Notes 10 largely plagiarized by %khc

1 Some Ideal Systems

Four major ideal systems that we will encounter all over the place are the ideal wire, the ideal differentiator, the ideal integrator, and the ideal delay. Their impulse and frequency responses are summarized in Figure 1.

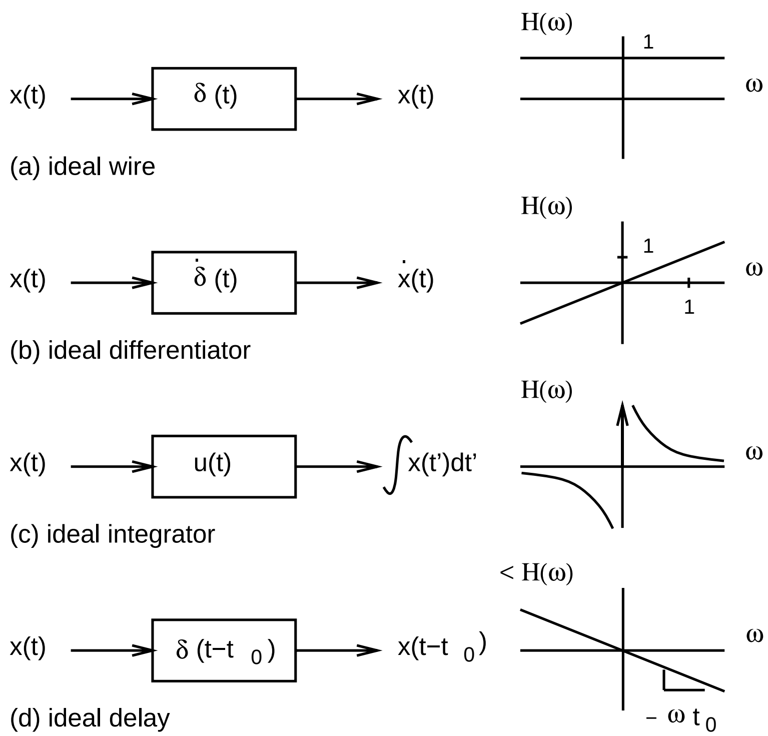

Figure 1: Some ideal systems and their frequency responses.

Exercise Verify the impulse and frequency responses.

2 Filtering

There are two ways of looking at filters: in the time domain and in the frequency domain.

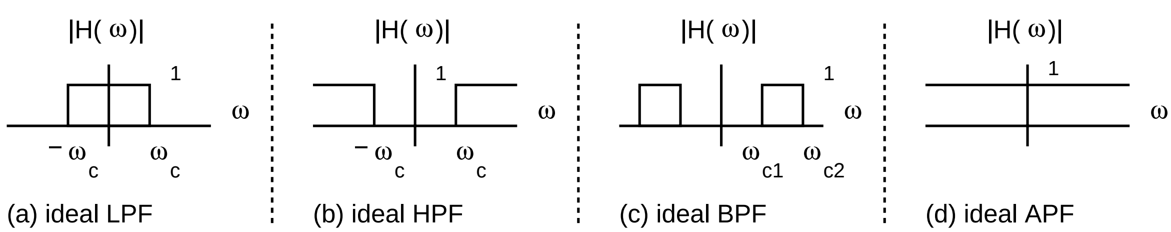

Figure 2: Magnitude responses of some ideal filters.

In the frequency domain, filters can be thought of as selecting frequencies of interest. It is useful to consider the magnitudes of the frequency response of various ideal filters.

- Ideal low pass filter (LPF) An LPF passes frequencies up to some cutoff frequency ω_c. Sanity checks:
  Does the LPF pass DC? Yes, since $H(\omega)$ is nonzero at $\omega = 0$.
  Does the LPF reject high frequencies (defined as $\omega > \omega_c$ for this filter)? Yes, since $H(\omega)$ is zero for $\omega > \omega_c$.
- Ideal high pass filter (HPF) An HPF passes freqencies above some cutoff frequency ω_c. Sanity checks:
  Does the HPF reject DC? Yes, since $H(\omega)$ is zero at $\omega = 0$. Does the HPF pass high frequencies (defined as $\omega > \omega_c$ for this filter)? Yes, since $H(\omega)$ is nonzero for $\omega > \omega_c$.
- Ideal band pass filter (BPF) A BPF passes frequencies between ω_{c1} and ω_{c2}. Sanity checks for this filter can be performed in the privacy of your own room.
- Ideal all pass filter (APF) An APF passes all frequencies. This looks pointless, but it is sometimes used to delay certain frequencies with respect to other frequencies, so its phase is usually an interesting function.[1] In other contexts, the APF is used to help construct invertible systems. More on this after we study the Laplace transform.

