61A Extra Lecture 5

Announcements

Data Representations

Functions with Shared Local State

Interactive Diagram

Functions with Shared Local State

```python
def box(contents):
    def get():
        return contents
    def put(value):
        nonlocal contents
        contents = value
    return get, put

get, put = box('Hello')
before = get()
put('Goodbye')
after = get()
```

Interactive Diagram

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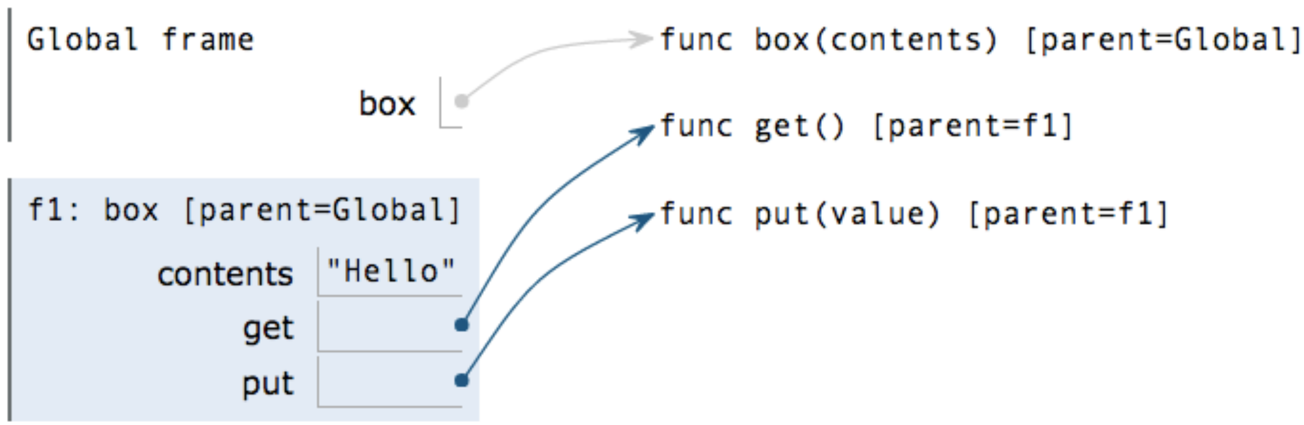

Interactive Diagram

Pairs Implemented as Functions

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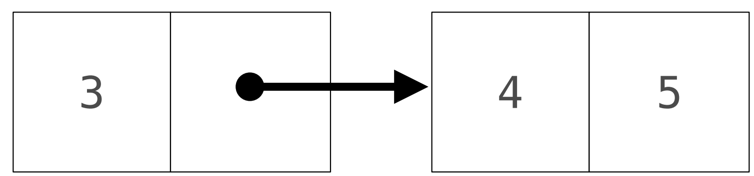

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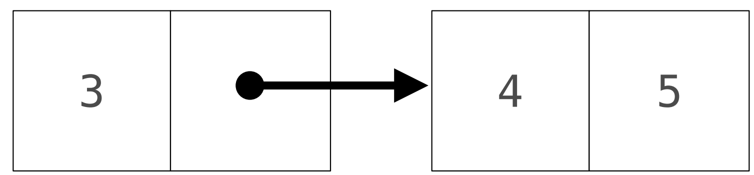

(Demo)

Linked Lists (Sneak Preview)

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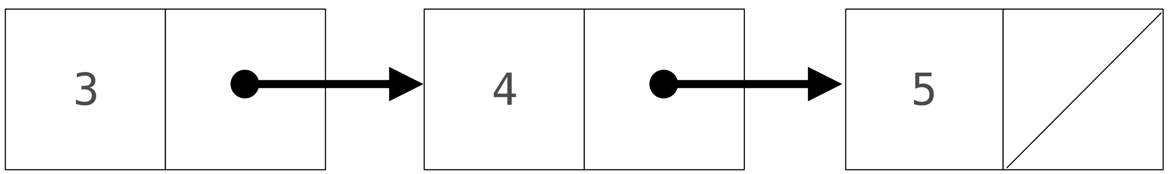

