Speeding Up Searching

• Linear search is OK for small data sets, bad for large.

• So linear search would be OK if we could rapidly narrow the search to a few items.

• Suppose that in constant time we could put any item in our data set into a numbered *bucket*, where # buckets stays within a constant factor of # keys.

• Suppose also that buckets contain roughly equal numbers of keys.

• Then search would be constant time in number of comparisons.
Hash functions

• To do this, must have way to convert key to bucket number: a hash function.

"hash /hæʃ/ 2a a mixture; a jumble. b a mess." Concise Oxford Dictionary, eighth edition

• Example:
  - \( N = 200 \) data items.
  - keys are longs, evenly spread over the range \( 0..2^{63} - 1 \).
  - Want to keep maximum search to \( L = 2 \) items.
  - Use hash function \( h(K) = K \mod M \), where \( M = N/L = 100 \) is the number of buckets: \( 0 \leq h(K) < M \).
  - So 100232, 433, and 10002332482 go into different buckets, but 10, 400210, and 210 all go into the same bucket.
External chaining

- Array of $M$ buckets.
- Each bucket is a list of data items.

- Not all buckets have same length, but average is $N/M = L$, the load factor.
- To work well, hash function must avoid too many collisions: keys that "hash" to equal values.
Ditching the Chains: Open Addressing

- Idea: Put one data item in each bucket.
- When there is a collision, and bucket is full, just use another.
- So you actually use a hash function with two arguments: $h(K, i)$, where $K$ is the key, and the buckets you try are $h(K, 0), h(K, 1), \ldots$.
- Various possibilities (here $S$ is the table size):
  - Linear probes: $h(K, i) = (h(K, 0) + C \cdot i) \mod S$, for constant $C$.
  - Quadratic probes: $h(K, i) = (h(K, 0) + C_1 \cdot i + C_2 \cdot i^2) \mod S$.
  - Double hashing: $h(K, i) = (h(K, 0) + i \cdot h'(K)) \mod S$, where $h'$ is a different hash function.
- Example: $h(K, i) = (K + i) \% M$, with $M = 10$:
  - Add 1, 2, 11, 3, 102, 9, 18, 108, 309 to empty table.

<table>
<thead>
<tr>
<th></th>
<th>108</th>
<th>1</th>
<th>2</th>
<th>11</th>
<th>3</th>
<th>102</th>
<th>309</th>
<th>18</th>
<th>9</th>
</tr>
</thead>
</table>

- Things can get slow, even when table is far from full.
- Lots of literature on this technique, but
- Personally, I just settle for external chaining.
Filling the Table

• To get (likely to be) constant-time lookup, need to keep \#buckets within constant factor of \#items.

• So resize table when load factor gets higher than some limit.

• In general, must \textit{re-hash} all table items.

• Still, this operation constant time per item,

• So by doubling table size each time, get constant \textit{amortized} time for insertion and lookup

• (Assuming, that is, that our hash function is good).
Sets vs. Dictionaries

- So far, we’ve really been talking about hash tables containing sets of keys. The important operations are testing for membership, adding keys, and iterating through the keys.

- The term “hash table” can also refer to a dictionary, in which each key is associated with a value.

- The associated value does not affect in the search process.

- The important operations are testing for whether a key is in the table, finding what value is associated with a given key, and setting the value associated with a key.

- Here, we’ll concentrate on sets of keys, since the addition of associated values is a conceptually small change.
Hash Functions: Strings

• For String, "s_0s_1 \cdots s_{n-1}" want function that takes all characters and their positions into account.

• What’s wrong with $s_0 + s_1 + \ldots + s_{n-1}$?

• For strings, Java uses

$$h(s) = s_0 \cdot 31^{n-1} + s_1 \cdot 31^{n-2} + \ldots + s_{n-1}$$

computed modulo $2^{32}$ as in Java int arithmetic.

• To convert to a table index in $0 \ldots N - 1$, compute $h(s) \% N$ (but don’t use table size that is multiple of 31!)

