Lecture #15

OUTLINE

The Bipolar Junction Transistor

- Fundamentals
- Ideal Transistor Analysis

Reading: Chapter 10, 11.1

Bipolar Junction Transistors (BJTs)

- Over the past 3 decades, the higher layout density and low-power advantage of CMOS technology has eroded away the BJT's dominance in integrated-circuit products.

  (higher circuit density → better system performance)

- BJTs are still preferred in some digital-circuit and analog-circuit applications because of their high speed and superior gain.

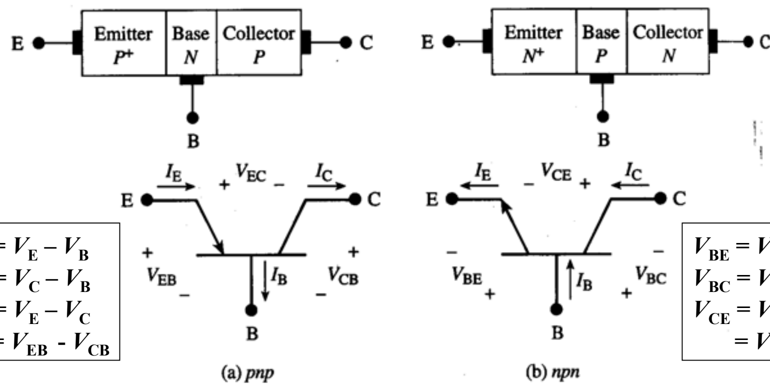
  ✓ faster circuit speed

  ✗ larger power dissipation

  → limits integration level to ~10^4 circuits/chip

Introduction

- The BJT is a 3-terminal device
  - 2 types: PNP and NPN



$V_{EB} = V_E - V_B$
$V_{CB} = V_C - V_B$
$V_{EC} = V_E - V_C$
$= V_{EB} - V_{CB}$

$V_{BE} = V_B - V_E$
$V_{BC} = V_B - V_C$
$V_{CE} = V_C - V_E$
$= V_{CB} - V_{EB}$

(a) *pnp* (b) *npn*

- The convention used in the textbook does not follow IEEE convention (currents defined as positive flowing into a terminal)
- We will follow the convention used in the textbook

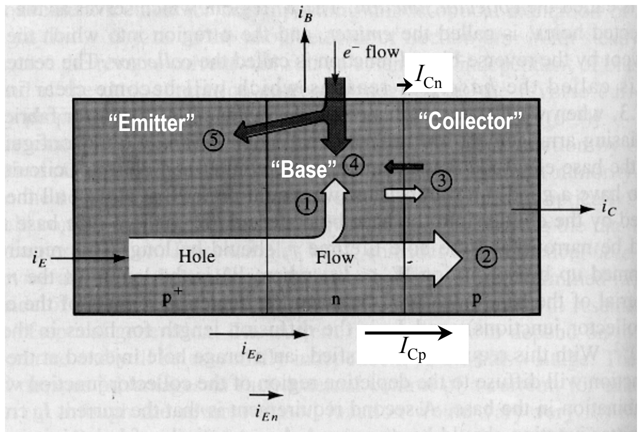
Charge Transport in a BJT

- Consider a reverse-biased pn junction:
  - Reverse saturation current depends on rate of minority-carrier generation near the junction

    ⇒ can increase reverse current by increasing rate of minority-carrier generation:
    - Optical excitation of carriers
    - Electrical injection of minority carriers into the neighborhood of the junction

PNP BJT Operation (Qualitative)

"Active Bias": $V_{EB} > 0$ (forward bias), $V_{CB} < 0$ (reverse bias)



$$\beta_{dc} \cong \frac{I_C}{I_B}$$

BJT Design

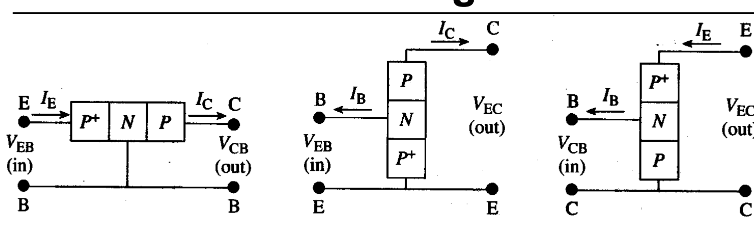
- Important features of a good transistor:
  - Injected minority carriers do not recombine in the neutral base region
  - Emitter current is comprised almost entirely of carriers injected into the base (rather than carriers injected into the emitter