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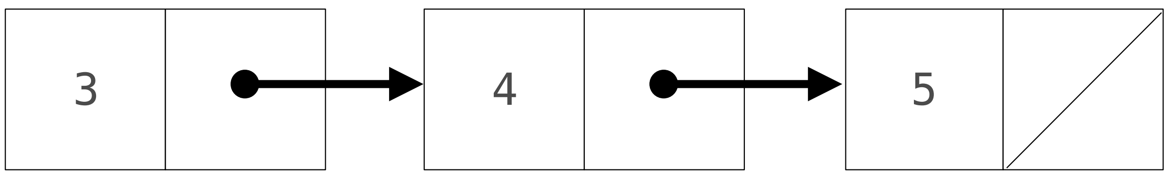


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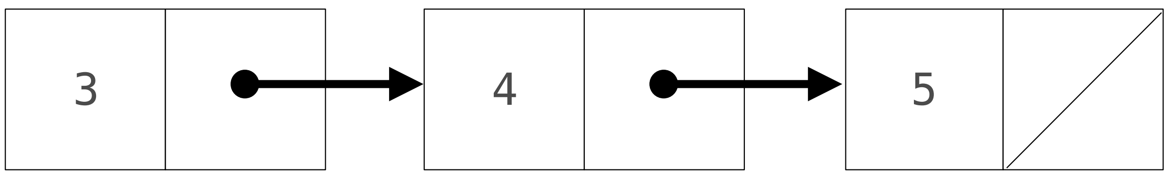


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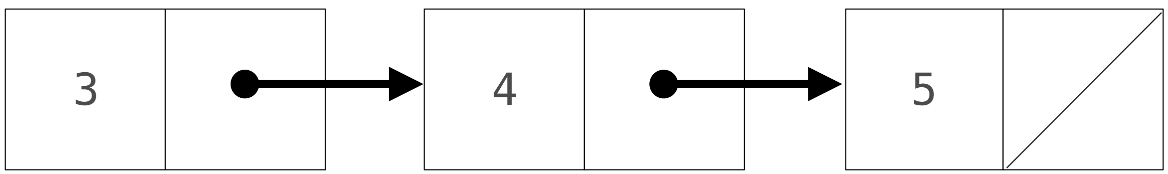


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def list_len(s):
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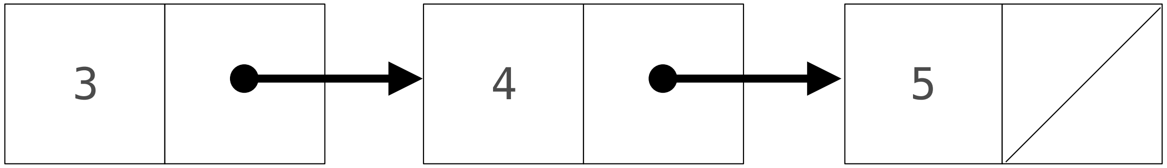


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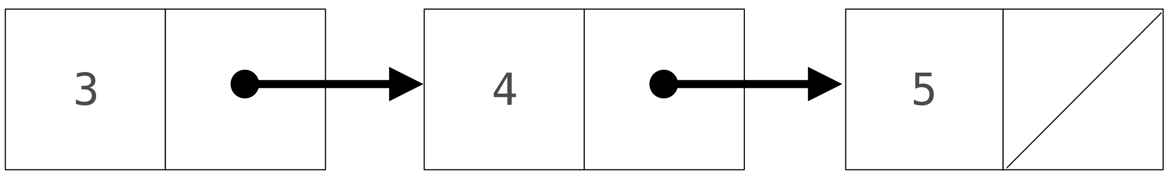


