CS61A Lecture 11
Immutable Trees

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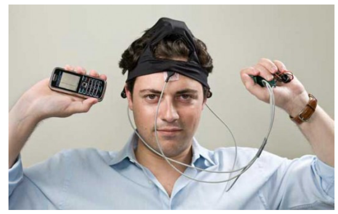
Computer Science in the News

iBrain to allow Stephen Hawking to communicate through brainwaves alone

By Kent Sutherland
11:52 July 4, 2012
3 Comments
3 Pictures



http://www.gizmag.com/ibrain-stephen-hawking-communicate-brainwaves/23182/ 2

Today

- Review: Immutable Dictionaries
- Deep Tuples
- Immutable Trees

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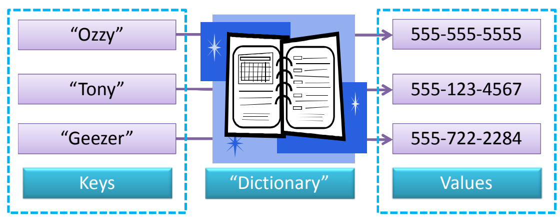
Review: Dictionaries

Often we want to associate pieces of data with other pieces of data.



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Review: Immutable Dictionaries

```
>>> phone_bk = make_idict((“Ozzy”, “555-5555”),
...                       (“Tony”, “123-4567”),
...                       (“Geezer”, “722-2284”))
>>> idict_select(phone_bk, “Ozzy”)
“555-5555”
>>> idict_select(phone_bk, “Geezer”)
“722-2284”
>>> idict_keys(phone_bk)
(“Ozzy”, “Tony”, “Geezer”)
```

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Review: Higher Order Functions For Sequences

```
>>> nums = (1, 2, 3, 4, 5)
>>> tuple(map(lambda x: x * x, nums))
(1, 4, 9, 16, 25)
>>> tuple(map(lambda x: x + 1, nums))
(2, 3, 4, 5, 6)
>>> tuple(filter(lambda x: x % 2 == 0, nums))
(2, 4)
>>> tuple(filter(lambda x: x <= 3, nums))
(1, 2, 3)
>>> from functools import reduce
>>> reduce(lambda x, y: x * y, nums, 1)
120
>>> reduce(lambda x, y: x + y, nums, 0)
15
```

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Practice: Higher Order Functions For Sequences

What are the outputs for each of the following lines of Python?

```
>>> from operator import add
>>> tuple(map(lambda x: reduce(add, x),
        ((2, 3), (5, 6), (8, 9))))
?????
>>> tuple(map(lambda x: x - 1,
              filter(lambda x: x % 2 == 0,
                     map(lambda x: x + 1,
                         range(10)))))
?????
```

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Practice: Higher Order Functions For Sequences

What are the outputs for each of the following lines of Python?

```
>>> from operator import add
>>> tuple(map(lambda x: reduce(add, x),
        ((2, 3), (5, 6), (8, 9))))
(5, 11, 17)
>>> tuple(map(lambda x: x - 1,
              filter(lambda x: x % 2 == 0,
                     map(lambda x: x + 1,
                         range(10)))))
(1, 3, 5, 7, 9)
```

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Announcements

- Homework 5 is due **July 6**.
- Project 2 is due **July 13**.
- Project 1 contest is on!
  - *How to submit*: Submit a file pig.py with your `final_strategy` to `proj1-contest`.
  - *Deadline*: Friday, **July 6** at **11:59pm**.
  - *Prize*: One of 3 copies of *Feynman* and 1 extra credit point.
  - *Metric*: We will simulate your strategy against everyone else's, and tally your win rate. Draws count as losses.

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Announcements: Midterm 1

- Midterm 1 is on **July 9**.
  - *Where*? 2050 VLSB.
  - *When*? 7PM to 9PM.
  - *How much*? Material covered until July 4.
- Closed book and closed electronic devices.
- One 8.5" x 11" 'cheat sheet' allowed.
- Group portion is 15 minutes long.
- Post-midterm potluck on Wednesday, **July 11**.

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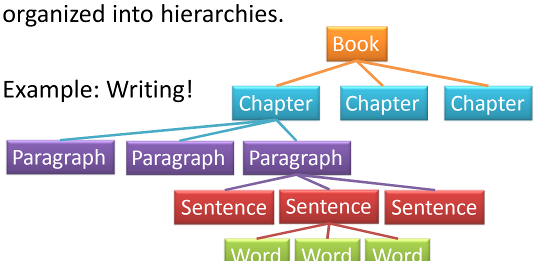
Hierarchical Data

Often we find that information is nicely organized into hierarchies.

Example: Writing!



