

Notes 16 largely plagiarized by %khc

1 Phase Modulation/Frequency Modulation

Before, we saw that, in amplitude modulation, the message was carried by the amplitude of the underlying sinusoidal waveform.

In exponential modulation, the modulated signal $x(t)$ is of the form:

$$x(t) = A \cos(\omega_c t + \theta(t))$$

and the message is carried not by the amplitude of the cosine, but by the phase of the cosine. Specifically,

$$\theta(t) = \begin{cases} \phi_\Delta m(t) & \text{in phase modulation (PM)} \\ \omega_\Delta \int_{-\infty}^t m(\tau) d\tau & \text{in frequency modulation (FM)} \end{cases}$$

Narrowband In narrowband exponential modulation, we assume that $\theta(t) \ll 1$. Then

$$\begin{aligned} x(t) &= A \cos(\omega_c t + \theta(t)) \\ &= \mathcal{R}e [A e^{j(\omega_c t + \theta(t))}] \\ &= \mathcal{R}e [A e^{j\omega_c t} e^{j\theta(t)}] \\ &\approx \mathcal{R}e [A e^{j\omega_c t} (1 + j\theta(t))] \\ &= \mathcal{R}e [A (\cos \omega_c t + j \sin \omega_c t) (1 + j\theta(t))] \\ &= A \cos \omega_c t - A\theta(t) \sin \omega_c t \end{aligned}$$

where we have used the small angle approximation for e^x . So

$$x(t) = \begin{cases} A \cos \omega_c t - A\phi_\Delta m(t) \sin \omega_c t & \text{in PM} \\ A \cos \omega_c t - A\omega_\Delta [\int_{-\infty}^t m(\tau) d\tau] \sin \omega_c t & \text{in FM} \end{cases}$$

The frequency domain representation of $x(t)$ is readily sketchable in the PM case. Note that the resulting spectrum is similar to that of an AM-DSB modulated signal.

Exercise Given that:

$$M(\omega) = \begin{cases} 1 - \frac{1}{\omega_m} |\omega| & \text{for } 0 < \omega < \omega_m \text{ and } -\omega_m < \omega < 0 \\ 0 & \text{otherwise} \end{cases}$$

Sketch $X(\omega)$ for NBPM and compare it to the spectrum of $m(t) \cos \omega_c t$.

For FM, we need to perform the integration of $m(t)$ first before multiplying by the sine. In the frequency domain, this is equivalent to multiplying $M(\omega)$ by $\frac{1}{j\omega} + \pi\delta(\omega)$, and then convolving the resulting mess by the transform of $\sin \omega_c t$.

Exercise Sketch the frequency domain representation of NBFM for the $M(\omega)$ above.

If the message has bandwidth ω_m , both narrowband PM and FM consume as much bandwidth as AM-DSB schemes; that is, $2\omega_m$.

Exercise Why must $m(t)$ not contain any DC component in the case of NBFM?

Wideband Without the restriction $\theta(t) \ll 1$, for arbitrary messages, wideband PM and FM become disasters to analyze in the frequency domain. However, in certain special cases, we can still make some useful comment about the frequency domain representation.

One of these special cases is tone modulation (using a sinusoid as the message), as discussed in the ee120L wideband FM handout, and consequently not covered here.

Another of these special cases is square wave modulation. Let's consider sending an even square wave with period T , 50% duty cycle, and ± 1 voltage as $m(t)$. In PM, this corresponds to:

$$x(t) = \begin{cases} \cos(\omega_c t + \phi_\Delta) & \text{for the positive-going portion of } m(t) \\ \cos(\omega_c t - \phi_\Delta) & \text{for the negative-going portion of } m(t) \end{cases}$$

