

# Section 1

Wednesday, June 26

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

Feel free to introduce yourself to your neighbors and discuss these problems together.

1. Logic is fun!

- 1a. Three logicians walk into a bar. The bartender asks, “Does everybody want a beer?” The first one says “I don’t know,” the second one says “I don’t know,” and the third one says “Yes.” What is going on? (Note: The phrasing of the bartender is very important.)
- 1b. You are on an island inhabited by two types of people: the Liars and the Truth-tellers. Liars always lie, and Truth-tellers always tell the truth. In all other respects, the two types are indistinguishable. You meet a very attractive local and ask him/her on a date. The local responds, “I will go on a date with you if and only if I am a Truth-teller.” Is this good news?

2. Write a proposition in logic notation expressing that “there is one and only one  $x$  such that  $P(x)$  holds.”

3. Prove or disprove each of the following:

- 3a.  $\forall x(P(x) \wedge Q(x)) \equiv \forall xP(x) \wedge \forall xQ(x)$
- 3b.  $\forall x(P(x) \vee Q(x)) \equiv \forall xP(x) \vee \forall xQ(x)$
- 3c.  $\forall x(P(x) \Rightarrow Q(x)) \equiv (\forall xP(x)) \Rightarrow (\forall xQ(x))$
- 3d.  $\exists x(P(x) \wedge Q(x)) \equiv \exists xP(x) \wedge \exists xQ(x)$
- 3e.  $\exists x(P(x) \vee Q(x)) \equiv \exists xP(x) \vee \exists xQ(x)$
- 3f.  $\exists x(P(x) \Rightarrow Q(x)) \equiv (\exists xP(x)) \Rightarrow (\exists xQ(x))$

4. A valid tiling for a chessboard is an arrangement of tiles such that no two tiles overlap and every square of the board is covered by a tile.

- 4a. A domino is a tile consisting of two contiguous squares. Is there a valid domino tiling for the  $8 \times 8$  chessboard where the squares in the bottom left and top right corners have been removed?
- 4b. A straight tetromino is a tile consisting of four contiguous squares. Prove or disprove: A  $10 \times 10$  chessboard can be tiled with straight tetrominoes.