61A Extra Lecture 2

Monday, September 14

Announcements

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You have a midterm in two days.

Decision Making

The Hog End Game

The Hog End Game

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Them: 99

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$$P(S_n \geq k) = \sum_{t=2}^{6} P(t) \cdot P(S_{n-1} \geq k - t)$$

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Can compute using tree recursion!!!

Memoization

(Sort of like memorization)

Computing Fibonacci

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    Return 0
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    return fib(n-2) + fib(n-1)
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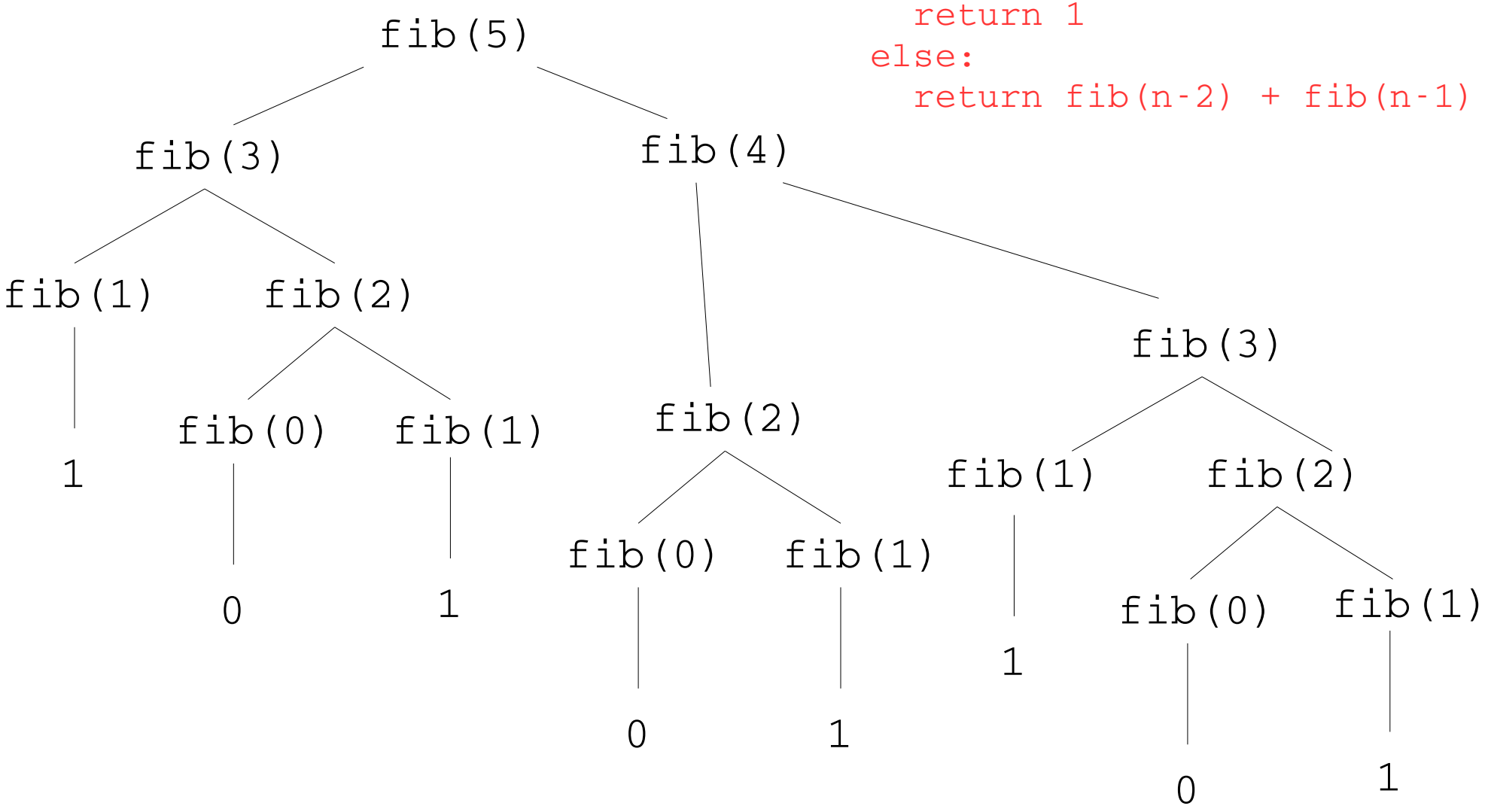

