

Today:

- Trees

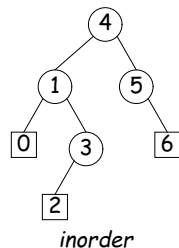
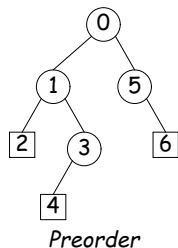
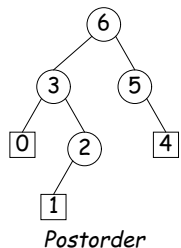
Readings for Today: *Data Structures*, Chapter 5

Readings for Next Topic: *Data Structures*, Chapter 6

- Trees naturally represent recursively defined, hierarchical objects with more than one recursive subpart for each instance.
- Common examples: expressions, sentences.
 - Expressions have definitions such as "an expression consists of a literal or two expressions separated by an operator."
- Also describe structures in which we recursively divide a set into multiple subsets.

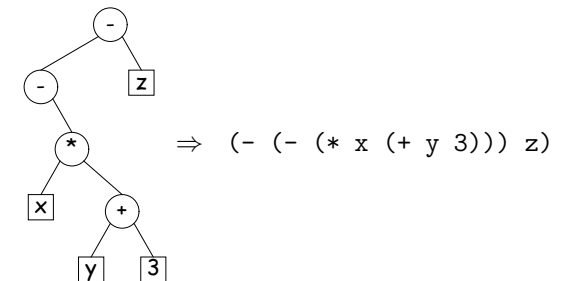
Fundamental Operation: Traversal

- *Traversing a tree* means enumerating (some subset of) its nodes.
- Typically done recursively, because that is natural description.
- As nodes are enumerated, we say they are *visited*.
- Three basic orders for enumeration (+ variations):
 - **Preorder**: visit node, traverse its children.
 - **Postorder**: traverse children, visit node.
 - **Inorder**: traverse first child, visit node, traverse second child (binary trees only).



Preorder Traversal and Prefix Expressions

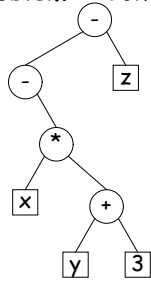
Problem: Convert



```
static String toLisp (Tree<String> T) {
    if (T == null)
        return "";
    else if (T.degree () == 0)
        return T.label ();
    else {
        String R; R = "";
        for (int i = 0; i < T.numChildren (); i += 1)
            R += " " + toLisp (T.child (i));
        return String.format ("%s%s", T.label (), R);
    }
}
```

Inorder Traversal and Infix Expressions

Problem: Convert



$\Rightarrow ((-(x*(y+3)))-z)$

To think about: how to get rid of all those parentheses.

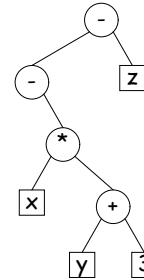
```
static String toInfix (Tree<String> T) {
    if (T == null)
        return "";
    if (T.degree () == 0)
        return T.label ();
    else {
        return String.format ("%s%s%s",
                               toInfix (T.left ()), T.label (), toInfix (T.right ()))
    }
}
```

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Postorder Traversal and Postfix Expressions

Problem: Convert



$\Rightarrow x y 3 +:2 *:2 -:1 z -:2$

```
static String toPolish (Tree<String> T) {
    if (T == null)
        return "";
    else {
        String R; R = "";
        for (int i = 0; i < T.numChildren (); i += 1)
            R += toPolish (T.child (i)) + " ";
        return String.format ("%s%s:%d", R, T.label (), T.degree ());
    }
}
```

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A General Traversal: The Visitor Pattern

```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

• What is Action?