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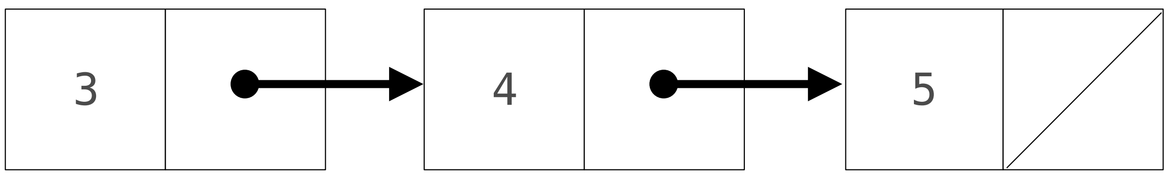


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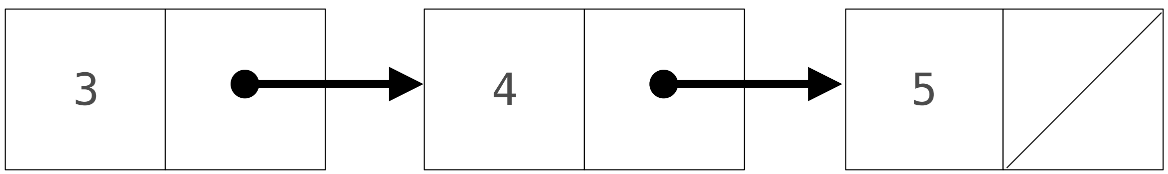


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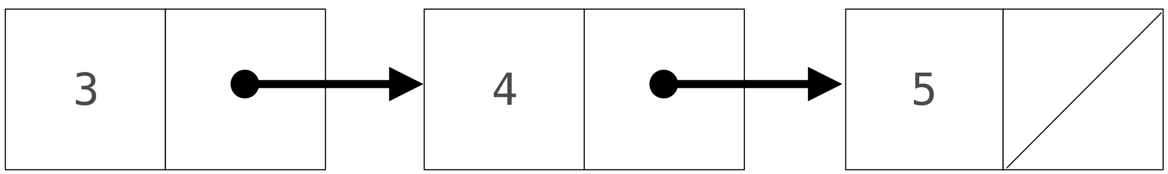


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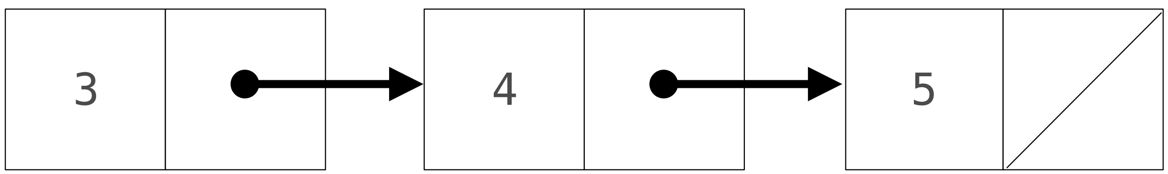


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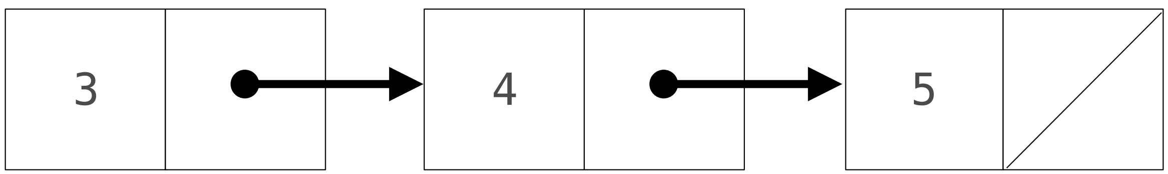


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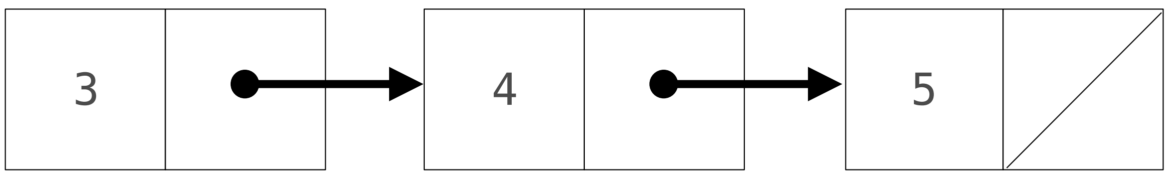


