Problem Set 8

1. *Sampling* You are interviewing families in a certain district. In order to ascertain the opinion held by a given family you sample two persons from the family. Recognizing that the order matters in which the two persons from one family are interviewed, how many ways can one sample two people from a six person family? If the two people are interviewed simultaneously so that order no longer matters, how many ways can one sample two people from a 6-person family?

2. *Win-loss Record* The Cal soccer team must play nine games during its season. In how many ways can the team end its season with six wins, one loss and two ties? What about three wins, three losses and three ties? Which is larger? Does it agree with your intuition?

3. *Numbers*
   (a) How many odd numbers in [100,9999] have distinct digits?

   (b) How many palindromes (numbers that are the same when you write them backward s) are in this range?

   (Note: You may not brute force this one by writing a program - show your work!)

4. *Mom vs. Girlfriend*
   Every evening a man either visits his mother, who lives downtown, or visits his girlfriend, who lives uptown. In order to be completely fair, he goes to the bus stop every evening at a random time and takes either the uptown or the downtown bus, whichever comes first. As it happens each of the two kinds of buses stop at the bus stop every 15 minutes with perfect regularity (according to a fixed schedule). Yet he visits his mother only around twice each month. How come?

5. *Independence*
   Let Ω be a sample space, and let $A, B \subseteq \Omega$ be two *independent* events. Let $\overline{A} = \Omega - A$ and $\overline{B} = \Omega - B$ (sometimes written $\neg A$ and $\neg B$) denote the complementary events.

   For the purposes of this question, you may use the following definition of independence: Two events A, B are *independent* if $\Pr[A \cap B] = \Pr[A]\Pr[B]$.

   (a) Prove or disprove: $\overline{A}$ and $\overline{B}$ are necessarily independent.
   (b) Prove or disprove: A and $\overline{B}$ are necessarily independent.
   (c) Prove or disprove: A and $\overline{A}$ are necessarily independent.
   (d) Prove or disprove: It is possible that $A = B$.

6. *Burnt pancakes*
   I have a bag containing three pancakes: One golden on both sides, one burnt on both sides, and one golden on one side and burnt on the other. You shake the bag, draw a pancake at random, look at one side, and notice that it is burnt. What is the probability that the other side is burnt? Show your work.

7. *Monty Hall revisited*

   In this variant of the Monty Hall problem, after the contestant has chosen a door, Monty asks another contestant to open one of the other two doors. That contestant, who also has no idea where the prize is, opens one of those two remaining doors at random, and (as it happens) you both see that there is no prize there. Monty now asks you if you wish to switch or stick with your original choice. What is your best strategy? Why?