CS61A Lecture 4
Higher Order Functions

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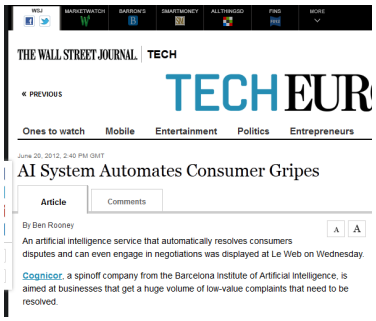
Computer Science in the News



Review

Write the function `filtered_count` that takes a number, n, and a function, `pred`*, and prints all the numbers 1 to n for which `pred` returns `True`.

*pred stands for *predicate*, a function that only returns True or False.

Review

Write the function `filtered_count` that takes a number, n, and a function, `pred`*, and prints all the numbers 1 to n for which `pred` returns `True`.

```
def filtered_count(n, pred):
    k = 1
    while k <= n:
        if pred(k):
            print(k)
        k = k + 1
```

*pred stands for *predicate*, a function that only returns True or False.

Today

Defining functions inside functions.

Returning functions from functions.

A Friendly Greek Letter.

http://gingerconsult.files.wordpress.com/2012/06/turn-it-up.png?w=300&h=300

Local (Helper) Functions

Python allows us to make functions *inside* other functions. This is useful for making helper functions.

$$f(a,b,c) = \begin{matrix} ab + \left(\frac{b}{a}\right) + a^b \\ ac + \left(\frac{c}{a}\right) + a^c \end{matrix}$$

```
def funny_function(a, b, c):
    a_funny_b = (a * b) + (b / a) + (a ** b)
    a_funny_c = (a * c) + (c / a) + (a ** c)
    return a_funny_b / a_funny_c
```

Local (Helper) Functions

Python allows us to make function
functions. This is useful for making

Locally defined "helper" functions help us avoid repetition!

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(a, b, c):
    def a_funny(b):
        return (a * b) + (b / a) + (a ** b)
    return a_funny(b) / a_funny(c)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(a, b, c):
    def a_funny(b):
        return (a * b) + (b / a) + (a ** b)
    return a_funny(b) / a_funny(c)

funny_function(2, 4, 6)
```


Local (Helper) Functions

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def funny_function(a, b, c):
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funny_function(2, 4, 6)
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Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, b, c):
    def a_funny(b):
        return (2 * b) + (b / 2) + (2 ** b)
    return a_funny(b) / a_funny(c)

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, b, c):
    def a_funny(b):
        return (2 * b) + (b / 2) + (2 ** b)
    return a_funny(b) / a_funny(c)

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(b):
        return (2 * b) + (b / 2) + (2 ** b)
    return a_funny(4) / a_funny(6)

funny_function(2, 4, 6)
```

Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(b):
        return (2 * b) + (b / 2) + (2 ** b)
    return a_funny(4) / a_funny(6)

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(b):
        return (2 * b) + (b / 2) + (2 ** b)
    return a_funny(4) / a_funny(6)

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(4):
        return (2 * 4) + (4 / 2) + (2 ** 4)
    return 26 / a_funny(6)

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(6):
        return (2 * 6) + (6 / 2) + (2 ** 6)
    return 26 / a_funny(6)

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(6):
        return (2 * 6) + (6 / 2) + (2 ** 6)
    return 26 / 79

funny_function(2, 4, 6)
```


Local (Helper) Functions

How does this work?

$$f(a,b,c) = \frac{ab + \left(\frac{b}{a}\right) + a^b}{ac + \left(\frac{c}{a}\right) + a^c}$$

```
def funny_function(2, 4, 6):
    def a_funny(b):
        return (2 * b) + (b / 2) + (2 ** b)
    return 26 / 79

funny_function(2, 4, 6) → 0.329…
```

Announcements

- You will be placed into study/exam groups in discussion section today.
  - Email your TA ASAP if you can't make section so they can assign you a group.
- Homework 1 is due tomorrow night.
- Homework 2 is out and due Tuesday night.
- Project 1 is out and due the night of 6/29.

Problem: Making Adders

```
def add_1_to_twice(n):
    return (2*n)+1
def add_2_to_twice(n):
    return (2*n)+2
def add_3_to_twice(n):
    return (2*n)+3
...
def add_65536_to_twice(n):
    return (2*n)+65536
def add_65537_to_twice(n):
    return (2*n)+65537
...
```


Problem: Making Adders

```
def add_1_to_twice(n):
    return (2*n)+1
def add_2_to_twice(n):
    return (2*n)+2
def add_3_to_twice(n):
    return (2*n)+3
...
def add_65536_to_twice(n):
    return (2*n)+65536
def add_65537_to_twice(n):
    return (2*n)+65537
...
```

There has to be a better way!

Problem: Making Adders

