Lecture #31

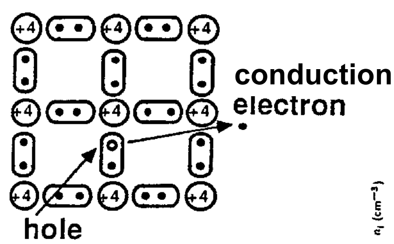
ANNOUNCEMENTS

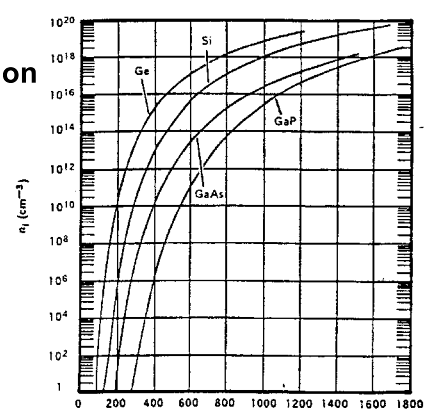
- TA's will hold a review session on Thursday May 15:
  - 2-5 PM, 277 Cory
- Final Exam will take place on Friday May 23:
  - 12:30-3:30 PM, Sibley Auditorium (Bechtel Bldg.)
  - Closed book; 7 pgs of notes + calculator allowed

OUTLINE

– Review of Fundamental Concepts

Intrinsic Carrier Concentration $n_i = \sqrt{N_c N_v} e^{-E_g/kT}$





Covalent (shared e$^-$) bonds exists between Si atoms in a crystal. Since the e$^-$ are loosely bound, some will be free at any T, creating hole electron pairs.

Si:

$$n_i = 3.9 \times 10^{16} T^{3/2} e^{-\frac{0.605\text{eV}}{kT}} /\text{cm}^3$$

$n_i \cong 10^{10}$ cm^{-3} at room temperature

Charge-Carrier Concentrations

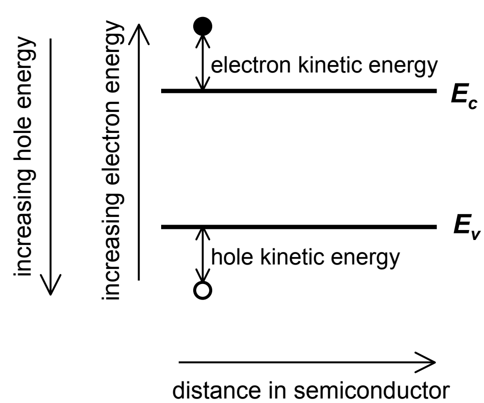
N_D: ionized donor concentration (cm^{-3})
N_A: ionized acceptor concentration (cm^{-3})

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

<u>Note</u>: Carrier concentrations depend on *net* dopant concentration (N_D - N_A) !

Energy Band Diagram

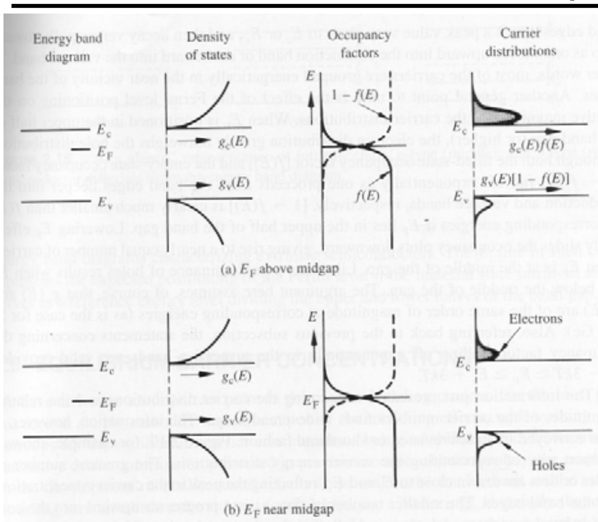


Fermi Function

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$f(E) \cong e^{-(E-E_F)/kT} \quad \text{if} \quad E\text{-}E_F > 3kT$$

Relationship between E_F and n,p



$$n = n_i e^{(E_F - E_i)/kT}$$
$$= N_c e^{(E_c - E_F)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$
$$= N_v e^{(E_F - E_v)/kT}$$

Free Carriers in Semiconductors

- Three primary types of carrier action occur inside a semiconductor:
  - drift
  - diffusion
  - recombination-generation

$$\frac{D}{\mu} = \frac{kT}{q}$$

Net Generation Rate

- The net generation rate is given by

$$\frac{\partial p}{\partial t} = \frac{\partial n}{\partial t} = \frac{n_i^2 - np}{\tau_p(n+n_1) + \tau_n(p+p_1)}$$

where $n_1 \equiv n_i e^{(E_T - E_i)/kT}$ and $p_1 \equiv n_i e^{(E_i - E_T)/kT}$

