levels

Reading: Chapter 3.4, 3.5

Clarification: Direct *vs.* Indirect Band Gap



Direct	Indirect
Small change in momentum required for recombination	Large change in momentum required for recombination
→ momentum is conserved by photon emission	→ momentum is conserved by phonon + photon emission

Example: Relaxation to Equilibrium State

Consider a semiconductor with no current flow in which thermal equilibrium is disturbed by the sudden creation of excess holes and electrons. The system will relax back to the equilibrium state via R-G mechanism:

$$\frac{dn}{dt} = -\frac{\Delta n}{\tau_n}$$ for electrons in p-type material

$$\frac{dp}{dt} = -\frac{\Delta p}{\tau_p}$$ for holes in n-type material

Net Recombination Rate (General Case)

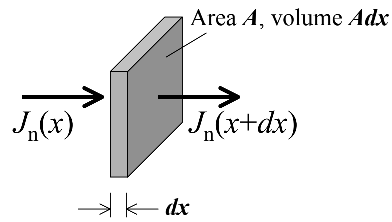
- For arbitrary injection levels and both carrier types in a non-degenerate semiconductor, the net rate of recombination is:

$$\frac{d\Delta n}{dt} = \frac{d\Delta p}{dt} = -\frac{pn - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

where $n_1 \equiv n_i e^{(E_T - E_i)/kT}$ and $p_1 \equiv n_i e^{(E_i - E_T)/kT}$

Derivation of Continuity Equation

- Accounting of carrier-flux into/out-of an infinitesimal volume:



$$Adx\left(\frac{\partial n}{\partial t}\right) = -\frac{1}{q}\left[J_n(x)A - J_n(x+dx)A\right] - \frac{\Delta n}{\tau_n}Adx$$

$$J_n(x+dx) = J_n(x) + \frac{\partial J_n(x)}{\partial x}dx$$

$$=> \frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n}$$

Continuity Equations:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x)}{\partial x} - \frac{\Delta p}{\tau_p} + G_L$$

Derivation of Minority-Carrier Diffusion Equations

- Simplifying assumptions:
  - 1-D
  - negligible electric field
  - n_0, p_0 are independent of x
  - low-level injection conditions

Minority Carrier Diffusion Length

- Consider the special case:
  - Constant minority-carrier (hole) injection at x=0
  - Steady state, no light

$$\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{D_P \tau_p} = \frac{\Delta p}{L_P^{\ 2}}$$

L_P is the ***hole diffusion length***

$$L_p \equiv \sqrt{D_p \tau_p}$$

- Physically, L_P and L_N represent the average distance that minority carriers can diffuse into a sea of majority carriers before being annihilated.

- Example: $N_D = 10^{16}$ cm^{-3}; $\tau_p = 10^{-6}$ s

Quasi-Fermi Levels

- Whenever $\Delta n = \Delta p \neq 0$, $np \neq n_i^2$. However, we would like to preserve and use the relations:

$$n = N_c e^{-(E_c - E_F)/kT} \qquad p = N_v e^{-(E_F - E_v)/kT}$$

- These equations imply $np = n_i^2$, however. The solution is to introduce two ***quasi-Fermi levels*** F_N and F_P such that

$$n = N_c e^{-(E_c - F_N)/kT} \qquad p = N_v e^{-(F_P - E_v)/kT}$$

Example: Quasi-Fermi Levels

Consider a Si sample with $N_D = 10^{17}$ cm^{-3} and $\Delta n = \Delta p = 10^{14}$ cm^{-3}.

(a) Find n:

$n = n_0 + \Delta n = N_D + \Delta n \approx 10^{17}$ cm^{-3}

(b) Find p:

$p = p_0 + \Delta p = (n_i^2 / N_D) + \Delta p \approx 10^{14}$ cm^{-3}

(c) Find the np product:

$np \approx 10^{17} \times 10^{14} = 10^{31}$ cm^{-6} >> n_i^2

(d) Find F_N:

$n = 10^{17}$ cm^{-3} = $N_c e^{-(E_c - F_N)/kT}$

$E_c - F_N = kT \times ln(N_c/10^{17})$

$= 0.026 \text{ eV} \times ln(2.8\times10^{19}/10^{17})$

$= 0.15$ eV

(e) Find F_P:

p = 10^{14} cm^{-3} = $N_v e^{-(F_P - E_v)/kT}$

$F_P – E_v = kT \times ln(N_v/10^{17})$

$= 0.026 \text{ eV} \times \ln(10^{19}/10^{14})$

$= 0.30$ eV