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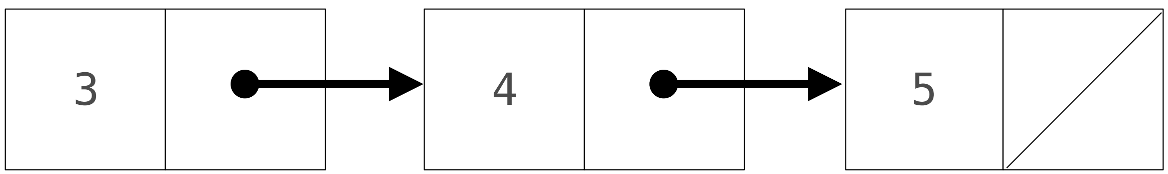


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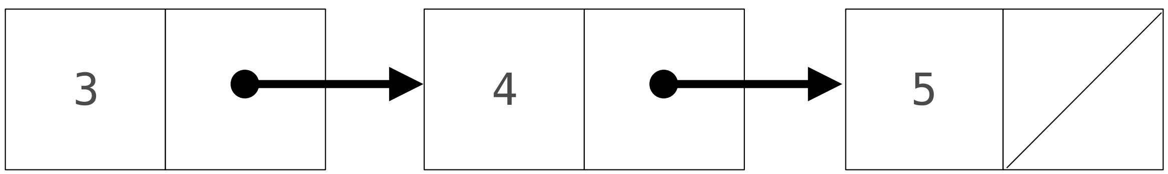


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(Demo)

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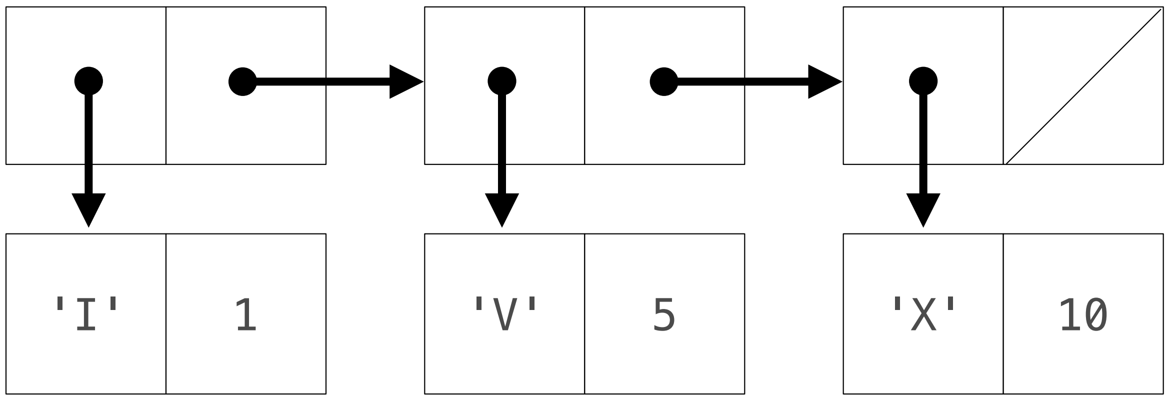


