Heaps & Hashing

Discussion 10
Announcements

- Homework 6 due Tuesday 03/29
- Week 9 Survey due Tuesday 03/29
- Project 2 due Friday April 04/01
- Test 2 Review Sessions
  - Wednesday 03/30
  - Friday 04/01
- Test 2 on Wednesday 04/06
Review
Heaps are special trees that follow a few basic rules:

1. Heaps are complete - the only empty parts of a heap are in the bottom row, to the right
2. In a min-heap, each node must be smaller than all of its child nodes. The opposite is true for max-heaps.
Insertion into Heaps
Deletion from Heaps
Hashing

Hash functions are functions that represent an object using an integer. We use them to figure out which bucket of our hashset the item should go in.

Once we have a hash for our object we use mod to find out which bucket it goes into.

In each bucket, we deal with having lots of items by chaining the items and using .equals to find what we are looking for.

![Diagram showing chaining in a hash table]

**It is important that your .equals() function matches the result of comparing hashcodes - if two items are equal, they must also have the same hashcode**
Open Addressing

An alternative to externally chained hashmaps. When there is a collision in bucket $h(k)$, use another box using the formula $h(k) + f(m)$ for some function $f$.

- **Linear Probing** → $h(k) + m$, $h(k) + 2m$, $h(k) + 3m$, ...
- **Quadratic Probing** → $h(k) + 1 \times m$, $h(k) + 4 \times m$, $h(k) + 9 \times m$, ...
- **Double Hashing** → $h(k) + h'(k)$, $h(k) + 2h'(k)$, $h(k) + 3h'(k)$, ...
# Heaps of Fun

What is the worst case runtime for each operation, with and without resizing?

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Heaps of Fun

What are the advantages of using an array-based heap over a pointer-based heap?
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Pointer-based heaps are less space efficient because array-based heaps only need to keep one unit of space per item, while pointer-based heaps need to store both the item itself and pointers to its children.
1C Heaps of Fun

How can you implement a max-heap of integers if you only have access to a min-heap?
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For every insert operation, negate the number and add it to the min-heap. To perform a removeMax operation, call removeMin on the min-heap and negate the number returned.
Given an array and a min-heap, describe an algorithm that would allow you to sort the elements of the array in ascending order. Give the best and worst case runtime of your algorithm.
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Insert all items into a min-heap then call $\text{removeMin}$ repeatedly to get items in order from smallest to largest.

**Best Case:** If all items are identical, adding and removing require no bubbling up/down so each operation takes $\Theta(1)$ time. Since we do each operation once per item in the array, the total run time is $\Theta(N)$.

**Worst Case:** All inserted items must bubble all the way up to the root and all removed items must bubble all the way down. Around $\Theta(\log N)$ per operation, so simplifies to $\Theta(N \log N)$. 
Describe a way to modify the usual max heap implementation so that finding the minimum element takes constant time without incurring more than a constant amount of additional time and space for the other operations.
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Simply add a variable that keeps track of the minimum value in the heap. When inserting a new value, simply update this variable if the new value is smaller than it. Since the max heap only supports removing the largest element, rather than arbitrary elements, the minimum element will only be removed when the heap becomes empty, at which point we will need to reset the variable keeping track of the minimum value.
2B Assorted Heap Questions

How do you insert a new element into a d-ary heap? What is the run time in terms of d and n?

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To insert, we add the new element on the last level of the tree and then bubble it up. Since the tree has $\Theta(\log_d(n))$ levels and we have to do at most one comparison (compare the node to its parent) and one swap at each level, insertion takes $\Theta(\log_d(n))$ time.

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What is the runtime of finding the minimum element in a d-ary heap with n nodes in terms of d and n?

The minimum element is simply the root, just as with a binary min-heap. So finding it takes $\Theta(1)$ time.

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If you modify a key that has been inserted into a HashMap, can you retrieve the entry again? Explain.

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4 Hashcode

Explain whether each of the following hashcodes are valid or invalid.

(1) public int hashcode() {
    return -1;
}

(2) public int hashcode() {
    return intValue() * intValue();
}

(3) public int hashcode() {
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apple

cherry
5 Hashing Practice

- apple
- fig
- cherry
5 Hashing Practice

apple

fig

cherry

guava
5 Hashing Practice

- apple
- cherry
- durian
- fig
- guava
5 Hashing Practice

apple → apricot

cherry

durian

fig

guava
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