Memoized Fibonacci

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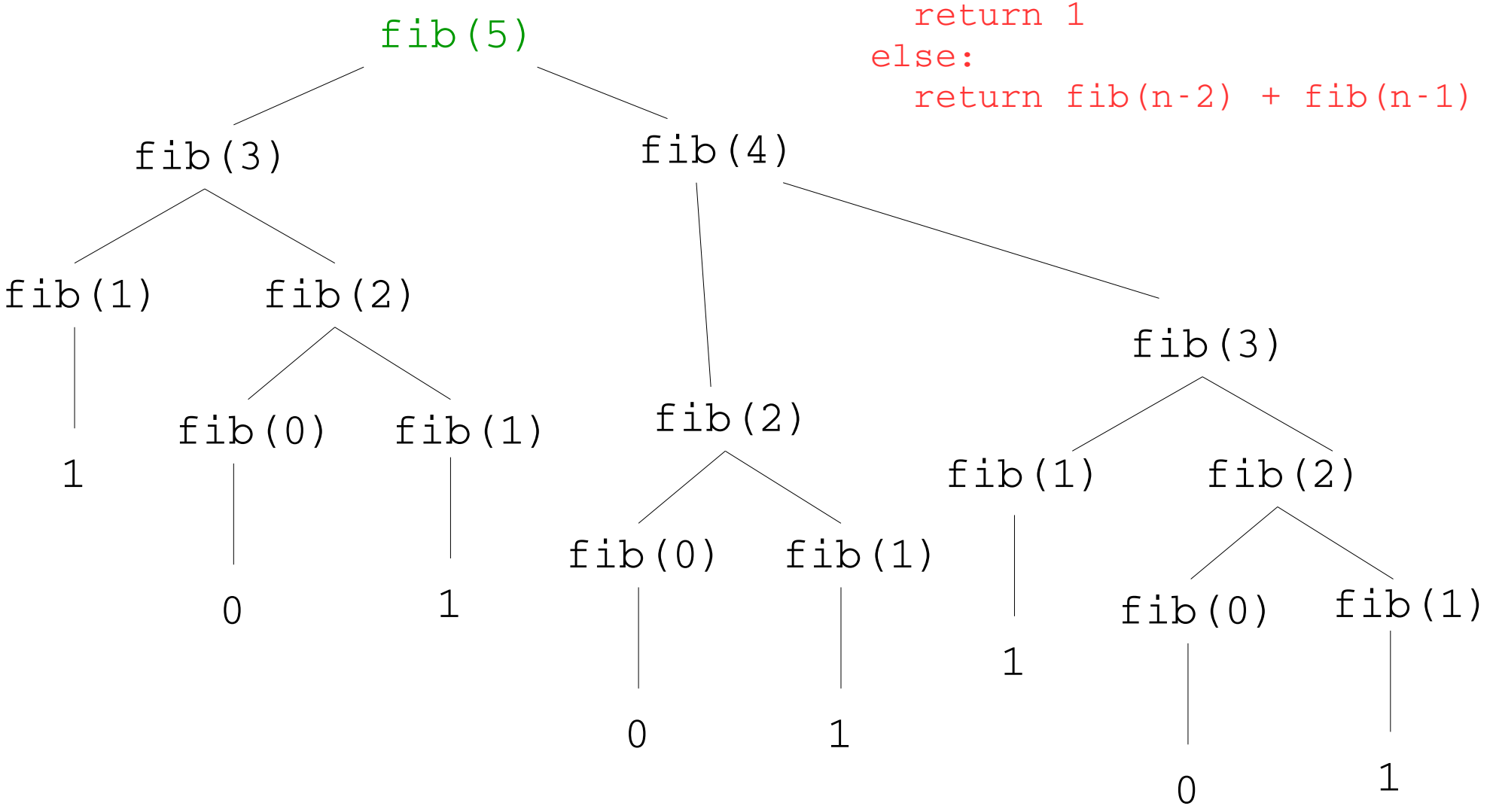