```
def add_1_to_twice(n):
    return (2*n)+1
def add_2_to_twice(n):
    return (2*n)+2
def add_3_to_twice(n):
    return (2*n)+3
...
def add_65536_to_twice(n):
    return (2*n)+65536
def add_65537_to_twice(n):
    return (2*n)+65537
...
```

Can we generalize?

Problem: Making Adders

http://farm8.staticflickr.com/7014/6766279501_4ddeb9193c_z.jpg

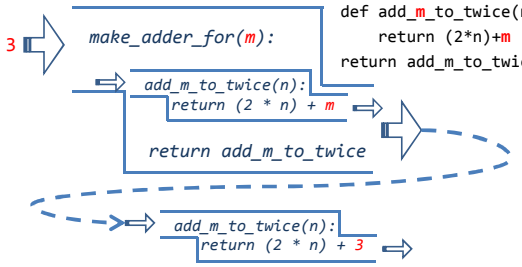
Problem: Making Adders

```
def make_adder_for(m):
    def add_m_to_twice(n):
        return (2*n)+m
    return add_m_to_twice

add_1_to_twice = make_adder_for(1)
add_2_to_twice = make_adder_for(2)
add_3_to_twice = make_adder_for(3)
...
add_65536_to_twice = make_adder_for(65536)
add_65537_to_twice = make_adder_for(65537)
...
```

Problem: Making Adders

```
def make_adder_for(m):
    def add_m_to_twice(n):
        return (2*n)+m
    return add_m_to_twice
```



Practice: Making Summation...ers

I want to make specialized sums for later use. Complete the definition below:

```
def make_summationer(term):
    def summation(n):



    return summation
```


Practice: Making Summation...ers

I want to make specialized sums for later use. Complete the definition below:

```
def make_summationer(term):
    def summation(n):
        k, sum = 1, 0
        while k <= n:
            sum = sum + term(k)
            k = k + 1
        return sum
    return summation
```


Practice: Making Adders for Summations

Say I wanted to get the sum of the numbers 4 through 17. How can I do this in one line using `summation` and `make_adder`?

```
def make_adder(n):
    def add_n(m):
        return n + m
    return add_n
def summation(n, term):
    sum, k = 0, 1
    while k <= n:
        sum = sum + term(k)
        k = k + 1
    return sum
>>> ??!?!?!???!?!
147
```


Practice: Making Adders for Summations

Say I wanted to get the sum of the numbers 4 through 17. How can I do this in one line using `summation` and `make_adder`?

```
def make_adder(n):
    def add_n(m):
        return n + m
    return add_n
def summation(n, "k + 3"):
    sum, k = 0, 1
    while k <= n:
        sum = sum + "k + 3"
        k = k + 1
    return sum
>>> summation(14, make_adder(3))
147
```


Anonymous Functions

What if we don't want to give a name to our function? Seems silly that we have to come up with a name that we're only using once in our program (in the return statement).

```
def make_adder(n):
    def add_n(m):
        return n + m
    return add_n
```

Can I do the same thing without providing a name?

- Yes!

λ Anonymous Functions

In Computer Science, we traditionally call anonymous function values "*lambda functions*."

In Python a lambda function has the form:

```
lambda <arguments>: <expression>
```

For example we could have done:

```
def make_adder(n):
    return lambda m: n + m
```


λ Anonymous Functions

Translating between lambdas and defined functions.

```
<name> = lambda <arguments>: <expression>

def <name>(<arguments>):
    return <expression>
```


λ Anonymous Functions

Translating between lambdas and defined functions.

```
square = lambda x: x * x

def square(x):
    return x*x
```


λ Anonymous Functions

Translating between lambdas and defined functions.

```
foo = lambda a, x, b: (a * x) + b

def foo(a, x, b):
    return (a * x) + b
```


λ Anonymous Functions

Translating between lambdas and defined functions.

```
def make_adder(n):
    return lambda m: m + n

def make_adder(n):
    def add_n(m):
        return m + n
    return add_n
```


λ Anonymous Functions

Key points about lambda in Python:

- A lambda in Python ***is an expression***, so you can use them anywhere you can use any other expression in Python.
- You can only use a single *expression* in the body of a lambda function.
- A lambda function always **returns** the value of the body expression.
- Lambdas should be used *sparingly*, only for very simple functions. Otherwise the code can quickly become **unreadable**.

Practice: Anonymous Functions

What would the following expression evaluate to?

```
>>> summation(3, lambda x: x + 5)
```

Practice: Anonymous Functions

What would the following expression evaluate to?

```
>>> summation(3, lambda x: x + 5)
21
```

Conclusion

- Functions can be defined inside other functions!
- Functions can *return* functions.
- Python has **lambda functions** for creating anonymous functions.
- **Next Time:** More practice with HOFs, and practical applications!

Extras: Making My Adder and Using It Too!

We can add 3 and 5, using only the values make_adder, 3, and 5.