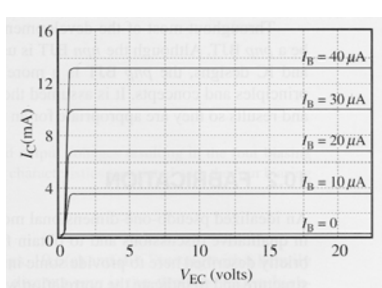
Base Current Components

The base current consists of majority carriers supplied for

1. Recombination of injected minority carriers in the base
2. Injection of carriers into the emitter
3. Reverse saturation current in collector junction
   - Reduces | I_B |
4. Recombination in the base-emitter depletion region

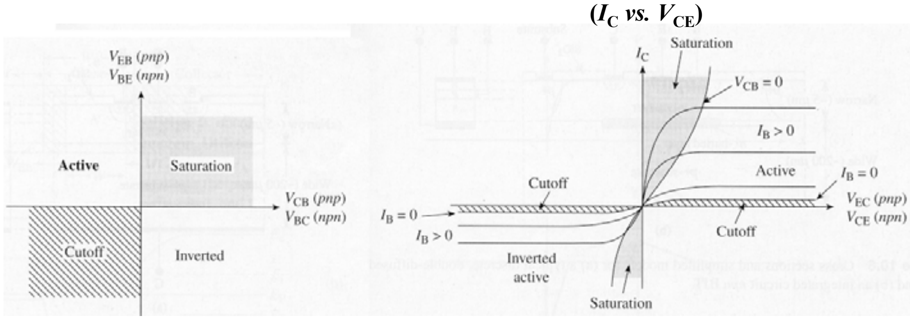
Circuit Configurations





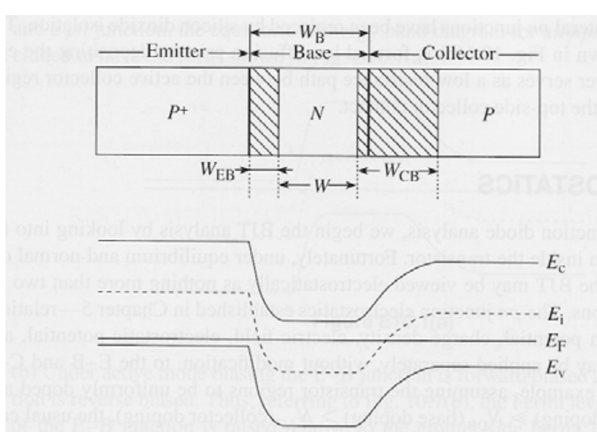
Modes of Operation

Common-emitter output characteristics
(I_C *vs.* V_{CE})

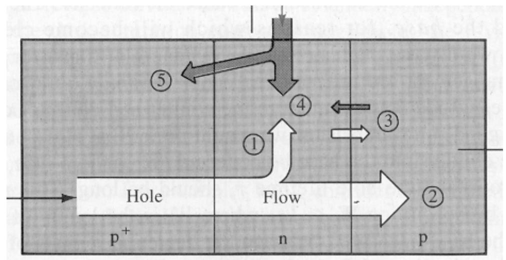


BJT Electrostatics

- Under normal operating conditions, the BJT may be viewed electrostatically as two independent pn junctions



BJT Performance Parameters (PNP)



- Emitter Efficiency:

$$\gamma = \frac{I_{Ep}}{I_{Ep}+I_{En}}$$

  – Decrease **(5)** relative to **(1+2)** to increase efficiency

- Base Transport Factor:

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}}$$

  – Decrease **(1)** relative to **(2)** to increase transport factor

- Common-Base d.c. Current Gain: $\alpha_{dc} \equiv \gamma\alpha_T$

Collector Current (PNP)

- The collector current is comprised of
  - Holes injected from emitter, which do not recombine in the base ← (2)
  - Reverse saturation current of collector junction ← (3)