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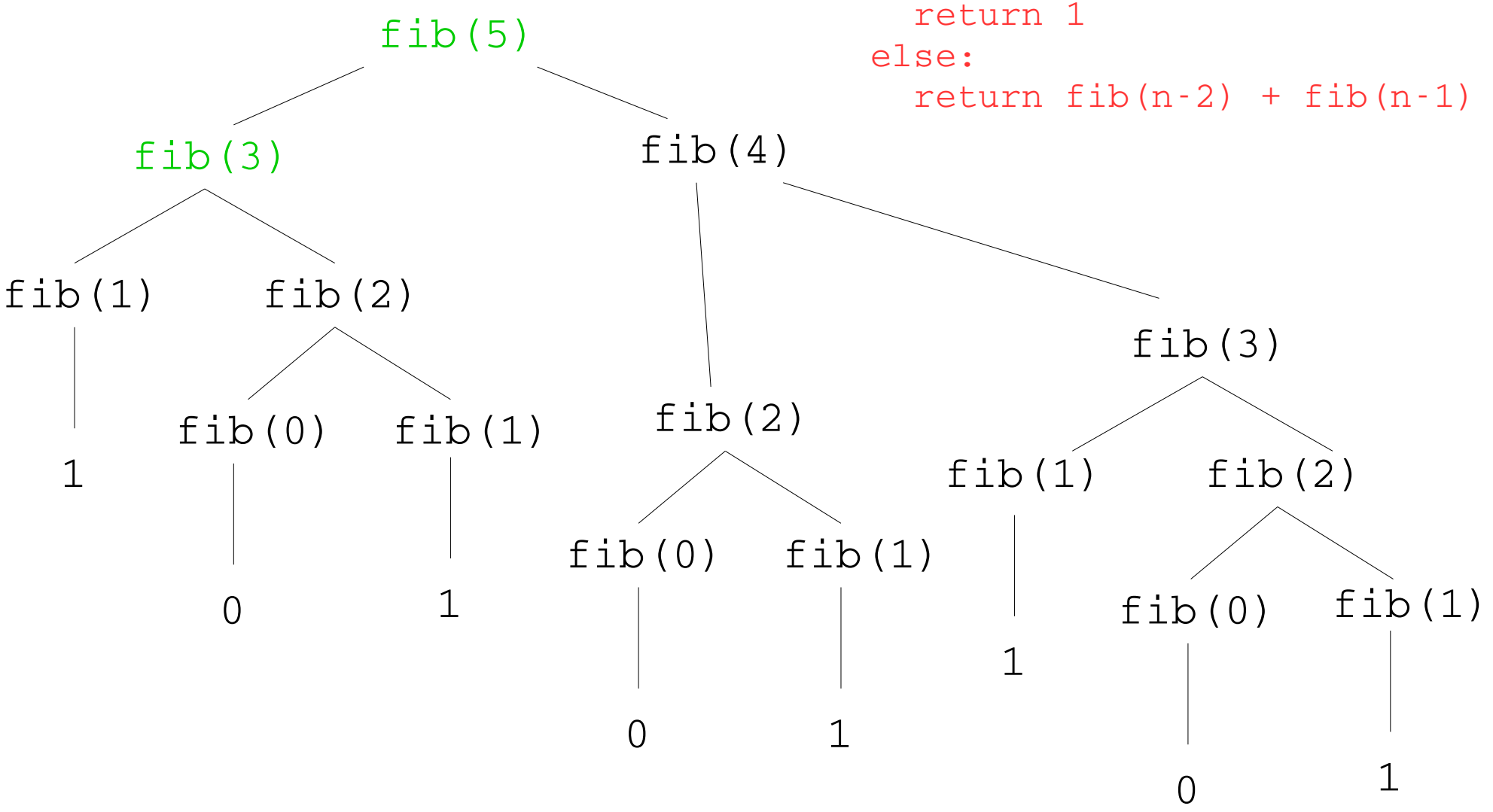

