61A Extra Lecture 11

Announcements

The Logic Language

The Logic Language

The *Logic* language was invented for Structure and Interpretation of Computer Programs

- Based on Prolog (1972)
- Expressions are facts or queries, which contain relations
- Expressions and relations are Scheme lists
- For example, **(likes john dogs)** is a relation

Simple Facts

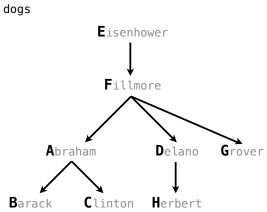
A simple fact expression in the Logic language declares a relation to be true

Let's say I want to track the heredity of a pack of dogs

Language Syntax:

- A relation is a Scheme list
- A fact expression is a Scheme list of relations

```
logic> (fact (parent delano herbert))
logic> (fact (parent abraham barack))
logic> (fact (parent abraham clinton))
logic> (fact (parent fillmore abraham))
logic> (fact (parent fillmore delano))
logic> (fact (parent fillmore grover))
logic> (fact (parent eisenhower fillmore))
```



Relations are Not Procedure Calls

In *Logic*, a relation is **not** a call expression.

- *Scheme*: the expression **(abs -3)** calls *abs* on -3. It returns 3.
- *Logic*: **(abs -3 3)** asserts that *abs* of -3 is 3.

To assert that 1 + 2 = 3, we use a relation: **(add 1 2 3)**

We can ask the Logic interpreter to complete relations based on known facts.

(add ? 2 3)	**1**
(add 1 ? 3)	**2**
(add 1 2 ?)	**3**
(? 1 2 3)	**add**

Queries

Queries

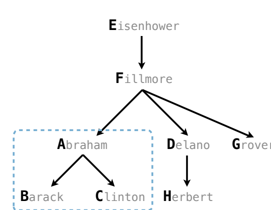
A *query* contains one or more relations that may contain variables.

Variables are symbols starting with **?**

```
logic> (fact (parent delano herbert))
logic> (fact (parent abraham barack))
logic> (fact (parent abraham clinton))
logic> (fact (parent fillmore abraham))
logic> (fact (parent fillmore delano))
logic> (fact (parent fillmore grover))
logic> (fact (parent eisenhower fillmore))
logic> (query (parent abraham ?puppy))
Success!
puppy: barack
puppy: clinton
```

A variable can have any name

Each line is an assignment of variables to values



(Demo)

Compound Facts and Queries

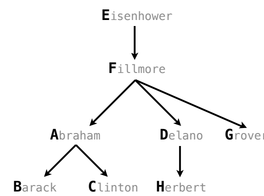
Compound Facts

A fact can include multiple relations and variables as well.

(fact <conclusion> <hypothesis$_0$> <hypothesis$_1$> ... <hypothesis$_N$>)

Means <conclusion> is true if all the <hypothesis$_K$> are true.

```
logic> (fact (child ?c ?p) (parent ?p ?c))

logic> (query (child herbert delano))
Success!

logic> (query (child eisenhower clinton))
Failure.

logic> (query (child ?kid fillmore))
Success!
kid: abraham
kid: delano
kid: grover
```



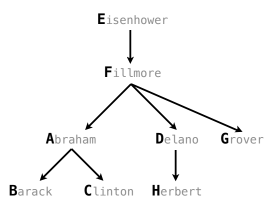
Compound Queries

An assignment must satisfy all relations in a query.

(query <relation$_0$> <relation$_1$> ... <relation$_N$>)

is satisfied if all the <relation$_K$> are true.

```
logic> (fact (child ?c ?p) (parent ?p ?c))

logic> (query (parent ?grampa ?kid)
              (child clinton ?kid))
Success!
grampa: fillmore    kid: abraham

logic> (query (child ?y ?x)
              (child ?x eisenhower))
Success!
y: abraham    x: fillmore
y: delano     x: fillmore
y: grover     x: fillmore
```

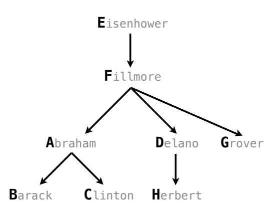


Recursive Facts

Recursive Facts

A fact is recursive if the same relation is mentioned in a hypothesis and the conclusion.

```
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))

logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower

logic> (query (ancestor ?a barack)
              (ancestor ?a herbert))
Success!
a: fillmore
a: eisenhower
```



Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))