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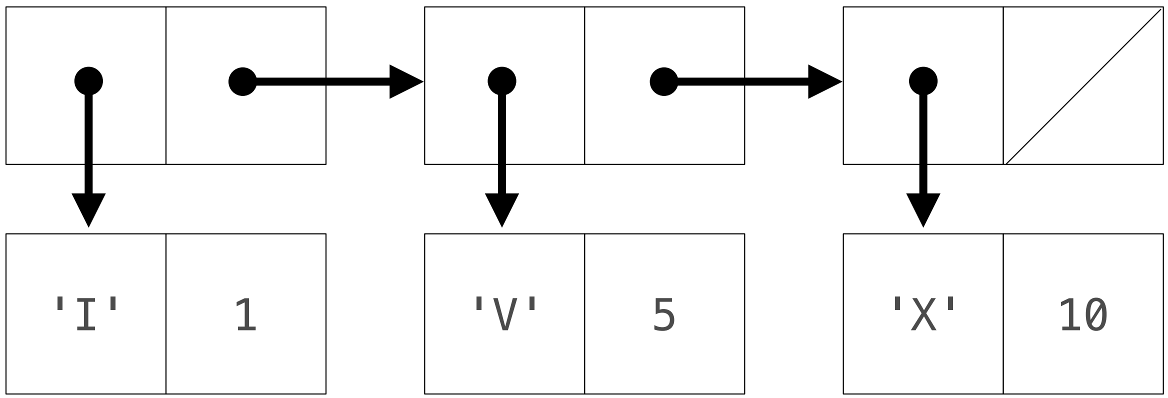


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Constraint Networks

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Solving for Variables in an Equation

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$$p * v = n * k * t$$
$$9 * c = 5 * (f - 32)$$

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A Constraint Network for Temperature Conversion

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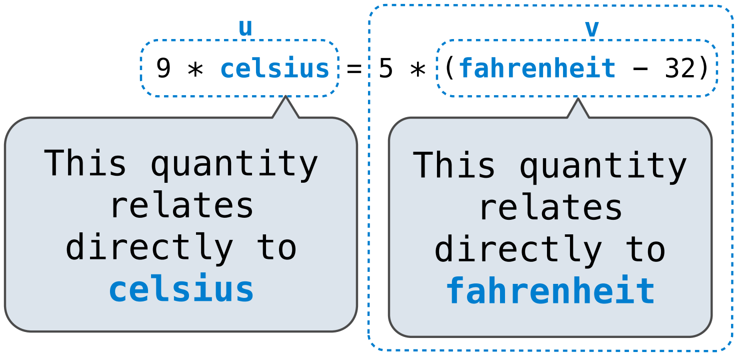