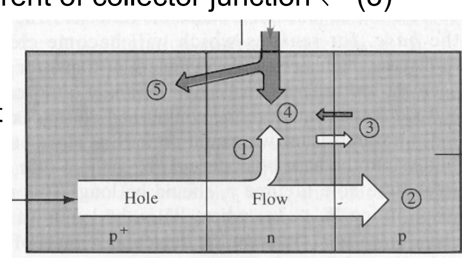
$$I_C = \alpha_{dc} I_E + I_{CB0}$$

where I_{CB0} is the collector current which flows when $I_E = 0$

$$I_C = \alpha_{dc}\left(I_C + I_B\right) + I_{CB0}$$

$$I_C = \frac{\alpha_{dc}}{1-\alpha_{dc}} I_B + \frac{I_{CB0}}{1-\alpha_{dc}}$$

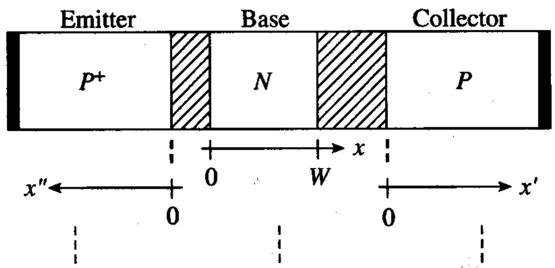
$$= \beta I_B + I_{CE0}$$



- Common-Emitter d.c. Current Gain:

$$\beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$$

Notation (PNP BJT)



$N_E = N_{AE}$	$N_B = N_{DB}$	$N_C = N_{AC}$
$D_E = D_N$	$D_B = D_P$	$D_C = D_N$
$\tau_E = \tau_n$	$\tau_B = \tau_p$	$\tau_C = \tau_n$
$L_E = L_N$	$L_B = L_P$	$L_C = L_N$
$n_{E0} = n_{p0} = n_i^2/N_E$	$p_{B0} = p_{n0} = n_i^2/N_B$	$n_{C0} = n_{p0} = n_i^2/N_C$

Ideal Transistor Analysis

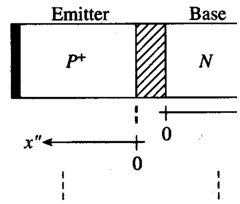
- Solve the minority-carrier diffusion equation in each quasi-neutral region to obtain excess minority-carrier profiles
  - different set of boundary conditions for each region
- Evaluate minority-carrier diffusion currents at edges of depletion regions

$$I_{En} = -qAD_E \left.\frac{d\Delta n_E}{dx''}\right|_{x''=0} \qquad I_{Ep} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=0}$$

$$I_{Cn} = qAD_C \left.\frac{d\Delta n_C}{dx'}\right|_{x'=0} \qquad I_{Cp} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=W}$$

- Add hole & electron components together → terminal currents

Emitter Region Formulation



- Diffusion equation:

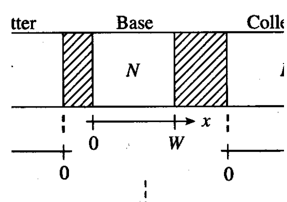
$$0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$$

- Boundary Conditions:

$$\Delta n_E(x'' \to \infty) = 0$$

$$\Delta n_E(x''=0) = n_{E0}(e^{qV_{EB}/kT} - 1)$$

Base Region Formulation



- Diffusion equation:

$$0 = D_B \frac{d^2 \Delta p_B}{dx^2} - \frac{\Delta p_B}{\tau_B}$$

- Boundary Conditions:

$$\Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1)$$

$$\Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1)$$

Collector Region Formulation

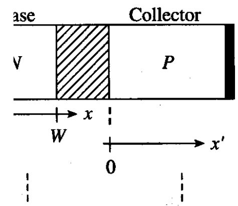
- Diffusion equation:

$$0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$$

- Boundary Conditions:

$$\Delta n_C(x' \to \infty) = 0$$

$$\Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1)$$



Current Formulation

$$I_{En} = -qAD_E \frac{d\Delta n_E}{dx''}\bigg|_{x''=0}$$

$$I_{Ep} = -qAD_B \frac{d\Delta p_B}{dx}\bigg|_{x=0}$$

$$I_{Cp} = -qAD_B \frac{d\Delta p_B}{dx}\bigg|_{x=W}$$

$$I_{Cn} = qAD_C \frac{d\Delta n_C}{dx'}\bigg|_{x'=0}$$

Emitter Region Solution

- The solution of $0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$ is:

$$\Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E}$$

- From the boundary conditions: $\Delta n_E(x'' \to \infty) = 0$

$$\Delta n_E(x''=0) = n_{E0}(e^{qV_{EB}/kT} - 1)$$

we have: $\Delta n_E(x'') = n_{E0}(e^{qV_{EB}/kT} - 1)e^{-x''/L_E}$

and: $I_{En} = qA \frac{D_E}{L_E} n_{E0}(e^{qV_{EB}/kT} - 1)$

Collector Region Solution

- The solution of $0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$ is:

$$\Delta n_C(x') = A_1 e^{-x'/L_C} + A_2 e^{x'/L_C}$$

- From the boundary conditions: $\Delta n_C(x' \to \infty) = 0$

$$\Delta n_C(x'=0) = n_{C0}(e^{qV_{CB}/kT} - 1)$$

- we have: $\Delta n_C(x') = n_{C0}(e^{qV_{CB}/kT} - 1)e^{-x'/L_C}$

and: $I_{Cn} = -qA \frac{D_C}{L_C} n_{C0}(e^{qV_{CB}/kT} - 1)$

Base Region Solution

- The solution of $0 = D_B \frac{d^2 \Delta n_B}{dx^2} - \frac{\Delta p_B}{\tau_B}$ is:

$$\Delta p_B(x) = A_1 e^{-x/L_B} + A_2 e^{x/L_B}$$

- From the boundary conditions: $\Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1)$

$$\Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1)$$

we have:

$$\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\left(\frac{e^{(W-x)/L_B} - e^{-(W-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$
$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

- Now, we know $\sinh(\xi) = \frac{e^{\xi} - e^{-\xi}}{2}$
- Therefore, we can write:

$$\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\left(\frac{e^{(W-x)/L_B} - e^{-(W-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$
$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

as $\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\frac{\sinh\left[(W-x)/L_B\right]}{\sinh\left(W/L_B\right)}$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\frac{\sinh\left[x/L_B\right]}{\sinh\left(W/L_B\right)}$$

- We know $\cosh(\xi) = \frac{e^{\xi} + e^{-\xi}}{2}$
- Therefore, we have:

$$I_{Ep} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1) - \frac{1}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

and:

$$I_{Cp} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{1}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

Terminal Currents

- We know:

$$I_{En} = qA\frac{D_E}{L_E}n_{E0}(e^{qV_{EB}/kT}-1)$$

$$I_{Ep} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1) - \frac{1}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_{Cp} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{1}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_{Cn} = -qA\frac{D_C}{L_C}n_{C0}(e^{qV_{CB}/kT}-1)$$

- Therefore:

$$I_E = qA\left[\left(\frac{D_E}{L_E}n_{E0} + \frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)(e^{qV_{EB}/kT}-1) - \left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)(e^{qV_{CB}/kT}-1)\right]$$

$$I_C = qA\left[\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)(e^{qV_{EB}/kT}-1) - \left(\frac{D_C}{L_C}n_{C0} + \frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)(e^{qV_{CB}/kT}-1)\right]$$

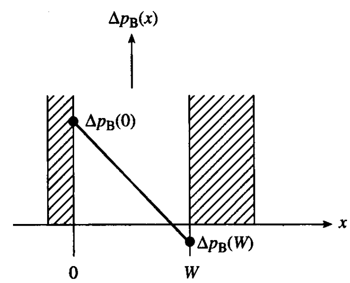
Simplification

- In real BJTs, we make $W << L_B$ for high gain. Then, since

$$\sinh(\xi) \to \xi \quad \text{for } \xi << 1$$
$$\cosh(\xi) \to 1 + \tfrac{\xi^2}{2} \quad \text{for } \xi << 1$$

we have:

$$\Delta p_B(x) \cong p_{B0}(e^{qV_{EB}/kT} - 1)\left(1 - \tfrac{x}{W}\right)$$
$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\tfrac{x}{W}\right)$$



Performance Parameters (Active Mode)

$$\gamma = \frac{1}{1 + \frac{n_{iE}^2}{n_{iB}^2}\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$$

$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\alpha_{dc} = \frac{1}{1 + \frac{n_{iE}^2}{n_{iB}^2}\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\beta_{dc} = \frac{1}{\frac{n_{iE}^2}{n_{iB}^2}\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

Assumptions:
- emitter junction forward biased, collector junction reverse biased
- $W << L_B$