E_T = trap-state energy level

Drift and Resistivity

- Electrons and holes moving under the influence of an electric field can be modelled as quasi-classical particles with average drift velocity

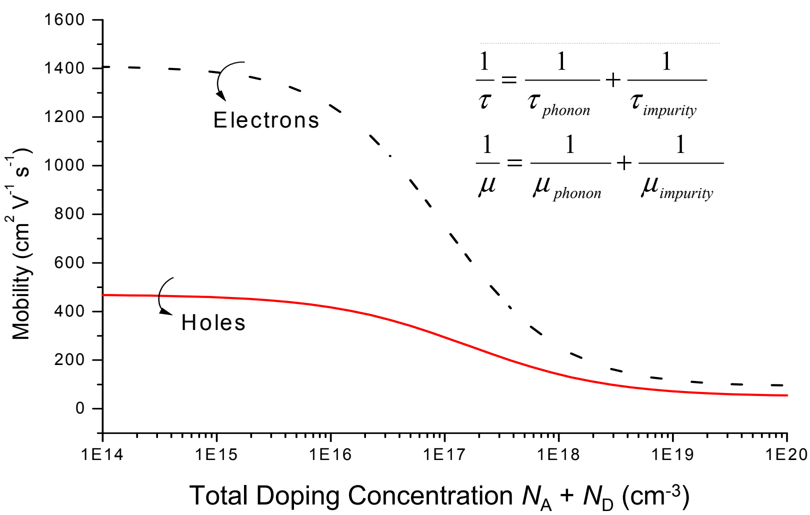
$$|v_\mathrm{d}| = \mu \mathcal{E}$$

- The conductivity of a semiconductor is dependent on the carrier concentrations and mobilities

$$\sigma = qn\mu_n + qp\mu_p$$

- Resistivity $\rho = \dfrac{1}{\sigma} = \dfrac{1}{qn\mu_n + qp\mu_p}$

Mobility Dependence on Doping



$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

Total Current

$$J = J_N + J_P$$

$$J_N = J_{N,\text{drift}} + J_{N,\text{diff}} = qn\mu_n \mathcal{E} + qD_N \frac{dn}{dx}$$

$$J_P = J_{P,\text{drift}} + J_{P,\text{diff}} = qp\mu_p \mathcal{E} - qD_P \frac{dp}{dx}$$

Electrostatic Variables

$$V = \frac{1}{q}(E_{\text{reference}} - E_c)$$

$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx}$$

$$\frac{\rho}{\varepsilon} = \frac{d\mathcal{E}}{dx}$$

Continuity Equations:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x)}{\partial x} - \frac{\Delta p}{\tau_p} + G_L$$

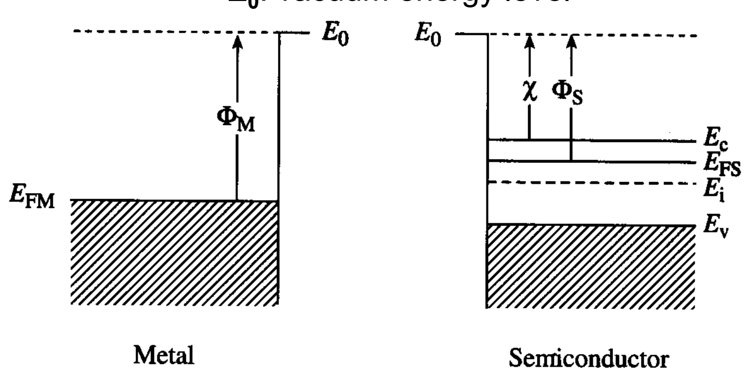
Minority Carrier Diffusion Equations:

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{1}{q}\frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

