Section 12

Monday, August 5

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. Compute the expectation and variance of a random variable X which is exactly 10 with probability 1/2, and otherwise is uniformly distributed on the values $\{1, 2, 3\}$.

2. A spider is expecting guests and wants to catch 500 flies for her dinner. Exactly 100 flies pass by her web every hour. Exactly 60 of these flies are quite small and are caught with probability $\frac{1}{6}$ each. Exactly 40 of the flies are big and are caught with probability $\frac{3}{4}$ each. Assume all fly interceptions are mutually independent. We are trying to find an upper bound on the probability that the spider catches at least 500 flies in 10 hours.

   2a. Markov's Inequality states that for a nonnegative random variable X with mean $\mathrm{E}(X) = \mu$, and any $\alpha > 0$, $\Pr[X \geq \alpha] \leq \frac{\mu}{\alpha}$. What would the Markov bound be on the probability that the spider will catch her quota of 500 flies?

   2b. What would the Chebyshev bound be on the probability that the spider will catch her quota of 500 flies?

3. Let's explore Markov's Inequality.

   3a. Give an example to show that Markov's Inequality becomes false if we remove the non-negativity assumption.

   3b. Give an example to show that Markov's Inequality is tight; i.e., for some nonnegative random variable X and some $\alpha > 0$, $\Pr[X \geq \alpha] = \frac{\mathrm{E}(X)}{\alpha}$.

4. The variance of Geom(p) is $\frac{1-p}{p^2}$ (you may take this for granted). Give an exact expression for the variance of the coupon collecting distribution (the number of boxes you buy until you have at least one of each of the n types of coupons). You do not need to simplify your expression.