Section 4

Monday, July 8

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. Some more practice problems.

   1a. Find the last digit of $9^{2938} - 5^{8460}$.

   1b. Show that if a is an odd natural number, then $a^2 \equiv 1 \pmod 8$.

2. Books are identified by an International Standard Book Number (ISBN), a 10-digit code $x_1 x_2 \cdots x_{10}$ assigned by the publisher. The first 9 digits contain information about the book, and the last digit is a "check digit" that is either a digit or the letter X (used to represent the number 10). This check digit is selected so that $\sum_{i=1}^{10} i \cdot x_i \equiv 0 \pmod{11}$.

   2a. The ISBN for a certain book starts with 702004001_. What is the last digit?

   2b. Wikipedia says that you can get the check digit by computing $(\sum_{i=1}^{9} i \cdot x_i) \bmod 11$. Show that Wikipedia's description is equivalent to the one above.

   2c. Prove that changing any single digit of the ISBN will render the ISBN invalid. That is, the check digit allows you to detect a single-digit substitution error.

   2d. Can you switch any two digits in an ISBN and still have it be a valid ISBN? (E.g., could both 01**2**34**5**678X and 01**5**34**2**678X both be valid ISBNs?)

   2e. Canada decides to change its ISBN system by doing the check digit computation modulo 12 rather than modulo 11 (if the check digit needs to be 11, they'll use "Y"). That is, the digits now have to satisfy $\sum_{i=1}^{10} i \cdot x_i \equiv 0 \pmod{12}$. Shall we blame Canada for reducing the error-detecting capabilities of the check digit?

3. Solve the following system of equations:

$$5x \equiv 8y \pmod{13}$$

$$x \equiv 9y - 11 \pmod{13}$$

4. Prove that $GCD(7n + 4, 5n + 3) = 1$ for all $n \in \mathbb{N}$.