

0. (0 pts) Warmup exercises
 OW problems 7.12, 7.15, 7.19 (answers in back of book). These do not need to be turned in.

1. (20 pts) Sampling and Reconstruction (OW 7.1)

In many cases of conversion from discrete to continuous time, we considered an ideal low-pass filter for reconstruction. In practice, a digital-to-analog converter, which can be modelled as a zero-order hold, is followed by a low-complexity analog filter. Consider the system shown in Fig. 1, where $x(t) = \cos(200\pi t)$, $T_s = 0.001$ sec, $T = T_s$, and $h(t) = 200\pi e^{-200\pi t}u(t)$.

- Sketch $x(t)$, $x_\delta(t)$, $x_z(t)$, $x_r(t)$ (for $0 \leq t \leq 10T_s$) and associated spectra. ($x_r(t)$ will be an approximate sketch.)
- Numerically estimate the fraction of the power in $x_r(t)$ which is not at the original signal frequency of 100 Hz.

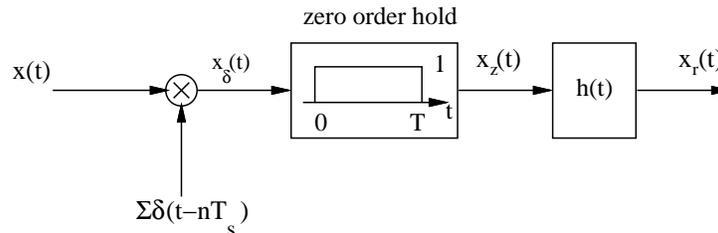


Fig. 1. Sampling and zero order hold.

2. (20 pts) Digital Differentiator (OW 7.4)

Let $x(t)$ be a band-limited signal such that $X(j\omega) = 0$ for $|\omega| \geq \frac{\pi}{T}$.

- If $x(t)$ is sampled using sampling period T , determine an interpolating function $g(t)$ such that:

$$\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} x(nT)g(t - nT)$$

- Is the function $g(t)$ unique?

3. (10 pts) DFT (DFT H.O. Lec 11,12, 13)

This problem considers digital interpolation using upsampling and a digital low pass filter. Consider a 1 KHz cosine sampled at 16 KHz for 16 samples.

- Determine $X[k]$ the DFT of $x[n]$. What is the spacing of the samples in the frequency domain? For example, what frequency does $k=4$ correspond to?
- Create a new sequence $y[n]$ of length 32 by upsampling, and sketch $y[n]$. That is $y[n] = x[n/2]$ for n even and $y[n] = 0$ for n odd. Determine $Y[k]$ the 32 point DFT of $y[n]$. What is the new spacing of samples in the frequency domain?

4. (20 pts) Laplace Transform Lec 16, OW 9.1-9.2

For each signal below calculate its bilateral Laplace Transform $X(s)$ and region of convergence, and determine whether the $j\omega$ axis is in the region of convergence.

- i) $x(t) = \Pi(t+1)$ ii) $x(t) = e^{-|t|}$ iii) $x(t) = e^{2t}\cos(4\pi t)u(t)$
 iv) $x(t) = e^{5t}u(t)$ v) $x(t) = e^{5t}u(-t)$

5. (20 pts) Laplace Transform Lec 17, OW 9.3, 9.5-9.7

For each part below, use Laplace transforms and LTI properties to find the output $y(t)$ for an LTI system with impulse response $h(t)$ and input $x(t)$.

- i. $x(t) = e^{-2t}u(t)$, $h(t) = u(t)$.
 ii. $x(t) = e^{-2t}u(t)$, $h(t) = e^{-3t}u(t)$.
 iii. $x(t) = u(t)$, $h(t) = \cos(2\pi t)u(t)$
 iv. $x(t) = \cos(2\pi t)u(t)$, $h(t) = \delta(t-1) + \delta(t-0.5)$
 v. $x(t) = \cos(2\pi t)u(t)$, $h(t) = \cos(2\pi t)u(t)$.

6. (10 pts) Laplace Transform OW 9.7

A system with input $x(t)$ and output $y(t)$ is described by the LDE:

$$\frac{d^2y(t)}{dt^2} + 300\frac{dy(t)}{dt} + 2 \times 10^4y(t) = 10^3x(t) \quad (1)$$

Use the Laplace transform to solve for $y(t)$ with input $x(t) = u(t)$, and initial conditions $y(0^-) = 1$ and $\dot{y}(0^-) = -2$.