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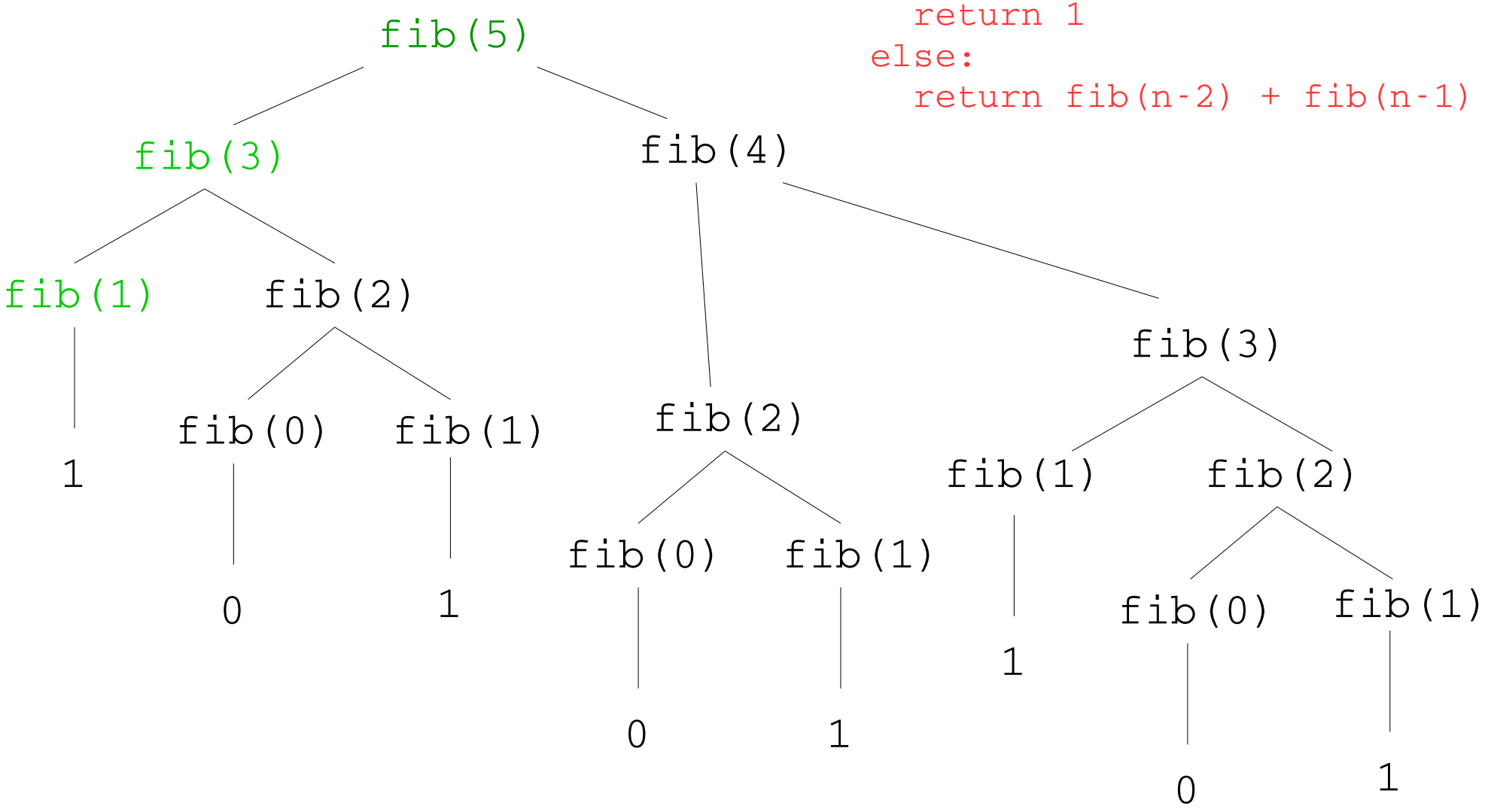

