Administrivia

- Unfortunately, we can only have 200 people in here. Please occupy only the seats we’ve reserved.
- Please make sure you have obtained a Unix account.
- If you decide not to take this course after all, please tell CalCentral ASAP, so that we can adjust the waiting list accordingly.
- HW #0 will be due next Friday at midnight. While you get credit for any submission, we strongly suggest that you give the problems a serious try.
- We strongly discourage taking this course P/NP (or S/U).
Lecture #2: Let's Write a Program: Prime Numbers

Problem: want java Primes \( U \) to print prime numbers through \( U \).

You type: java Primes 101

It types: 2 3 5 7 11 13 17 19 23 29

31 37 41 43 47 53 59 61 67 71

73 79 83 89 97 101

Definition: A prime number is an integer greater than 1 that has no divisors smaller than itself other than 1.

(Alternatively: \( p > 1 \) is prime iff \( \gcd(p, x) = 1 \) for all \( 0 < x < p \).)

Useful Facts:

- \( k \leq \sqrt{N} \) iff \( N/k \geq \sqrt{N} \), for \( N, k > 0 \).
- If \( k \) divides \( N \) then \( N/k \) divides \( N \).

So: Try all potential divisors up to and including the square root.
public class Primes {
    /** Print all primes up to ARGS[0] (interpreted as an integer), 10 to a line. */
    public static void main(String[] args) {
        printPrimes(Integer.parseInt(args[0]));
    }

    /** Print all primes up to and including LIMIT, 10 to a line. */
    private static void printPrimes(int limit) {
        /*{ For every integer, x, between 2 and LIMIT, print it if isPrime(x), 10 to a line. }*/
    }

    /** True iff X is prime */
    private static boolean isPrime(int x) {
        return /*( X is prime )*/;
    }
}
Testing for Primes

private static boolean isPrime(int x) {
    if (x <= 1)
        return false;
    else
        return !isDivisible(x, 2);  // "!" means "not"
}

/** True iff X is divisible by any positive number >=K and < X,
   * given K > 1. */
private static boolean isDivisible(int x, int k) {
    if (k >= x)  // a "guard"
        return false;
    else if (x % k == 0)  // "%" means "remainder"
        return true;
    else  // if (k < x && x % k != 0)
        return isDivisible(x, k+1);
}
Thinking Recursively

Understand and check \texttt{isDivisible(13,2)} by \textit{tracing one level}.

/** True iff X is divisible by
 * some number >=K and < X,
 * given K > 1. */
private static boolean isDivisible...
  if (k >= x)
    return false;
  else if (x % k == 0)
    return true;
  else
    return isDivisible(x, k+1);
}

Lesson: Comments aid understanding. Make them \textit{count}!

- Call assigns \( x=13, \ k=2 \)
- Body has form `if (k >= x) \( S_1 \)`
  \( \text{else } S_2 \).
- Since \( 2 < 13 \), we evaluate the first \textit{else}.
- Check if \( 13 \mod 2 = 0 \); it's not.
- Left with \texttt{isDivisible(13,3)}.
- Rather than tracing it, instead \textit{use the comment}:
  - Since 13 is \textit{not} divisible by any integer in the range 3..12 (and \( 3 > 1 \)), \texttt{isDivisible(13,3)} must be \textit{false}, and we're done!
- Sounds like that last step begs the question. Why doesn't it?
Iteration

- `isDivisible` is *tail recursive*, and so creates an *iterative process*.

- Traditional “Algol family” production languages have special syntax for iteration. Four equivalent versions of `isDivisible`:

  ```
  if (k >= x)
    return false;
  else if (x % k == 0)
    return true;
  else
    return isDivisible(x, k+1);
  ```

  ```
  while (k < x)
    // !(k >= x)
    if (x % k == 0)
      return true;
    k = k+1; // or k += 1, or (yuch) k++
  return false;
  ```

  ```
  int k1 = k;
  while (k1 < x)
    if (x % k1 == 0)
      return true;
    k1 += 1;
  return false;
  ```

  ```
  for (int k1 = k; k1 < x; k1 += 1)
    if (x % k1 == 0)
      return true;
  return false;
  ```

Last modified: Thu Aug 26 12:43:58 2021
Using Facts about Primes

- We haven’t used the Useful Facts from an earlier slide. Only have to check for divisors up to the square root.

- So, reimplement the iterative version of `isDivisible`:

```java
/** True iff X is divisible by some number >=K and < X,
* given that K > 1, and that X is not divisible by
* any number >1 and <K. */
private static boolean isDivisible(int x, int k) {
    int limit = (int) Math.round(Math.sqrt(x));
    for (int k1 = k; k1 <= limit; k1 += 1) {
        if (x % k1 == 0)
            return true;
    }
    return false;
}
```

- Why the additional (blue) condition in the comment?
Cautionary Aside: Floating Point

• In the last slide, we had

```java
int limit = (int) Math.round(Math.sqrt(x));
for (int k1 = k; k1 <= limit; k1 += 1) {
  ...
```

intending that this would check all values of \( k1 \) up to and including the square root of \( x \).

• Since floating-point operations yield approximations to the corresponding mathematical operations, you might ask the following about \((\text{int}) \text{Math.round}(\text{Math.sqrt}(x))\):

  - Is it always at least \( \lfloor \sqrt{x} \rfloor \)? (\( \lfloor z \rfloor \) means “the largest integer \( \leq z \).”)

    If not, we might miss testing \( \sqrt{x} \) when \( x \) is a perfect square.

• As it happens, the answer is “yes” for IEEE floating-point square roots.

• Just an example of the sort of detail that must be checked in edge cases.
Final Task: printPrimes (Simplified)

/** Print all primes up to and including LIMIT. */
private static void printPrimes(int limit) {

}
Simplified printPrimes Solution

//** Print all primes up to and including LIMIT. */
private static void printPrimes(int limit) {
    for (int p = 2; p <= limit; p += 1) {
        if (isPrime(p)) {
            System.out.print(p + " ");
        }
    }
    System.out.println();
}

printPrimes (full version)

/** Print all primes up to and including LIMIT, 10 to
 * a line. */

private static void printPrimes(int limit) {
    int np;
    np = 0;
    for (int p = 2; p <= limit; p += 1) {
        if (isPrime(p)) {
            System.out.print(p + " ");
            np += 1;
            if (np % 10 == 0)
                System.out.println();
        }
    }
    if (np % 10 != 0)
        System.out.println();
}