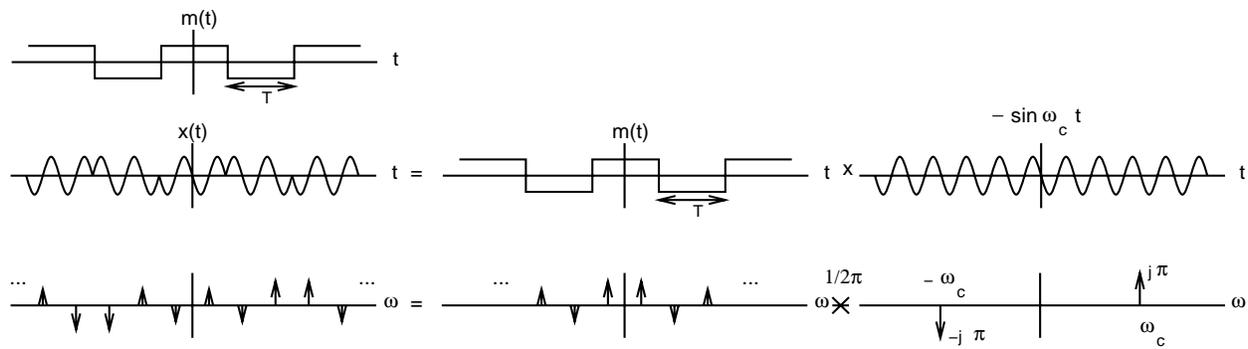


Figure 1: Frequency analysis of PM square wave modulation.

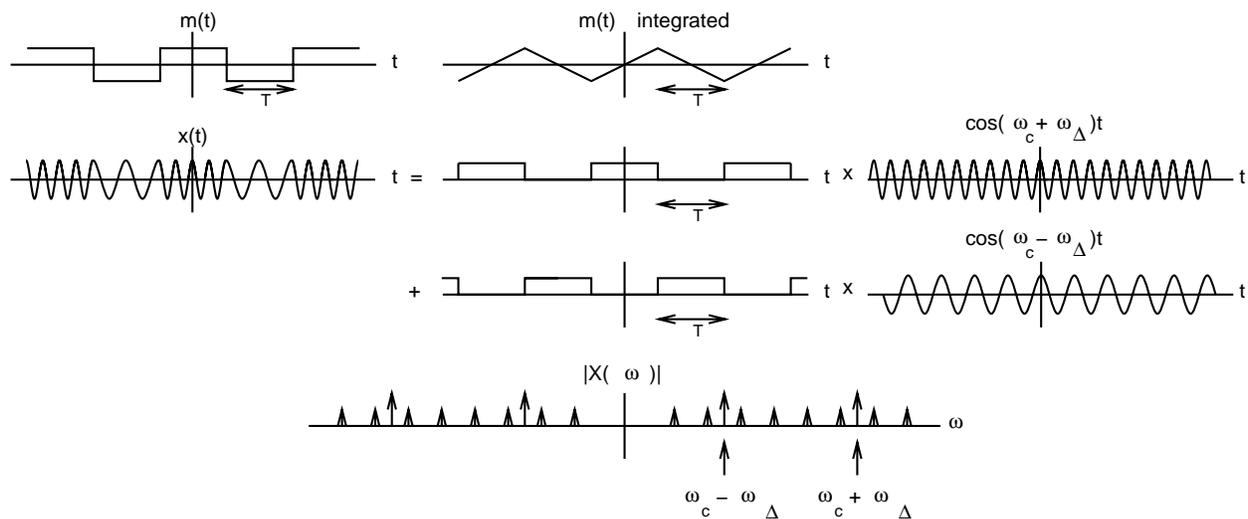


Figure 2: Frequency analysis of FM square wave modulation.

For a ϕ_Δ of $\frac{\pi}{2}$, this becomes:

$$x(t) = \begin{cases} -\sin \omega_c t & \text{for the positive-going portion of } m(t) \\ \sin \omega_c t & \text{for the negative-going portion of } m(t) \end{cases}$$

In the time domain, this looks like $-\sin \omega_c t$ multiplied by $m(t)$, so in the frequency domain we convolve the two impulses from the transform of the sine with the sinc-shaped train of impulses from the transform of $m(t)$, as illustrated in Figure 1.

In FM, using the same $m(t)$ as above, we notice that $m(t)$ gets integrated into a triangle wave. Thus,

$$x(t) = \begin{cases} \cos(\omega_c t + \omega_\Delta t) & \text{for the positive-going portion of } m(t) \\ \cos(\omega_c t - \omega_\Delta t) & \text{for the negative-going portion of } m(t) \end{cases}$$

It looks messy, but we can simplify the analysis by noting that $x(t)$ is the sum of: the product of an even square wave and a cosine at a higher frequency, and the product of a shifted square wave and a cosine at a lower frequency, as seen in the upper half of Figure 2. This has the magnitude spectrum $|X(\omega)|$ in the lower half of the same figure.

Exercise Verify this.

Exercise Also not covered here is Carson's rule; it's on the back of the wideband FM handout. This might come in handy on an exam when it comes to determining bandwidth...

2 Digital Formats: CW, FSK, PSK

Let's assume that you want to transmit a sequence of bits from one place to another. If you wish to transmit a bit every T seconds, one way you could do it is to:

- send a portion of a cosine at some carrier ω_c for T seconds and let it represent a "1".
- send nothing for T seconds and let it represent a "0".