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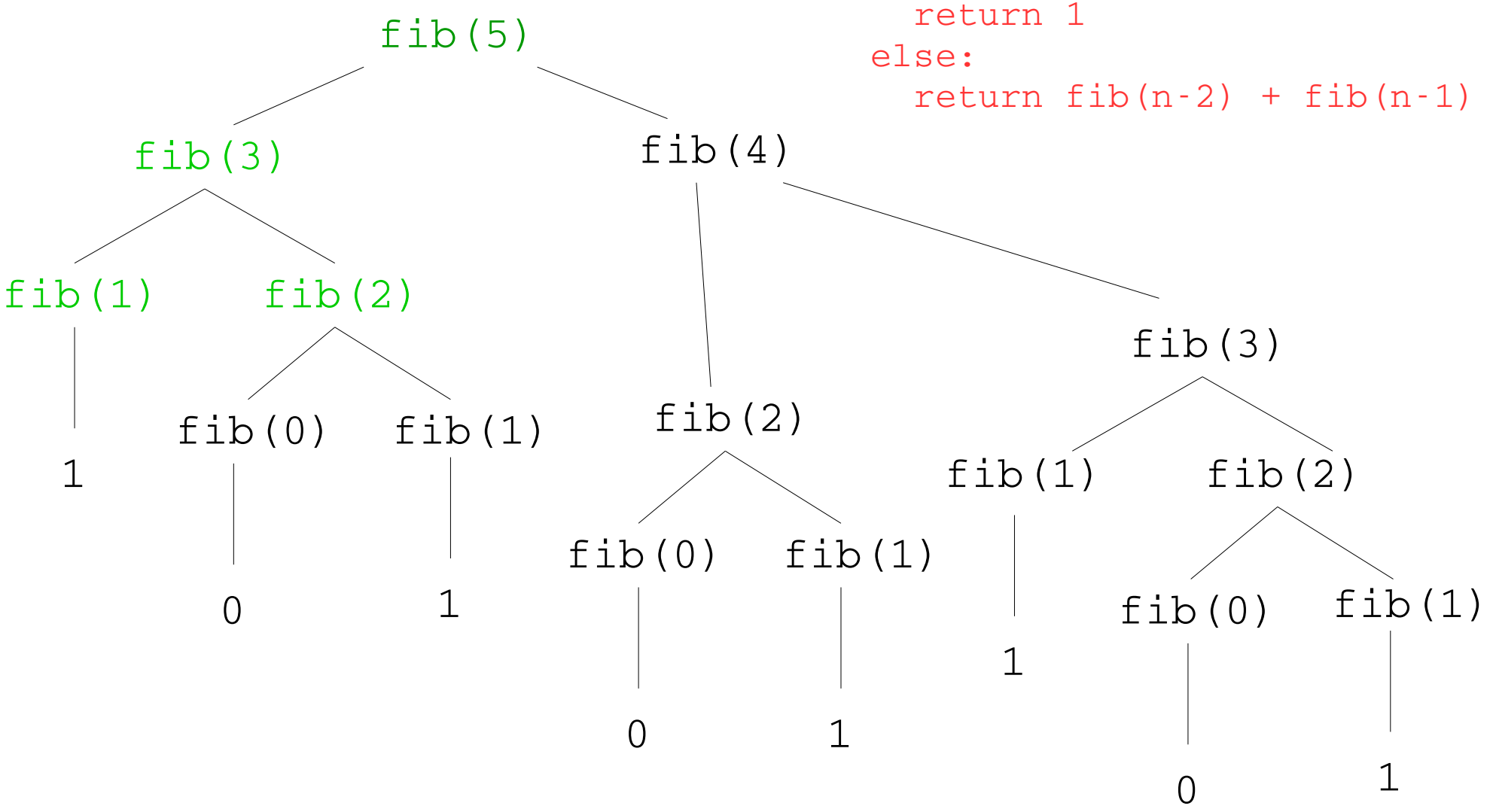