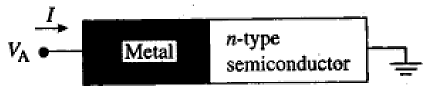
Work Function

E_0: vacuum energy level



Φ_M: metal work function Φ_S: semiconductor work function

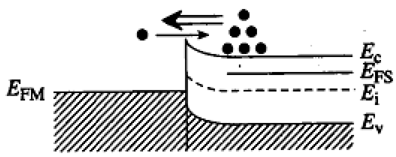
Schottky Diode

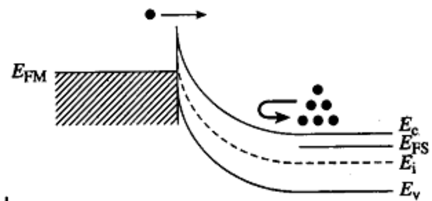


$V_A > 0$

$\Phi_{Bn} = \Phi_M - \chi$

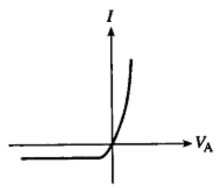
$V_A < 0$





$$W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_A)}{qN_D}}$$

$$C = \frac{\varepsilon_s}{W} A$$

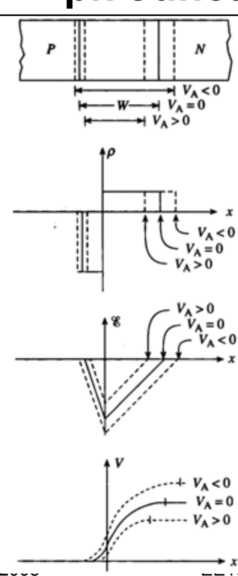


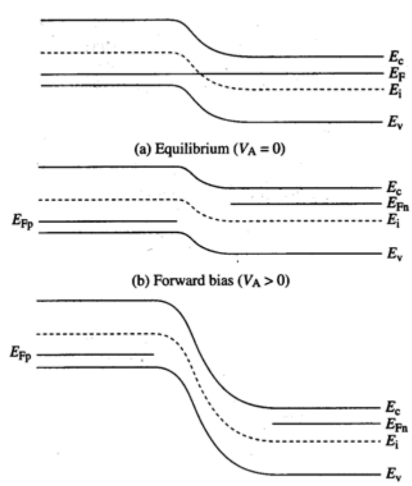
$$I = AT^2 J_S (e^{qV_A/kT} - 1)$$

where $J_S = 120 e^{-q\Phi_B/kT}$ A/cm^2

Fermi level splits into two levels (E_{FM} and E_{FS}) separated by qV_A

pn Junction Electrostatics





(a) Equilibrium ($V_A = 0$)

(b) Forward bias ($V_A > 0$)

(c) Reverse bias ($V_A < 0$)

pn Junction Electrostatics, $V_A \neq 0$

- Built-in potential V_{bi} (non-degenerate doping):

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) + \frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right) = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

- Depletion width W :

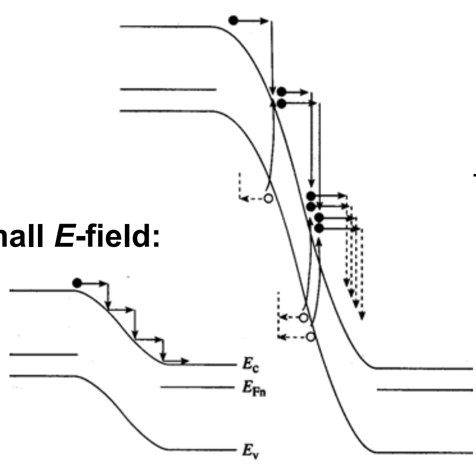
$$W = x_p + x_n = \sqrt{\frac{2\varepsilon_s}{q}(V_{bi} - V_A)\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$x_p = \frac{N_D}{N_A + N_D}W \qquad x_n = \frac{N_A}{N_A + N_D}W$$

Avalanche Breakdown Mechanism

High *E*-field:

Small *E*-field:



$$V_{BR} \approx \frac{\varepsilon_s \mathcal{E}_{crit}^2}{2qN} \text{ if } V_{BR} >> V_{bi}$$

$\mathcal{E}_{crit}$ increases slightly with *N*:

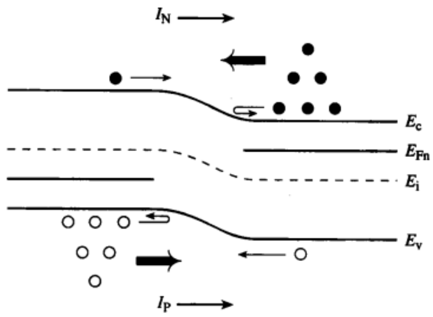
For 10^{14} cm^{-3} < *N* < 10^{18} cm^{-3},

10^5 V/cm < $\mathcal{E}_{crit}$ < 10^6 V/cm

"Law of the Junction"

The voltage V_A applied to a pn junction falls mostly across the depletion region (assuming that low-level injection conditions prevail in the quasi-neutral regions).

