Due at 1700, Fri. Feb. 6 in homework box under stairs, first floor Cory .

Note: up to 2 students may turn in a single writeup. Reading Nise 3, 4-4.5.

1. (20 pts) State Space (Nise 2.4, 3.4, 3.5)

a. Find the transfer function relating input $v_i(t)$ to output $i_3(t)$ for the circuit in Fig. 1.

b. Write the state space equations for this system in phase-variable form and find A, B, C, D.

2. (25 pts) State Space (Nise 2.6, 2.7, 3.4, 3.5)

Use the following values for the parameters: $\frac{N_1}{N_2} = 0.5$, $\frac{N_3}{N_4} = 0.5$, $J_a = 1\, kg{-}m^2$, $J_1 = 1\, kg{-}m^2$, $J_2 = 1\, kg{-}m^2$, $J_3 = 1\, kg{-}m^2$, $J_4 = 1\, kg{-}m^2$, $J_L = 3\, kg{-}m^2$ and $K = 4\, N{-}m$, $D = 2\, N{-}m{-}s$, $D_L = 4\, N{-}m{-}s$

a. Find the transfer function relating input $T(t)$ to output $\theta_1(t)$ for the rotary mechanical system in Fig. 2.

b. Write the state space equations for this system in phase-variable form and find A, B, C, D.

c. Draw the equivalent block diagram of the system in phase variable form using integrator, scale, and summing blocks.

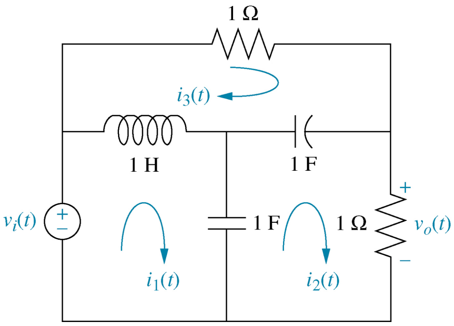

Fig. 1

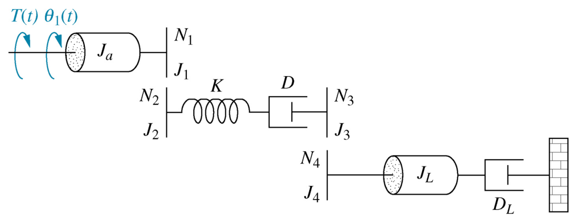

Fig. 2

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3. (20 pts) Transfer function from state space (Nise 3.6)

Find the transfer function $Y(s)/U(s)$ for the following systems:

a.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u(t) \qquad \text{and} \quad y = [1 \quad 0 \quad 0]\mathbf{x}$$

b.

$$\dot{\mathbf{x}} = A\mathbf{x}+Bu = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -6 & -12 & -9 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \qquad \text{and} \quad y = [-3 \quad -9 \quad -7] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u$$

4. (20 pts) Linearization (Nise 3.7)

For the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \beta(x_3 + kx_1) \\ \frac{-\alpha}{x_4^2} u \\ \beta x_1^2 \end{bmatrix} \tag{1}$$

Linearize the system about $x_1 = x_2 = x_3 = 1, x_4 = 2$, $u = 0$, and express in state space form: $\dot{\delta}\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u$. ($\beta, k$ and α are constants.)

5. (15 pts) Second order systems (Nise 4.5)

For each of the following transfer functions $H(s)$, find and plot the pole-zero diagram on the s-plane, then write an expression for the general form of the step response without explicitly solving for the inverse Laplace transform. Approximately sketch the general shape step response.

a. $H(s) = \frac{s+2}{(s+1)(s+4)}$

b. $H(s) = \frac{101}{s^2+2s+101}$

c. $H(s) = \frac{10}{(s+1)(s+10)}$