CS61B Lecture #31

Today:

- More balanced search structures (DS(IJ), Chapter 9)
Really Efficient Use of Keys: the Trie

• We haven’t said much about the cost of comparisons, generally treating the cost as constant.

• For strings, the worst case is the length of string.

• Therefore we should throw an extra factor of the key length, $L$, into costs:
  - $\Theta(M)$ comparisons really means $\Theta(ML)$ operations.
  - So to look for key $X$, we keep looking at same chars of $X$ $M$ times.

• Can we do better? Can we get search cost to be $O(L)$?

**Idea:** Make a *multi-way decision tree*, with one decision per character of key.
The Trie: Example

- Set of keys
  \{a, abase, abash, abate, abbas, axolotl, axe, fabric, facet\}
- Ticked lines show paths followed for “abash” and “fabric”
- Each internal node corresponds to a possible prefix.
- Characters in path to node = that prefix.
Adding Item to a Trie

- Result of adding bat and faceplate.
- New edges ticked.
A Side-Trip: Scrunching

- For speed, obvious implementation for internal nodes is array indexed by character.

- Gives $O(L)$ performance, $L$ length of search key.

- [Looks as if independent of $N$, number of keys. Is there a dependence?]

- **Problem:** arrays are *sparsely populated* by non-null values—waste of space.

**Idea:**  Put the arrays on top of each other!

- Use null (0, empty) entries of one array to hold non-null elements of another.

- Use extra markers to tell which entries belong to which array.
Scrunching Example

**Small example:** (unrelated to Tries on preceding slides)

- Three arrays, each indexed 0..9

```
<table>
<thead>
<tr>
<th>A1:</th>
<th>0 1 2 3 4 5 6 7 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bass  trout  pike</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A2:</th>
<th>0 1 2 3 4 5 6 7 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ghee  milk  oil</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A3:</th>
<th>0 1 2 3 4 5 6 7 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>salt  cumin  mace</td>
</tr>
</tbody>
</table>
```

- Now overlay them, but keep track of the original index of each item:

```
<table>
<thead>
<tr>
<th>Check:</th>
<th>0 1 2 3 4 5 6 7 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A123:</td>
<td>bass  trout  pike  ghee  milk  oil  mace</td>
</tr>
</tbody>
</table>
```

- Starred items are null in uncompressed array
- Index in original array or -1 if null in all arrays

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Scrunching Example (contd.)

A1: bass trout pike
A2: ghee milk oil
A3: salt cumin mace

Index in original array or -1 if null in all arrays

Starred items are null in uncompressed array

/* A2[i] == */ (Check[i + 2] == i) ? A123[i + 2] : null;
/* A3[i] == */ (Check[i + 1] == i) ? A123[i + 1] : null;
Practicum

- The scrunching idea is cute, but
  - Not so good if we want to expand our trie.
  - A bit complicated.
  - Actually more useful for representing large, sparse, fixed tables with many rows and columns.

- Furthermore, number of children in trie tends to drop drastically when one gets a few levels down from the root.

- So in practice, might as well use linked lists to represent set of node’s children...

- ...but use arrays for the first few levels, which are likely to have more children.
Probabilistic Balancing: Skip Lists

- A skip list can be thought of as a kind of n-ary search tree in which we choose to put the keys at “random” heights.

- More often thought of as an ordered list in which one can skip large segments.

- Typical example:

- To search, start at top layer on left, search until next step would overshoot, then go down one layer and repeat.

- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.

- Heights of the nodes were chosen randomly so that there are about 1/2 as many nodes that are > k high as there are that are k high.

- Makes searches fast with high probability.
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![Diagram of a skip list with keys at various heights]

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```
    -∞ 10 20 25 30 40 50 55 60 90 95 100 115 120 125 130 140 150 ∞

⇒
```

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  $$\begin{array}{c}
  \infty & 10 & 20 & 25 & 30 & 40 & 50 & 55 & 60 & 90 & 95 & 100 & 115 & 120 & 125 & 130 & 140 & 150 & \infty \\
  \hline
  3 & & & & & & & & & & & & & & & & & \\
  1 & & & & & & & & & & & & & & & & & \\
  0 & & & & & & & & & & & & & & & & & \\
  \infty & 10 & 20 & 25 & 30 & 40 & 50 & 55 & 60 & 90 & 95 & 100 & 115 & 120 & 125 & 130 & 140 & 150 & \infty \\
  \hline
  \end{array}$$

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• Makes searches fast *with high probability.*
Example: Adding and deleting

- Starting from initial list:

- In any order, we add 126 and 127 (choosing random heights for them), and remove 20 and 40:

- Shaded nodes here have been modified.
Summary

• Balance in search trees allows us to realize $\Theta(\lg N)$ performance.

• B-trees, red-black trees:
  - Give $\Theta(\lg N)$ performance for searches, insertions, deletions.
  - B-trees good for external storage. Large nodes minimize # of I/O operations

• Tries:
  - Give $\Theta(B)$ performance for searches, insertions, and deletions, where $B$ is length of key being processed.
  - But hard to manage space efficiently.

• Interesting idea: scrunched arrays share space.

• Skip lists:
  - Give probable $\Theta(\lg N)$ performance for searches, insertions, deletions
  - Easy to implement.
  - Presented for interesting ideas: probabilistic balance, randomized data structures.
Summary of Collection Abstractions

- **Multiset**: contains, iterator
- **List**: get(n)
- **Set**: Ordered Set (first), Unordered Set
  - **Priority Queue**: Blue: Java has corresponding interface
  - **Sorted Set**: subset
  - **Map**: contains, iterator, get
    - **Unordered Map**: Green: Java has no corresponding interface
    - **Ordered Map**
Data Structures that Implement Abstractions

**Multiset**
- **List**: arrays, linked lists, circular buffers
- **Set**
  - **OrderedSet**
    - *Priority Queue*: heaps
    - *Sorted Set*: binary search trees, red-black trees, B-trees, sorted arrays or linked lists
  - **Unordered Set**: hash table

**Map**
- **Unordered Map**: hash table
- **Ordered Map**: red-black trees, B-trees, sorted arrays or linked lists
Corresponding Classes in Java

**Multiset** (Collection)
- **List**: ArrayList, LinkedList, Stack, ArrayBlockingQueue, ArrayDeque
- **Set**
  - **OrderedSet**
    - **Priority Queue**: PriorityQueue
    - **Sorted Set (SortedSet)**: TreeSet
  - **Unordered Set**: HashSet

**Map**
- **Unordered Map**: HashMap
- **Ordered Map (SortedMap)**: TreeMap