(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
(parent fillmore delano)     ; (3), a simple fact
(ancestor fillmore herbert) ; (4), from (2), (3), & the 2nd ancestor fact
```

Hierarchical Facts

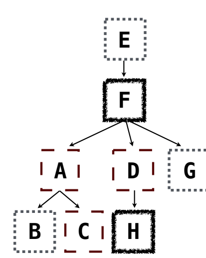
Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

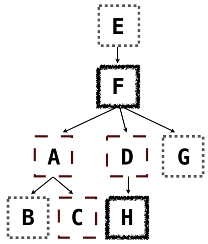
```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long

logic> (query (dog (name clinton) ?stats))
Success!
stats: (fur long)
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack     fur: short    y: eisenhower
x: clinton    fur: long     y: abraham
x: grover     fur: short    y: eisenhower
x: herbert    fur: curly    y: fillmore
```



Appending Lists

(Demo)

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y        ?z ))
```

Simple fact: Conclusion

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

```
      ?x      ?x
() (c d) => (c d)

(b) (c d) => (b c d)
?r   ?y        ?z

(e b) (c d) => (e b c d)
(e . (b)) (c d) => (e . (b c d))
?a    ?r    ?y     ?a     ?z
(?a . ?r)          (?a . ?z)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

(Demo)

Which Hypotheses Complete append-3?

I can append (a b) and (1 2) and (x y) to form (a b 1 2 x y).

```
(fact (append-3 ?x ?y ?z ?xyz)

  1:  (append-to-form ?x ?y ?xy)
      (append-to-form ?y ?z ?yz) )

  2:  (= (append-to-form ?x ?y) ?xy)
      (= (append-to-form ?y ?z) ?yz) )

  3:  (append-to-form ?x ?y ?xy)
      (append-to-form ?xy ?z ?xyz) )

  4:  (= (append-to-form ?x ?y) ?xy)
      (= (append-to-form ?xy ?z) ?xyz) )

  5:  None of the above
```


Define Base Fact of append-to-form So That No Lists Can Be Empty

~~; (append-to-form () (1 2 3) (1 2 3))~~

; (append-to-form (1) (2 3) (1 2 3))

; (append-to-form (1 2) (3) (1 2 3))

~~; (append-to-form (1 2 3) () (1 2 3))~~

1: (fact (append-to-form () ?x ?x))

2: (fact (append-to-form ?a ?x (?a . ?x)))

3: (fact (append-to-form ?a (?b . ?x) (?a ?b . ?x)))

4: (fact (append-to-form (?a) ?x (?a . ?x)))

5: (fact (append-to-form (?a) (?b . ?x) (?a ?b . ?x)))

Recursive fact:
```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y        ?z))
```


Permuting Lists

Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element | List | List with ?a in front | Bigger list with ?a somewhere | List with ?a somewhere

```
(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
      (insert ?a       ?r        ?s))

(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
      (insert   ?a   ?s  ?b)
      (anagram  ?r   ?s))
```

```
a | r t

  r t
ar t
 rat
 r ta

  t r
at r
 tar
 t ra
```

(Demo)

Unification

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
(   ?x   c    ?x   )      True, {x: (a b)}

( (a  b) c  (a  b) )
( (a ?y) ?z (a  b) )      True, {y: b, z: c}

( (a  b) c  (a  b) )
(   ?x  ?x    ?x   )      False
```

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

```
( (a  b) c (a  b) )          ( (a  b) c  (a  b) )
(   ?x   c   ?x   )          (   ?x  ?x    ?x   )
          Lookup                     Lookup
         (a  b)                        c
         (a  b)                      (a b)
{ x: (a b) }                 { x: (a b) }
  Success!                     Failure.
```

Symbols/relations without variables only unify if they are the same

Unifying Variables

Two relations that contain variables can be unified as well.

```
(   ?x        ?x    )
((a ?y c) (a b ?z))      True, {x: (a ?y c),
      Lookup                    y: b,
   (a ?y  c)                    z: c}
   (a  b ?z)
```

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ (a ?y c)   **lookup('?y')** ⇨ b   **ground('?x')** ⇨ (a b c)

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a  b) c (a  b) )
(   ?x   c   ?x   )
          Lookup
         (a  b)
         (a  b)
env: { x: (a b) }
```

Search

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))
(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))
(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}       (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}       (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}      (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

?left: (e . (b)) ⇨ (e b)

?r: (b . ()) ⇨ (b)

Depth-First Search

The space of facts is searched exhaustively, starting from the query and following a *depth-first* exploration order.

Depth-first search: Each proof approach is explored exhaustively before the next.

```
def search(clauses, env):
  for fact in facts:
    env_head = an environment extending env
    if unify(conclusion of fact, first clause, env_head):
      for env_rule in search(hypotheses of fact, env_head):
        for result in search(rest of clauses, env_rule):
          yield each successful result
```

Environment now contains new unifying bindings

- Limiting depth of the search avoids infinite loops.
- Each time a fact is used, its variables are renamed.
- Bindings are stored in separate frames to allow backtracking.

(Demo)

Addition

(Demo)