This is called continuous wave (CW).

Analyzing this in the frequency domain not only builds character, but can be used for fun and profit. Let's consider sending the bit pattern $\dots 1010 \dots$. In the time domain, we can think of this as multiplying $\cos \omega_c t$ by a square wave with period $2T$, 50% duty cycle, and 0-1 peak value. In the frequency domain, we have two impulses convolved with a sampled sinc, giving two copies of the sampled sinc. The final spectrum looks quite a lot like that in Figure 1, except that there is an impulse at both ω_c and $-\omega_c$, and each of the impulses there should be divided by a factor of two.

Exercise Why?

Another way to send a sequence of bits is to:

- send a portion of a cosine at some carrier ω_c for T seconds and let it represent a "1".
- send a portion of a cosine at another carrier $2\omega_c$ for T seconds and let it represent a "0".

This is called frequency shift keying (FSK).

This can be analyzed in a fashion similar to that of CW. Let's consider sending the same bit pattern $\dots 1010 \dots$. In the time domain, we send two different frequencies... Hey! We already did an analysis similar to this when we looked at FM square wave modulation.

Exercise Convince yourself that this is true. In fact, convince yourself that the diagrams we got in Figure 2 are applicable, and that the final spectrum is similar to that in the figure.

A third way to send a sequence of bits is to:

- send a portion of a cosine at some carrier ω_c for T seconds and let it represent a "1".
- send a portion of a cosine at the same carrier ω_c , but phase shifted by some convenient amount, for T seconds and let it represent a "0".

This is called phase shift keying (PSK).

Let's phase shift the waveform for "0" by π and reconsider the bit pattern $\dots 1010 \dots$. This is the same as multiplying our $\cos \omega_c t$ by a square wave with period $2T$, 50% duty cycle, and ± 1 peak voltage...

Exercise By now, it shouldn't surprise you that i rigged all the examples so that i only had to draw two sets of figures. It's sort of late at night right now... Anyway, complete this analysis. Your final spectrum should have the same shape as that in Figure 1.

3 AM Receivers

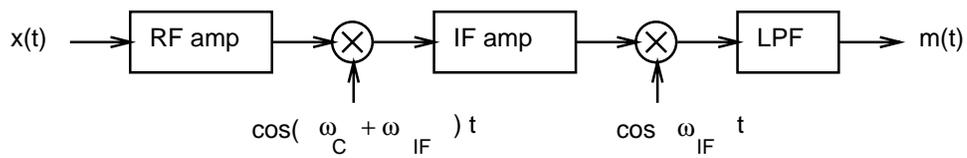
If we only had one radio station to listen to, life would be much simpler. We wouldn't have to design a sharp tunable filter. Unfortunately, life would be extremely boring and/or painful, especially if you loved classical music and all there was was the heavy metal station.

So we need a sharp tunable filter. As the saying goes: good, fast, and cheap; pick two. One way to get around this is to use a tunable RF amplifier (or image reject filter; the RF stands for radio frequency), demodulate down to an intermediate frequency (IF) by multiplying by $\cos(\omega_c + \omega_{IF})t$, pass everything through a sharp IF amplifier (bandpass filter), and then drop the results into your favorite demodulator scheme (asynchronous or synchronous), all as in Figure 3. Since you're asked to do this on problem set 8, i'll let you work it out.

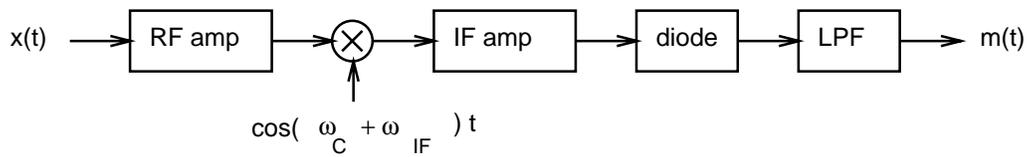
Exercise Verify that the setup above also works when $\cos(\omega_c - \omega_{IF})t$ is used to demodulate down to the intermediate frequency.

4 A Look Ahead

We've covered Fourier transforms and modulation. Now we tackle the Laplace transform and accompanying linear systems analysis. Since these are the three main topics on the midterm, you might want to take a few minutes out to review the first two subjects.



(a) AM superheterodyne receiver with synchronous demodulator



(b) AM superheterodyne receiver with asynchronous demodulator

Figure 3: AM receivers.