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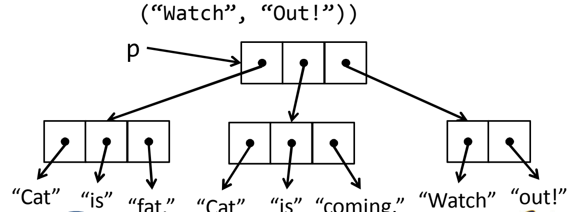
Hierarchical Data: Deep Tuples

We already had a way of representing something like this.

```
>>> p = (("Cat", "is", "fat."),
         ("Cat", "is", "coming."),
         ("Watch", "Out!"))
```



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Operating on Deep Tuples

So we already have a simple way of organizing data into hierarchies using "deep tuples."

How do we manipulate deep tuples?

- Not that different from working with regular tuples.
- Use ***tree recursion***!

Example: Operating on Deep Tuples

Let's say I want to write a function, `evens_count`, that counts the number of even numbers found in a deep tuple containing numbers (or tuples).

```
>>> woah_deep = ((1, 2), 3, ((4, 5), 6))
>>> evens_count(woah_deep)
3
```


Example: Operating on Deep Tuples

How we would have solved this if we were handed a simple tuple of numbers?

Now extend this to handle deep tuples!

```
# Iterative
def evens_count(t):
    total = 0
    for num in t:
        if num % 2 == 0:
            total += 1
    return total
```

```
# Recursive
def evens_count(t):
    if len(t) == 0:
        return 0
    if t[0] % 2 == 0:
        return 1 + evens_count(t[1:])
    return 0 + evens_count(t[1:])
```


Example: Operating on Deep Tuples

```
# Recursive
def evens_count(t):
    if len(t) == 0:
        return 0
    if is_tuple(t[0]):
        return evens_count(t[0]) + evens_count(t[1:])
    if t[0] % 2 == 0:
        return 1 + evens_count(t[1:])
    return 0 + evens_count(t[1:])
```

```
def is_tuple(x):
    return type(x) is tuple
```

First check if the first item in the sequence is also a tuple. If so, use ***tree recursion*** and count the evens in both the first item and the rest of t.

Practice: Operating on Deep Tuples

Write the procedure `deep_filter`, which takes a predicate and a deep tuple and returns a new deep tuple with only the items for which predicate returns True.

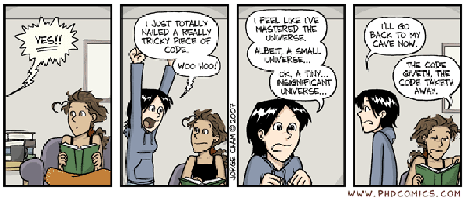
```
>>> woah_deep = ((1, 2), 3, ((4, 5), 6))
>>> deep_filter(lambda x: x % 2 == 0,
                woah_deep)
((2,), ((4,), 6))
>>> deep_filter(lambda x: x >= 2 and x <= 3,
                woah_deep)
((2,), 3, ((),))
```


Practice: Operating on Deep Tuples

Write the procedure `deep_filter`, which takes a predicate and a deep tuple and returns a new deep tuple with only the items for which predicate returns True.

```
def deep_filter(pred, dt):
    if len(dt) == 0:
        return dt
    if is_tuple(dt[0]):
        return (deep_filter(pred, dt[0]),) +\
               deep_filter(pred, dt[1:])
    if pred(dt[0]):
        return (dt[0],) + deep_filter(pred, dt[1:])
    return deep_filter(pred, dt[1:])
```

Break



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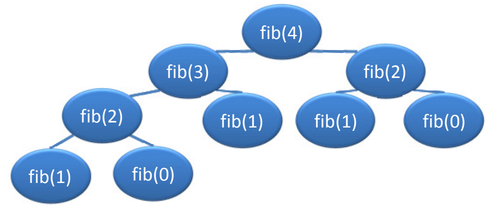
Hierarchical Data: Trees

Often, deep tuples aren't quite expressive enough for our purposes. Sometimes we want values in the middle too!



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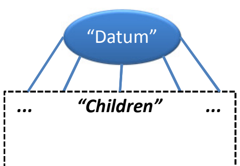
Hierarchical Data: Trees

A ***tree*** data structure traditionally has 2 parts:

1. A ***datum*** – The data stored in the top point of the tree.
2. Some ***children*** – Any trees that appear below this tree.