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```
      u                    v
9 * celsius = 5 * (fahrenheit - 32)
```

This quantity relates directly to **celsius**

This quantity relates directly to **fahrenheit**

Both sides of the equation are equal: they must be the same quantity

A Constraint Network for Temperature Conversion

Combination idea: All intermediate quantities have values too.

$$\overbrace{9 * \texttt{celsius}}^{u} = \overbrace{5 * \overbrace{(\texttt{fahrenheit} - 32)}^{v}}^{u}$$

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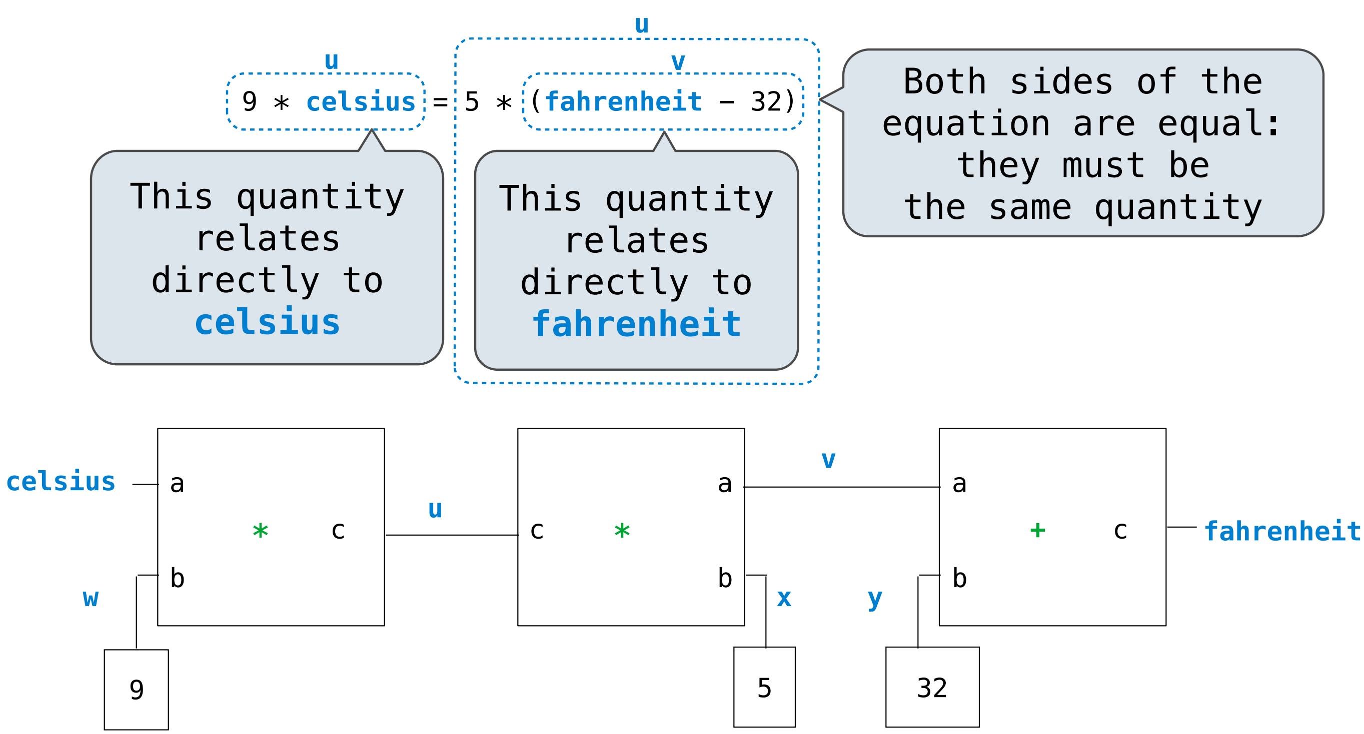
A Constraint Network for Temperature Conversion

Combination idea: All intermediate quantities have values too.



A Constraint Network for Temperature Conversion

Combination idea: All intermediate quantities have values too.

$$\overset{u}{\boxed{9 * \textbf{celsius}}} = \overset{u}{\boxed{5 * \overset{v}{\boxed{(\textbf{fahrenheit} - 32)}}}}$$

This quantity relates directly to **celsius**

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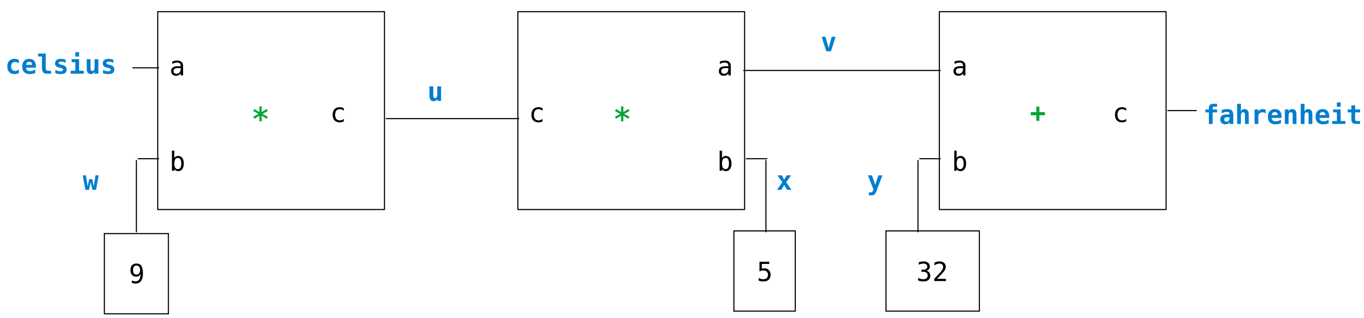

(Demo)