Section 9

Wednesday, July 24

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. The following are variants of the famous "boy or girl paradox" / "two child problem".

   1a. Mr. Smith has two children, at least one of whom is a boy. What is the probability that both children are boys?

   1b. Mr. Smith has two children, one of whom is a boy born on a Tuesday. What is the probability that both children are boys?

   Note: For both parts, assume that the probability of a boy or girl being born is the same, a child is equally likely to be born on any day of the week, and the genders of all children are independent of each other and independent of the day of the week.

2. Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year, so the prior probability of rain is just 5/365. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 5% of the time. What is the probability that it will rain on the day of Marie's wedding?

3. A doctor assumes that a patient has one of three diseases d_1, d_2, or d_3. Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has d_1, 0.6 if the patient has disease d_2, and 0.4 if the patient has disease d_3. Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?

4. Prove that in every probability space, if A and B are independent events, then $\overline{A}$ and $\overline{B}$ are also independent.

   Note: The most general version of this theorem states that if $A_1, \ldots, A_n$ are a fully independent collection of events, then you can take any subcollection and replace them by their complements, and it will still be fully independent. Intuitively, this should not come as a surprise, and you're free to use this fact throughout the course. It can be proven using the inclusion-exclusion principle, though we do not ask you to do that.