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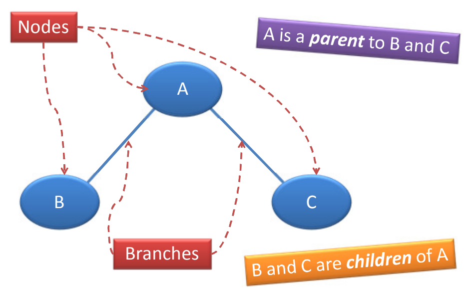
Hierarchical Data: Trees



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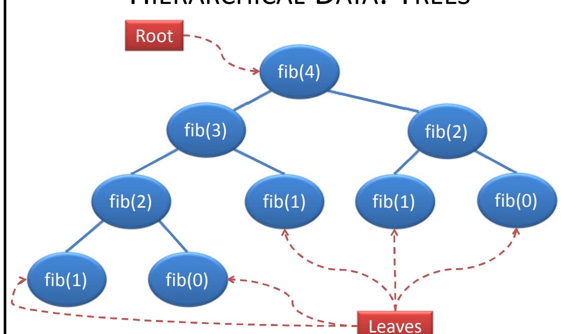
Hierarchical Data: Trees



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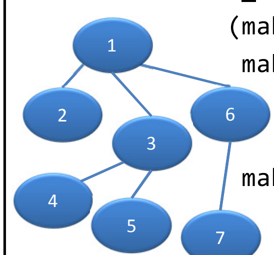
ITrees

```
>>> fir = make_tree(1,
              (make_tree(2),
               make_tree(3,
                   (make_tree(4),
                    make_tree(5))),
               make_tree(6,
                   (make_tree(7),))))
```



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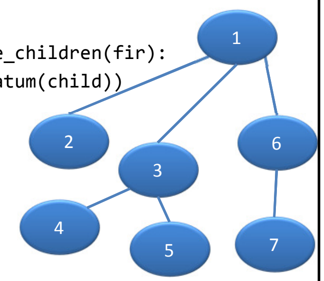
ITREES

```
>>> itree_datum(fir)
1
>>> for child in itree_children(fir):
        print(itree_datum(child))
2
3
6
```



ITREES

```
def make_itree(datum, children=()):
    return (datum, children)

def itree_datum(t):
    return t[0]

def itree_children(t):
    return t[1]
```

Constructor

Selectors

EXAMPLE: OPERATING ON ITREES

Suppose I want to write the function `itree_prod`, which takes an ITree of numbers and returns the product of all the numbers in the ITree.

```
>>> t = make_itree(1, (make_itree(2),
                       make_itree(3),
                       make_itree(4)))
>>> itree_prod(t)
24
```

Look! I got it right this time!

EXAMPLE: OPERATING ON ITREES

Idea: split the problem into 2 different parts: handling a single tree and handling a group of trees (a *forest*).

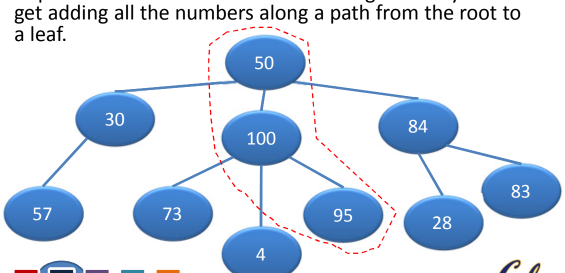
```
def itree_prod(t):
    return itree_datum(t) * forest_prod(itree_children(t))

def forest_prod(f):
    if len(f) == 0:
        return 1
    return itree_prod(f[0]) * forest_prod(f[1:])
```

This is called ***mutual recursion*** because it involves two functions recursively calling each other!

PRACTICE: OPERATING ON ITREES

Write the function `max_path_sum`, which takes an ITree of positive numbers and returns the largest sum you can get adding all the numbers along a path from the root to a leaf.



PRACTICE: OPERATING ON ITREES

Write the function `max_path_sum`, which takes an ITree of positive numbers and returns the largest sum you can get adding all the numbers along a path from the root to a leaf.

```
def max_path_sum(t):
    ??????


def max_forest_sum(f):
    if len(f) == 0:
        return 0
    ??????
```

Practice: Operating on ITrees

Write the function `max_path_sum`, which takes an ITree of positive numbers and returns the largest sum you can get adding all the numbers along a path from the root to a leaf.

```
def max_path_sum(t):
    max_child_sum = max_forest_sum(itree_children(t))
    return itree_datum(t) + max_child_sum

def max_forest_sum(f):
    if len(f) == 0:
        return 0
    return max(max_path_sum(f[0]),
               max_forest_sum(f[1:]))
```

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Conclusion

- Organizing data into hierarchies is ***very*** useful and ***very*** common in Computer Science
- We can think of nested tuples as a simple form of a tree structure that only has leaves.
- ITrees are useful for representing general tree-structures.
- ***Preview:*** binary search trees!

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