CS61B Lecture #27

Today: Sorting, continued

- Quicksort
- Selection
- Distribution counting
- Radix sorts

Next topic readings: *Data Structures*, Chapter 9.

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Quicksort: Speed through Probability

Idea:

- *Partition* data into pieces: everything $>$ a *pivot* value at the high end of the sequence to be sorted, and everything $\leq$ on the low end.
- Repeat recursively on the high and low pieces.
- For speed, stop when pieces are "small enough" and do insertion sort on the whole thing.
- Reason: insertion sort has low constant factors. By design, no item will move out of its will move out of its piece [why?], so when pieces are small, #inversions is, too.
- Have to choose pivot well. E.g.: *median* of first, last and middle items of sequence.

Example of Quicksort

- In this example, we continue until pieces are size ≤ 4.
- Pivots for next step are starred. Arrange to move pivot to dividing line each time.
- Last step is insertion sort.

16 10 13 18 -4 -7 12 -5 19 15 0 22 29 34 -1*

-4 -5 -7 | -1 | 18 13 12 10 19 15 0 22 29 34 16*

-4 -5 -7 | -1 | 15 13 12* 10 0 | 16 | 19* 22 29 34 18

-4 -5 -7 | -1 | 10 0 | 12 | 15 13 | 16 | 18 | 19 | 29 34 22

- Now everything is "close to" right, so just do insertion sort:

-7 -5 -4 -1 0 10 12 13 15 16 18 19 22 29 34

Performance of Quicksort

- Probabalistic time:
  - If choice of pivots good, divide data in two each time: $\Theta(N \lg N)$ with a good constant factor relative to merge or heap sort.
  - If choice of pivots bad, most items on one side each time: $\Theta(N^2)$.
  - $\Omega(N \lg N)$ in best case, so insertion sort better for nearly ordered input sets.
- Interesting point: randomly shuffling the data before sorting makes $\Omega(N^2)$ time *very* unlikely!

Quick Selection

The Selection Problem: for given k, find k^{th} smallest element in data.

- Obvious method: sort, select element #k, time $\Theta(N \lg N)$.
- If $k \le$ some constant, can easily do in $\Theta(N)$ time:
  - Go through array, keep smallest k items.
- Get *probably* $\Theta(N)$ *time* for all k by adapting quicksort:
  - Partition around some pivot, p, as in quicksort, arrange that pivot ends up at dividing line.
  - Suppose that in the result, pivot is at index m, all elements $\le$ pivot have indicies $\le m$.
  - If $m = k$, you're done: p is answer.
  - If $m > k$, recursively select k^{th} from left half of sequence.
  - If $m < k$, recursively select $(k - m - 1)^{\text{th}}$ from right half of sequence.

Selection Example

Problem: Find just item #10 in the sorted version of array:

Initial contents:

51	60	21	-4	37	4	49	10	40*	59	0	13	2	39	11	46	31
0																

Looking for #10 to left of pivot 40:

13	31	21	-4	37	4*	11	10	39	2	0	40	59	51	49	46	60
0																

Looking for #6 to right of pivot 4:

-4	0	2	4	37	13	11	10	39	21	31*	40	59	51	49	46	60
				4												

Looking for #1 to right of pivot 31:

-4	0	2	4	21	13	11	10	31	39	37	40	59	51	49	46	60
									9							

Just two elements; just sort and return #1:

-4	0	2	4	21	13	11	10	31	37	39	40	59	51	49	46	60
									9							

Result: 39

Selection Performance

- For this algorithm, if m roughly in middle each time, cost is

$$
\begin{aligned}
C(N) &= \begin{cases} 1, & \text{if } N = 1, \\ N + C(N/2), & \text{otherwise.} \end{cases} \\
&= N + N/2 + \ldots + 1 \\
&= 2N - 1 \in \Theta(N)
\end{aligned}
$$

- But in worst case, get $\Theta(N^2)$, as for quicksort.
- By another, non-obvious algorithm, can get $\Theta(N)$ worst-case time for all k (take CS170).

Better than N lg N?