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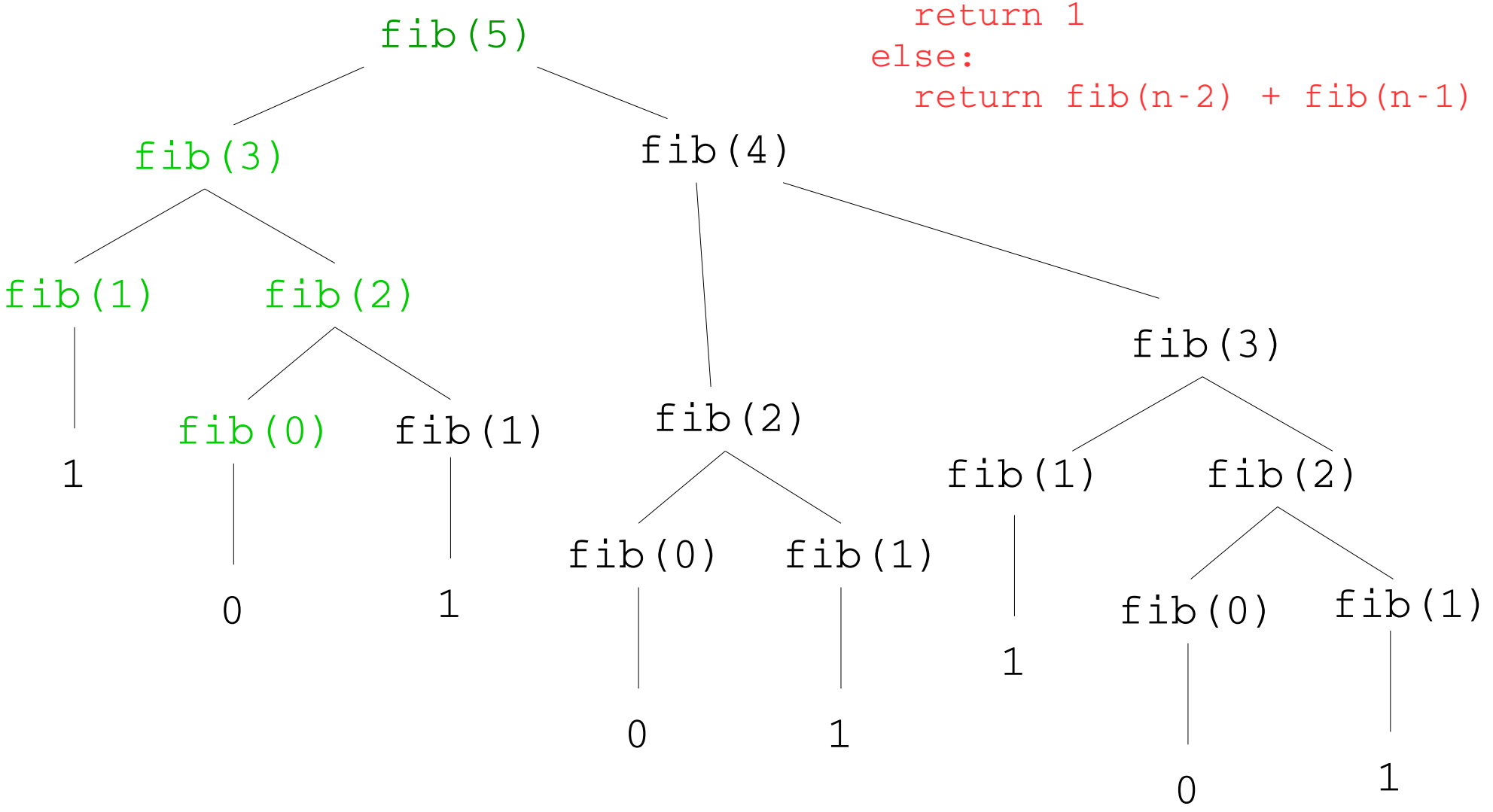

