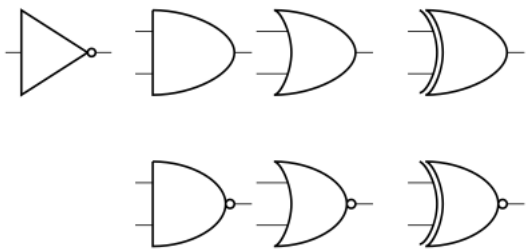
# CS61c Summer 2014 Discussion 10 – Synchronous Digital Systems and Boolean Algebra

July 23, 2014

## 1 Logic Gates

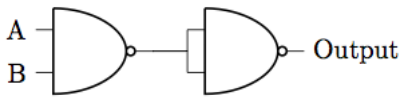
1. Label the following logic gates:



not, and, or, xor, nand, nor, xnor

2. Convert the following to boolean expressions:
   (a) NAND $\bar{A}\bar{B} + \bar{A}B + A\bar{B}$
   (b) XOR $\bar{A}B + A\bar{B}$
   (c) XNOR $\bar{A}\bar{B} + AB$

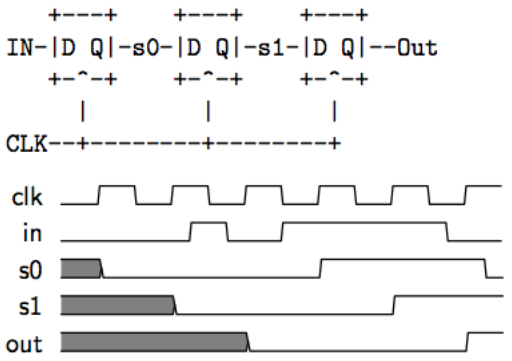
3. Create an AND gate using only NAND gates.



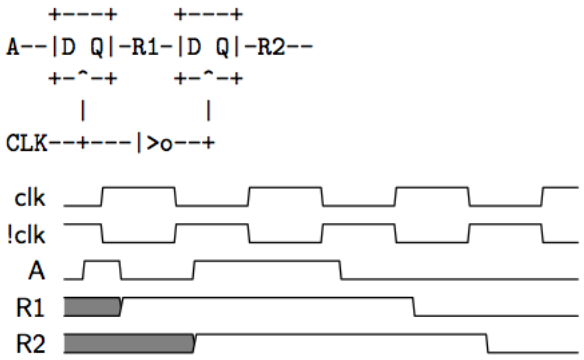

4. How many different two-input logic gates can there be? How many n-input logic gates?
   A truth table with $n$ inputs has $2^n$ rows. Each logic gate has a 0 or a 1 at each of these rows. Imagining a function as a $2^n$-bit number, we count $2^{2^n}$ total functions, 04 16 in the case of $n = 2$

## 2 State

1. Fill out the timing diagram for the circuit below:




2. Fill out the timing diagram for the circuit below:




## 3 Boolean Logic

- $A + \bar{A} = 1$
- $1 + A = 1$
- $A + AB = A$
- $0B = 0$
- $B\bar{B} = 0$
- $A + \bar{A}B = A + B$
- $(A + B)(A + C) = A + BC$
- $\overline{\text{AB}} = \bar{A} + \bar{B}$
- $\overline{\text{A+B}} = \bar{A}\bar{B}$

1. Minimize the following boolean expressions:

   (a) Standard: $(A + B)(A + \bar{B})C$

   $$(AA + A\bar{B} + AB + B\bar{B})C = (A + A(\bar{B} + B))C = AC$$

   (b) Grouping & Extra Terms: $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C$

   $$\begin{aligned} \bar{A}\bar{C}(\bar{B} + B) + A\bar{C}(\bar{B} + B) + AC(\bar{B} + B) &= \bar{A}\bar{C} + A\bar{C} + AC \\ &= \bar{A}\bar{C} + A\bar{C} + A\bar{C} + AC \\ &= (\bar{A} + A)C + A(\bar{C} + C) \\ &= A + \bar{C} \end{aligned}$$

(c) DeMorgan's: $\overline{\mathrm{A}(\bar{B}\bar{C} + BC)}$

$$\begin{aligned}
\overline{\mathrm{A}(\bar{B}\bar{C} + BC)} &= \bar{A} + \overline{\bar{B}\bar{C} + BC} \\
&= \bar{A} + \overline{\bar{B}\bar{C}}\,\overline{BC} \\
&= \bar{A} + (B + C)(\bar{B} + \bar{C}) \\
&= \bar{A} + B\bar{C} + \bar{B}C
\end{aligned}$$