- Can prove that *if all you can do to keys is compare them* then sorting must take $\Omega(N \lg N)$.
- Basic idea: there are $N!$ possible ways the input data could be scrambled.
- Therefore, your program must be prepared to do $N!$ different combinations of move operations.
- Therefore, there must be $N!$ possible combinations of outcomes of all the **if** tests in your program (we're assuming that comparisons are 2-way).
- Since each **if** test goes two ways, number of possible different outcomes for k **if** tests is 2^k.
- Thus, need enough tests so that $2^k > N!$, which means $k \in \Omega(\lg N!)$.
- Using Stirling's approximation,

$$m! \in \sqrt{2\pi m}\left(\frac{m}{e}\right)^m \left(1 + \Theta\left(\frac{1}{m}\right)\right),$$

  this tells us that

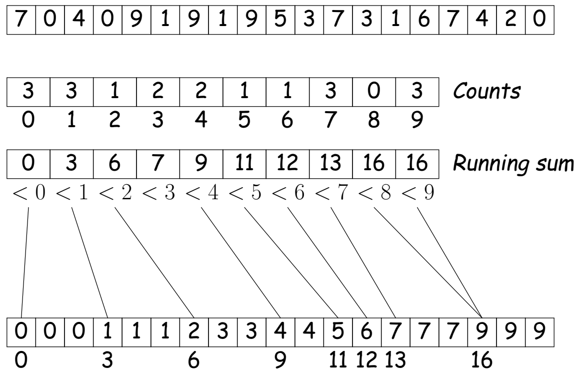
$$k \in \Omega(N \lg N).$$

Beyond Comparison: Distribution Counting

- But suppose can do more than compare keys?
- For example, how can we sort a set of N integer keys whose values range from 0 to kN, for some small constant k?
- One technique: *count* the number of items < 1, < 2, etc.
- If M_p =#items with value $< p$, then in sorted order, the j^{th} item with value p must be #$M_p + j$.
- Gives *linear-time* algorithm.

Distribution Counting Example

- Suppose all items are between 0 and 9 as in this example:



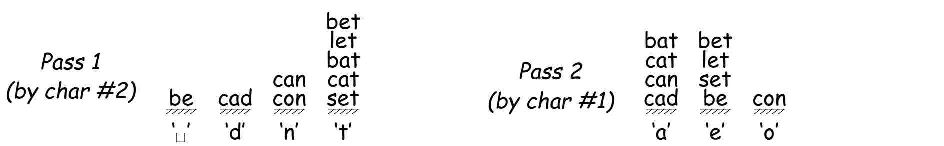
- "Counts" line gives # occurrences of each key.
- "Running sum" gives cumulative count of keys $\leq$ each value...
- ...which tells us where to put each key:
- The first instance of key k goes into slot m, where m is the number of key instances that are $< k$.

Radix Sort

Idea: Sort keys *one character at a time.*

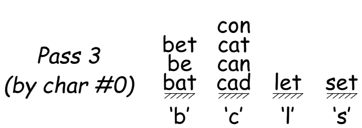
- Can use distribution counting for each digit.
- Can work either right to left (LSD radix sort) or left to right (MSD radix sort)
- LSD radix sort is venerable: used for punched cards.

Initial: set, cat, cad, con, bat, can, be, let, bet



be, cad, con, can, set, cat, bat, let, bet

cad, can, cat, bat, be, set, let, bet, con



bat, be, bet, cad, can, cat, con, let, set

MSD Radix Sort

- A bit more complicated: must keep lists from each step separate
- But, can stop processing 1-element lists

A	posn
⋆ set, cat, cad, con, bat, can, be, let, bet	0
⋆ bat, be, bet / cat, cad, con, can / let / set	1
bat / ⋆ be, bet / cat, cad, con, can / let / set	2
bat / be / bet / ⋆ cat, cad, con, can / let / set	1
bat / be / bet / ⋆ cat, cad, can / con / let / set	2
bat / be / bet / cad / can / cat / con / let / set	

Performance of Radix Sort

- Radix sort takes $\Theta(B)$ time where B is *total size of the key data*.
- Have measured other sorts as function of #records.
- How to compare?
- To have N different records, must have keys at least $\Theta(\lg N)$ long [why?]
- Furthermore, comparison actually takes time $\Theta(K)$ where K is size of key in worst case [why?]
- So $N \lg N$ comparisons really means $N(\lg N)^2$ operations.
- While radix sort takes $B = N \lg N$ time.
- On the other hand, must work to get good constant factors with radix sort.

And Don't Forget Search Trees

Idea: A search tree is in sorted order, when read in inorder.

- Need *balance* to really use for sorting [next topic].
- Given balance, same performance as heapsort: N insertions in time $\lg N$ each, plus $\Theta(N)$ to traverse, gives

$$\Theta(N + N \lg N) = \Theta(N \lg N)$$

Summary

- Insertion sort: $\Theta(Nk)$ comparisons and moves, where k is maximum amount data is displaced from final position.
  - Good for small datasets or almost ordered data sets.
- Quicksort: $\Theta(N \lg N)$ with good constant factor if data is not pathological. Worst case $O(N^2)$.
- Merge sort: $\Theta(N \lg N)$ guaranteed. Good for external sorting.
- Heapsort, treesort with guaranteed balance: $\Theta(N \lg N)$ guaranteed.
- Radix sort, distribution sort: $\Theta(B)$ (number of bytes). Also good for external sorting.