Section 3

Wednesday, July 3

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. In a group of n men and n women, Bob, one of the men, gets tipped off that he is the second-highest preference on every woman's list. Bob is pretty happy to hear this. Assuming we use the male-optimal algorithm, we can guarantee that at worst he will be matched to the kth highest woman on his list for some $k \leq n$. What is k? Give a bad example where Bob is indeed matched to the kth woman on his list.

2. Suppose that preferences in a stable marriage instance are universal: all n men share the preferences $W_1 > W_2 > \cdots > W_n$ and all women share the preferences $M_1 > M_2 > \cdots > M_n$.

   2a. What result do we get from running the algorithm with men proposing?

   2b. What result do we get from running the algorithm with women proposing?

   2c. What does this tell us about the number of stable matchings?

3. This problem is about GCDs.

   3a. Find $GCD(120, 111)$ using prime factorizations.

   3b. Find $GCD(120, 111)$ using the Euclidean algorithm.

   3c. Find integers a, b such that $GCD(120, 111) = a \cdot 120 + b \cdot 111$.

   3d. Find $GCD(240, 222)$. (Hint: Is there a shortcut, using what you've already computed?)

4. (Optional challenge) There are n boys and n girls at a middle school dance. Each boy has a short list of the girls that he would like to dance with, in order of preference, and he regards the girls that are not on his list as unacceptable. Similarly for the girls. Obviously, there might not be any stable pairings involving everyone (for example, if everyone regards you as unacceptable, you will never have a partner). Given a pairing P, we define a rogue couple (M, W) to be one in which all of the following conditions are satisfied:

   - M is not paired with W in P,
   - M and W find each other acceptable,
   - M is either unpaired in P, or prefers W to his partner in P,
   - W is either unpaired in P, or prefers M to her partner in P.

   A pairing P is called *stable* if there are no rogue couples. Prove that the middle schoolers can be partitioned into 2 sets: the *cool kids* who have partners in every stable pairing, and the *late-bloomers* who are unpaired in every stable pairing.