Why are these frequency responses ideal? Because the transition between passband and stopband is quite sharp. In real life™, this transition can only be achieved by increasing filter order (ie spending more $), and you will end up with ripples in the passband and stopband (but ripples of lower amplitude with increasing filter order). Also, the filters are not causal. [You are asked to prove this on ps4, problem 10. Have fun.] One more problem is that the impulse response of each of these filters is of infinite extent. This is going to require some truncation, because really long impulse responses mean really long convolutions that we'll have to perform.

So how do we get these filters then? We'll take a filter frequency response and find the time function to which it corresponds. This will be the filter's impulse response. We'll then need to truncate and delay the filter's impulse response. The appropriate delay combined with the correct truncation makes the filter causal. However, the truncation is going to introduce ripple in the frequency response. We have to sit down and decide whether or not we can live with this ripple. More on this later after we have delved more deeply into the Fourier transform.

We have also been ignoring the phase response. Ideally, we would love to have zero phase. Zero phase corresponds to zero delay in the filter, but we usually don't get to perform anything instantaneously.

We also need to have causal filters, which are realizable in real time. This means that we will have to delay the impulse response by some time T, which corresponds to multiplying the frequency response by e^{-jwT}. If our filter has the $|H(\omega)|$ of, say, the LPF, and has time delay T, then our frequency response would have phase $-\omega T$.

Nonlinear phase delay is interesting. Quoting from OWY, first edition, page 229: "if the phase shift is a nonlinear function of w, then each complex exponential will be shifted in a manner that results in a change of the relative phase. When these exponentials are superposed, we obtain a signal that may look considerably different..." The figure on page 227 illustrates this (and i will not attempt to draw it here). Besides, you should really get around to opening up your book at some point in time before the midterm.

3 An Interesting Result

In Figure 3, we have an interesting system. In fact, i claim that it's one way to get an HPF from an LPF.

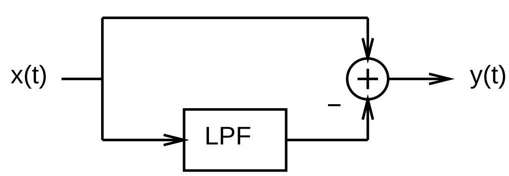

Figure 3: How to make an HPF from an LPF.

Why does it work? Let's assume that we have an ideal LPF that has amplitude 1 over the passband and cutoff frequency w_c. Let's also assume that we have an ideal HPF that has amplitude 1 over the passband and cutoff frequency

[1] The APF magnitude response closely approximated the walls of my room when i stayed at the International House. Story available upon request.

w_c. So

$$\begin{aligned} |H_{lpf}(\omega)| + |H_{hpf}(\omega)| &= 1 \\ |H_{hpf}(\omega)| &= 1 - |H_{lpf}(\omega)| \end{aligned}$$

This last equation is realized by the system in Figure 3. Neat, huh?[2]

Another way of looking at this problem is just to notice that the output of the low pass filter is going to have just the low frequency portion of the input $x(t)$, whereas the wire is carrying the entire frequency content of $x(t)$. If we subtract the low frequency portion of $x(t)$ from $x(t)$ itself, all we have left over is the high frequency content of $x(t)$, which is our output $y(t)$.

4 A Look Behind

The much-promised Fourier transform has arrived. But you should also be familiar with:

- system properties: linearity, time-invariance, causal, memoryless.
- derivation of the superposition integral for both L and LTI systems.
- $e^{j\omega t}$ is an eigenfunction for all LTI systems.
- sinusoidal steady-state response of real-world LTI systems.
- Fourier series analysis and synthesis.
- response of systems to Fourier series inputs.
- how to use the FS properties to avoid calculating that painful FS analysis integral.

[2] i really need to get out more often.