1  DFS, BFS, Dijkstra’s, A*
For the following questions, use the graph below and assume that we break ties by visiting lexicographically earlier nodes first.

(a) Give the depth first search preorder traversal starting from vertex A.

(b) Give the depth first search postorder traversal starting from vertex A.

(c) Give the breadth first search traversal starting from vertex A.

(d) Give the order in which Dijkstra’s Algorithm would visit each vertex, starting from vertex A. Sketch the resulting shortest paths tree.

(e) Give the path A* search would return, starting from A and with G as a goal.

Let \( h(u, v) \) be the valued returned by the heuristic for nodes \( u \) and \( v \).

\[
\begin{array}{c|c|c}
  u & v & h(u, v) \\
  A & G & 9 \\
  B & G & 7 \\
  C & G & 4 \\
  D & G & 1 \\
  E & G & 10 \\
  F & G & 3 \\
  H & G & 5 \\
\end{array}
\]
Graph Conceptuals

Answer the following questions as either True or False and provide a brief explanation:

1. If a graph with \( n \) vertices has \( n - 1 \) edges, it must be a tree.

2. The adjacency matrix representation is typically better than the adjacency list representation when the graph is very connected.

3. Every edge is looked at exactly twice in every iteration of DFS on a connected, undirected graph.

4. In BFS, let \( d(v) \) be the minimum number of edges between a vertex \( v \) and the start vertex. For any two vertices \( u, v \) in the fringe, \( |d(u) - d(v)| \) is always less than 2.

5. Given a fully connected, directed graph (a directed edge exists between every pair of vertices), a topological sort can never exist.
3 Conceptual Shortest Paths

Answer the following questions regarding shortest path algorithms for a weighted, undirected graph. If the statement is true, provide an explanation. If the statement is false, provide a counterexample.

(a) (T/F) If all edge weights are equal and positive, the breadth-first search starting from node A will return the shortest path from a node A to a target node B.

(b) (T/F) If all edges have distinct weights, the shortest path between any two vertices is unique.

(c) (T/F) Adding a constant positive integer $k$ to all edge weights will not affect any shortest path between two vertices.

(d) (T/F) Multiplying a constant positive integer $k$ to all edge weights will not affect any shortest path between two vertices.