• Not as hard to compute as you might think; don’t even need multiplication!

```java
int r; r = 0;
for (int i = 0; i < s.length(); i += 1)
    r = (r << 5) - r + s.charAt(i);
```
Hash Functions: Other Data Structures I

- Lists (ArrayList, LinkedList, etc.) are analogous to strings: e.g., Java uses

  ```java
  int hashCode = 1; Iterator i = list.iterator();
  while (i.hasNext()) {
      Object obj = i.next();
      hashCode =
          31*hashCode
          + (obj==null ? 0 : obj.hashCode());
  }
  ```

- Can limit time spent computing hash function by not looking at entire list. For example: look only at first few items (if dealing with a List or SortedSet).

- Causes more collisions, but does **not** cause equal things to go to different buckets.
Hash Functions: Other Data Structures II

• Recursively defined data structures ⇒ recursively defined hash functions.

• For example, on a binary tree, one can use something like

```java
hash(T):
    if (T == null)
        return 0;
    else return someHashFunction (T.label ())
        ^ hash(T.left ()) ^ hash(T.right ());
```
Identity Hash Functions

• We can use the address of object as its hashed value ("hash on identity") if distinct (\(!=\)) objects are never considered equal.

• But careful! Won't work for Strings, because \(\text{.equals}\) Strings could be in different buckets:

```java
String H = "Hello",
    S1 = H + ", world!",
    S2 = "Hello, world!";
String H = "Hello",
    S1 = H + ", world!",
    S2 = "Hello, world!";
```

• Here \(\text{S1.equals(S2)}\), but \(\text{S1} \neq \text{S2}\).

• Java provides the type \texttt{java.util.IdentityHashMap} for this purpose:

```java
IdentityHashMap<Object, String> map = new IdentityHashMap<>();
Thing x = ...;
map.put(x, "Thing #1"); // Works if Thing does not override \texttt{.equals}.
map.put("Frank", "President"); // Probably not a good idea.
```
What Java Provides

• The class `Object` defines function `hashCode()`.

• By default, returns the identity hash function, or something similar. [Why is this OK as a default?]

• Can override it for your particular type.

• For reasons given on last slide, it is overridden for type `String`, as well as many types in the Java library, like all kinds of `List`.

• The types `Hashtable`, `HashSet`, and `HashMap` use `hashCode` to give you fast look-up of objects.

```java
HashMap<KeyType,ValueType> map =
    new HashMap<>(approximate size, load factor);
map.put(key, value);   // Map KEY -> VALUE.
... map.get(someKey)   // VALUE last mapped to by SOMEKEY.
... map.containsKey(someKey) // Is SOMEKEY mapped?
... map.keySet()       // All keys in MAP (a Set)
```
Special Case: Monotonic Hash Functions

- Suppose our hash function is *monotonic*: either nonincreasing or nondecreasing.

- So, e.g., if key \( k_1 > k_2 \), then \( h(k_1) \geq h(k_2) \).

- Example:
  - Items are time-stamped records; key is the time.
  - Hashing function is to have one bucket for every hour.

- In this case, you *can* use a hash table to speed up range queries [How?]

- Could this be applied to strings? When would it work well?
Perfect Hashing

- Suppose set of keys is fixed.
- A tailor-made hash function might then hash every key to a different value: perfect hashing.
- In that case, there is no search along a chain or in an open-address table: either the element at the hash value is or is not equal to the target key.
- One technique: use a hash table of hash tables with two hash functions. One chooses a small hash table that uses the second hash function. With a few random selections of hash functions, you can soon find one where there is never a collision in the second hash table.
- For example, suppose we store key/value pairs in type Pair. Then:

```java
Pair[][] table = ...;
// Find which table X must be in:
int h0 = hash0(X);
// Use different hash function for each h0.
Pair[] pair = table[h0][hash2(h0, X)];
return pair.key.equals(X) ? pair.value : null;
```
Characteristics

- Assuming good hash function, add, lookup, deletion take $\Theta(1)$ time, amortized.
- Good for cases where one looks up equal keys.
- For range queries such as “Give me every name between Martin and Napoli,” hash tables are usually bad. [Why?]
- Hashing is probably not a good idea for small sets that you rapidly create and discard. [Why?]