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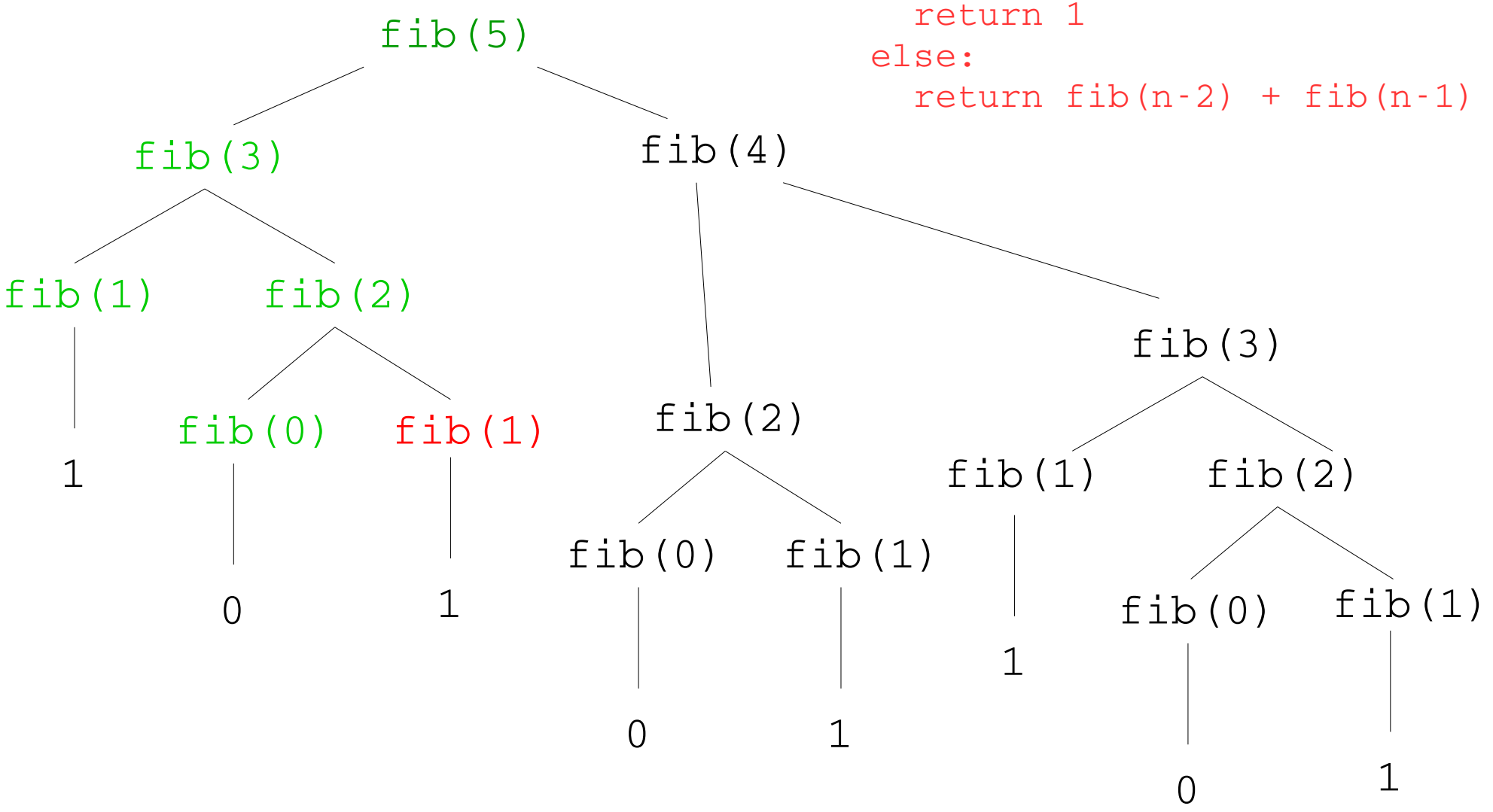

