

# Section 15

Wednesday, August 14

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. For each of the following functions from  $\mathbb{R}$  to  $\mathbb{R}$ , determine whether it is an injection, surjection, bijection, none of the above, or more than one of the above.
  - 1a.  $f(x) = 2^x$
  - 1b.  $f(x) = x^2$
  - 1c.  $f(x) = 2x + 1$
  
2. For each of the following sets, decide whether it is countable or uncountable, and justify your answer.
  - 2a. The set of all prime numbers
  - 2b. The set of all finite sequences of integers
  - 2c.  $P(\{1, 2, 3\})$
  - 2d. The set of all real numbers in the range  $[0, 0.1]$
  - 2e. The set of all real numbers that are roots of polynomials with natural number coefficients
  
3. Prove that if  $A$  is uncountable and  $B$  is a countable subset of  $A$ , then  $A - B$  is uncountable.
  
4. Prove that the following problem is not computable: Given a program (in your favorite programming language) and an input to the program, will it cause a runtime error? (Hint: don't reinvent the wheel.)