```
interface Action<Label> {
    void action (Tree<Label> T);
}

class Print implements Action<String> | preorderTraverse (myTree,
    void action (Tree<String> T) { | new Print ();
        System.out.print (T.label ()); |
    } |
} |
```

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Times

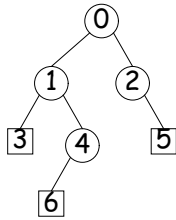
- The traversal algorithms have roughly the form of the boom example in §1.3.3 of *Data Structures*—an exponential algorithm.
- However, the role of M in that algorithm is played by the *height* of the tree, not the number of nodes.
- In fact, easy to see that tree traversal is *linear*: $\Theta(N)$, where N is the # of nodes: Form of the algorithm implies that there is one visit at the root, and then one visit for every *edge* in the tree. Since every node but the root has exactly one parent, and the root has none, must be $N - 1$ edges in any non-empty tree.
- In positional tree, is also one recursive call for each empty tree, but # of empty trees can be no greater than kN , where k is arity.
- For k -ary tree (max # children is k), $h + 1 \leq N \leq \frac{k^{h+1}-1}{k-1}$, where h is height.
- So $h \in \Omega(\log_k N) = \Omega(\lg N)$ and $h \in O(N)$.
- Many tree algorithms look at one child only. For them, time is proportional to the *height* of the tree, and this is $\Theta(\lg N)$, assuming that tree is *bushy*—each level has about as many nodes as possible.

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Level-Order (Breadth-First) Traversal

Problem: Traverse all nodes at depth 0, then depth 1, etc:



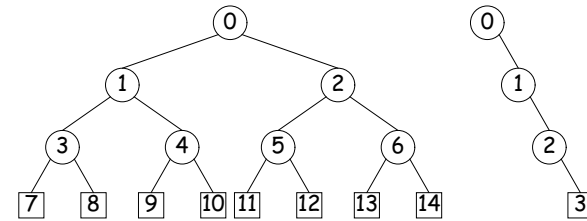
- One technique: *Iterative Deepening*. For each level, k , from 0 to h , call `doLevel(T,k)`

```
void doLevel (Tree T, int lev) {
    if (lev == 0)
        visit T
    else
        for each non-null child, C, of T {
            doLevel (C, lev-1);
        }
}
```

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Iterative Deepening Time?



- Let h be height, N be # of nodes.
- Count # edges traversed (i.e, # of calls, not counting null nodes).
- First (full) tree: 1 for level 0, 3 for level 1, 7 for level 2, 15 for level 3.
- Or in general $(2^1 - 1) + (2^2 - 1) + \dots + (2^{h+1} - 1) = 2^{h+2} - h \in \Theta(N)$, since $N = 2^{h+1} - 1$ for this tree.
- Second (right leaning) tree: 1 for level 0, 2 for level 1, 3 for level 2.
- Or in general $(h+1)(h+2)/2 = N(N+1)/2 \in \Theta(N^2)$, since $N = h+1$ for this kind of tree.

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Iterative Traversals

- Tree recursion conceals data: a *stack* of nodes (all the T arguments) and a little extra information. Can make the data explicit, e.g.:

```
void preorderTraverse2 (Tree T<T>, Action whatToDo) {
    Stack s = new Stack ();
    s.push (T);
    while (! s.isEmpty ()) {
        Tree node = (Tree) s.pop ();
        if (node == null)
            continue;
        whatToDo.action (node);
        for (int i = node.numChildren ()-1; i >= 0; i --= 1)
            s.push (node.child (i));
    }
}
```

- To do a breadth-first traversal, use a queue instead of a stack, replace push with add, and pop with removeFirst.
- Makes breadth-first traversal worst-case linear time in all cases, but also linear space for "bushy" trees.

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Iterators for Trees

- Frankly, iterators are not terribly convenient on trees.
- But can use ideas from iterative methods.

```
class PreorderTreeIterator<T> implements Iterator<T> {
    private Stack<Tree<T>> s = new Stack<Tree<T>> ();

    public PreorderTreeIterator (Tree<T> T) { s.push (T); }

    public boolean hasNext () { return ! s.isEmpty (); }
    public T next () {
        Tree<T> result = s.pop ();
        for (int i = result.numChildren ()-1; i >= 0; i --= 1)
            s.push (result.child (i));
        return result.label ();
    }

    void remove () { throw new UnsupportedOperationException (); }
}
```

Example: (what do I have to add to class `Tree` first?)

```
for (String label : aTree) System.out.print (label + " ");
```

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