We can draw 2 quasi-Fermi levels in the depletion region:



$$p = n_i e^{(E_i - F_P)/kT}$$

$$n = n_i e^{(F_N - E_i)/kT}$$

$$pn = n_i^2 e^{(E_i - F_P)/kT} e^{(F_N - E_i)/kT}$$

$$= n_i^2 e^{(F_N - F_P)/kT}$$

$$\boxed{pn = n_i^2 e^{qV_A/kT}}$$

Excess Carrier Concentrations at $-x_p$, x_n

p-side

$$p_p(-x_p) = N_A$$

$$n_p(-x_p) = \frac{n_i^2 e^{qV_A/kT}}{N_A} = n_{p0} e^{qV_A/kT}$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$$

n-side

$$n_n(x_n) = N_D$$

$$p_n(x_n) = \frac{n_i^2 e^{qV_A/kT}}{N_D} = p_{n0} e^{qV_A/kT}$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

pn Diode *I-V* Characteristic

p-side: $J_n = -qD_n \frac{d\Delta n_p(x'')}{dx''} = q\frac{D_n}{L_n} n_{p0}(e^{qV_A/kT} - 1)e^{-x''/L_n}$

n-side: $J_p = -qD_p \frac{d\Delta p_n(x')}{dx'} = q\frac{D_p}{L_p} p_{n0}(e^{qV_A/kT} - 1)e^{-x'/L_p}$

$$J = J_n\big|_{x=-x_p} + J_p\big|_{x=x_n} = J_n\big|_{x''=0} + J_p\big|_{x'=0}$$

$$J = qn_i^2\left[\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right](e^{qV_A/kT} - 1)$$

pn Junction Capacitance

2 types of capacitance associated with a pn junction:

1. C_J ***depletion capacitance***

$$C_J \equiv \left|\frac{dQ_{dep}}{dV_A}\right| = A\frac{\varepsilon_s}{W}$$

2. C_D ***diffusion capacitance*** (due to variation of stored minority charge in the quasi-neutral regions)

For a one-sided p^+n junction ($Q_P >> Q_N$):

$$C_D = \left|\frac{dQ}{dV_A}\right| = \tau_p \frac{dI}{dV_A} = \tau_p G = \frac{\tau_p I_{DC}}{kT / q}$$

Deviations from the Ideal *I-V* Behavior

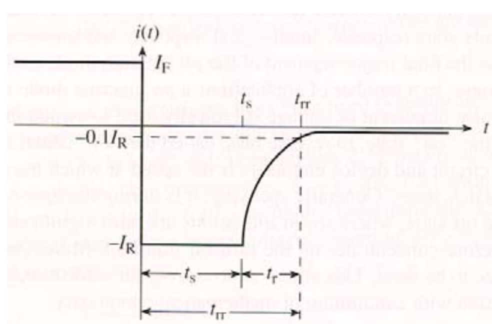
Resulting from
- recombination/generation in the depletion region
- series resistance
- high-level injection

Transient Response of pn Diode

- Because of C_D, the voltage across the pn junction depletion region cannot be changed instantaneously.

(The delay in switching between the ON and OFF states is due to the time required to change the amount of excess minority carriers stored in the quasi-neutral regions.)

Turn-off transient:

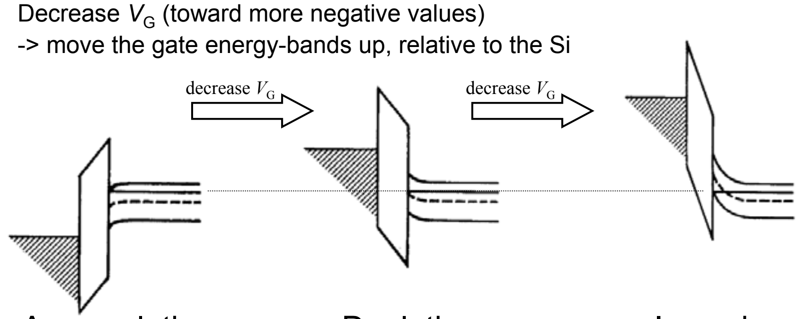


Minority-Carrier Injection & Collection

- Under **forward** bias, minority carriers are **injected** into the quasi-neutral regions of the diode.
  - → Current flowing across junction is comprised of hole and electron components
- Under **reverse** bias, minority carriers are **collected** into the quasi-neutral regions of the diode. (Minority carriers within a diffusion length of the depletion region will diffuse into the depletion region and then be swept across the junction by the electric field)
  - → Current flowing depends on the rate at which minority carriers are supplied

MOS Band Diagrams (n-type Si)

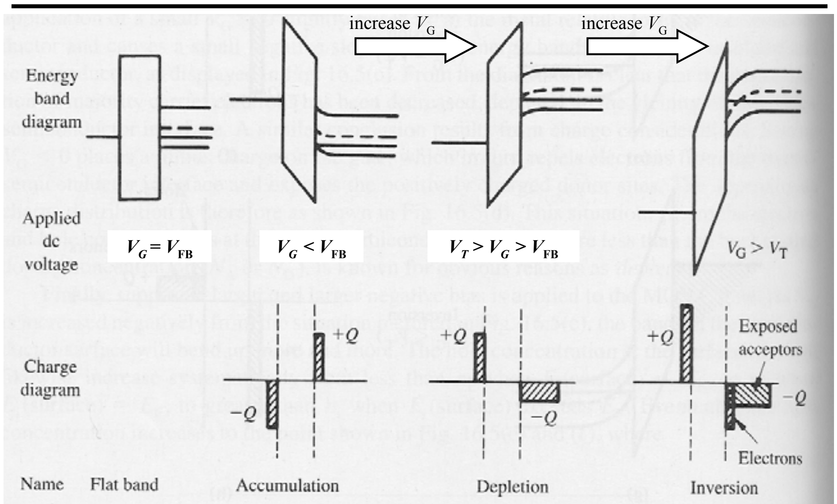
Decrease V_G (toward more negative values)
-> move the gate energy-bands up, relative to the Si



- Accumulation
  - $V_G > V_{FB}$
    - Electrons accumulate at surface
- Depletion
  - $V_G < V_{FB}$
    - Electrons repelled from surface
- Inversion
  - $V_G < V_T$
    - Surface becomes p-type

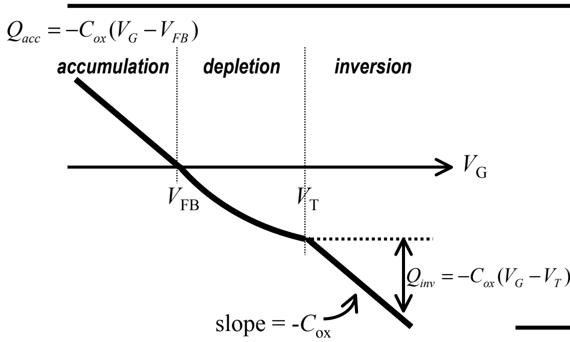
$$V_G = V_{FB} + V_{ox} + \psi_s$$

Biasing Conditions for p-type Si



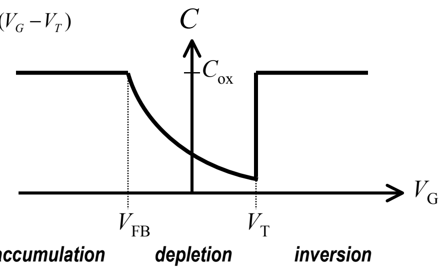
$$W_d = \sqrt{\frac{2\varepsilon_{Si}\psi_s}{qN_A}}$$

MOS Charge & Capacitance (p-type Si)



$$C = \left|\frac{dQ_s}{dV_G}\right|$$

Ideal *C-V* curve

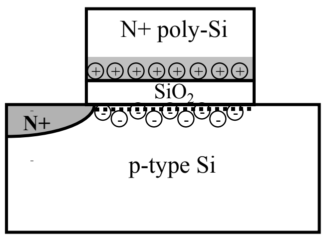


$$V_{FB} = \phi_{MS} - \frac{Q_F}{C_{ox}}$$

$$V_T = V_{FB} + 2\psi_B + \frac{\sqrt{2qN_A\varepsilon_{Si}(2\psi_B)}}{C_{ox}}$$

V_T Adjustment by Back Biasing

- In some IC products, V_T is dynamically adjusted by applying a back bias:
  - When a MOS capacitor is biased into inversion, a pn junction exists between the surface and the bulk.
  - If the inversion layer contacts a heavily doped region of the same type, it is possible to apply a bias to this pn junction



- V_G biased so surface is inverted
- Inversion layer contacted by N+ region
- Bias V_C applied to channel
  → Reverse bias V_B-V_C applied btwn channel & body

Effect of V_{CB} on ψ_s and V_T

- Application of reverse bias -> non-equilibrium
  - 2 Fermi levels (one for n-region, one for p-region)
    - Separation = qV_{BC} ➔ **ψ_s increased by V_C**
- Reverse bias widens W_d, increases Q_{dep}
  ➔ Q_{inv} decreases with increasing V_{CB}, for a given V_{GB}

$$V_T = V_{FB} + V_C + 2\psi_B + \frac{\sqrt{2qN_A\varepsilon_{Si}(2\psi_B + V_{CB})}}{C_{ox}}$$