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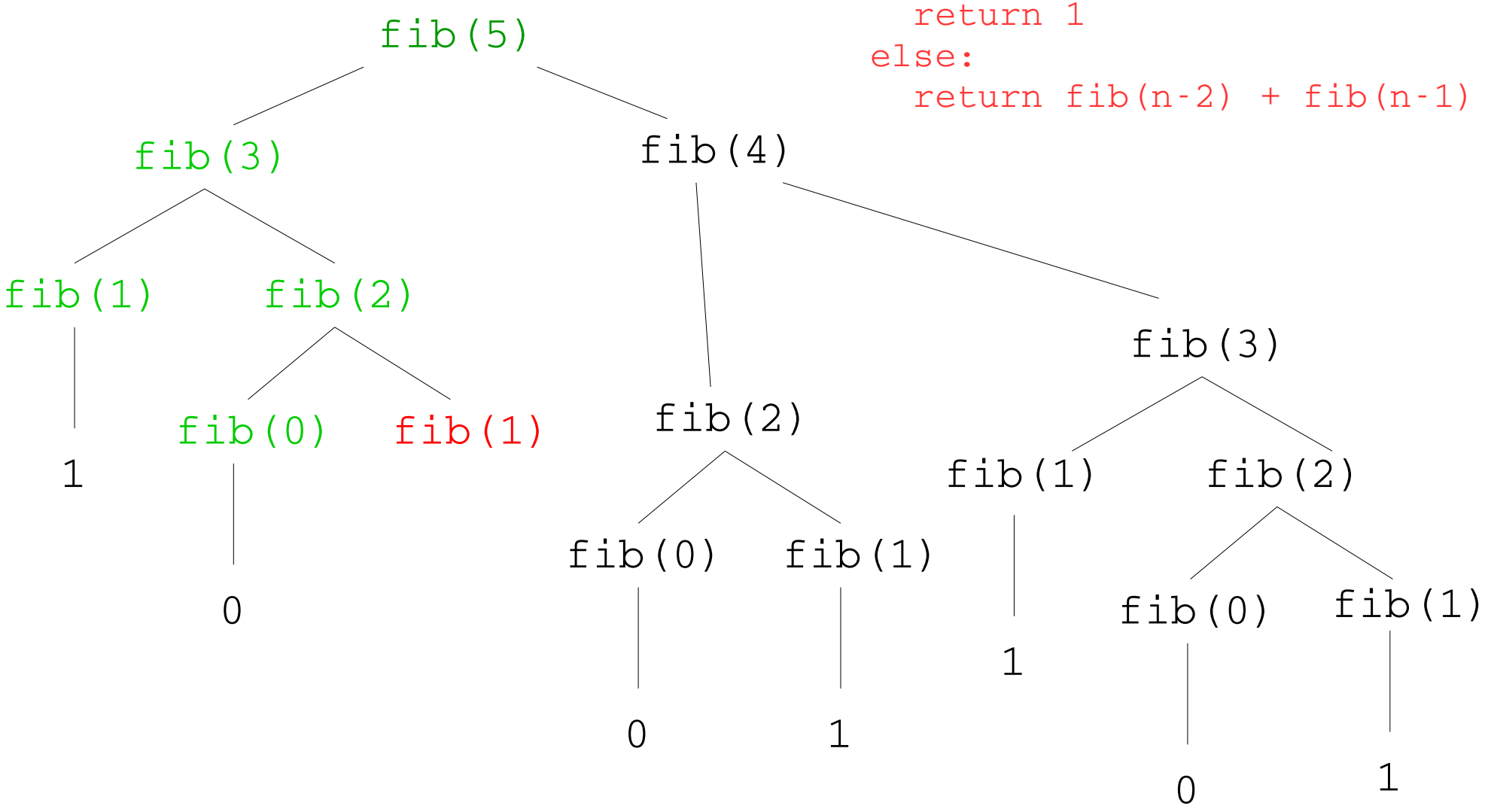

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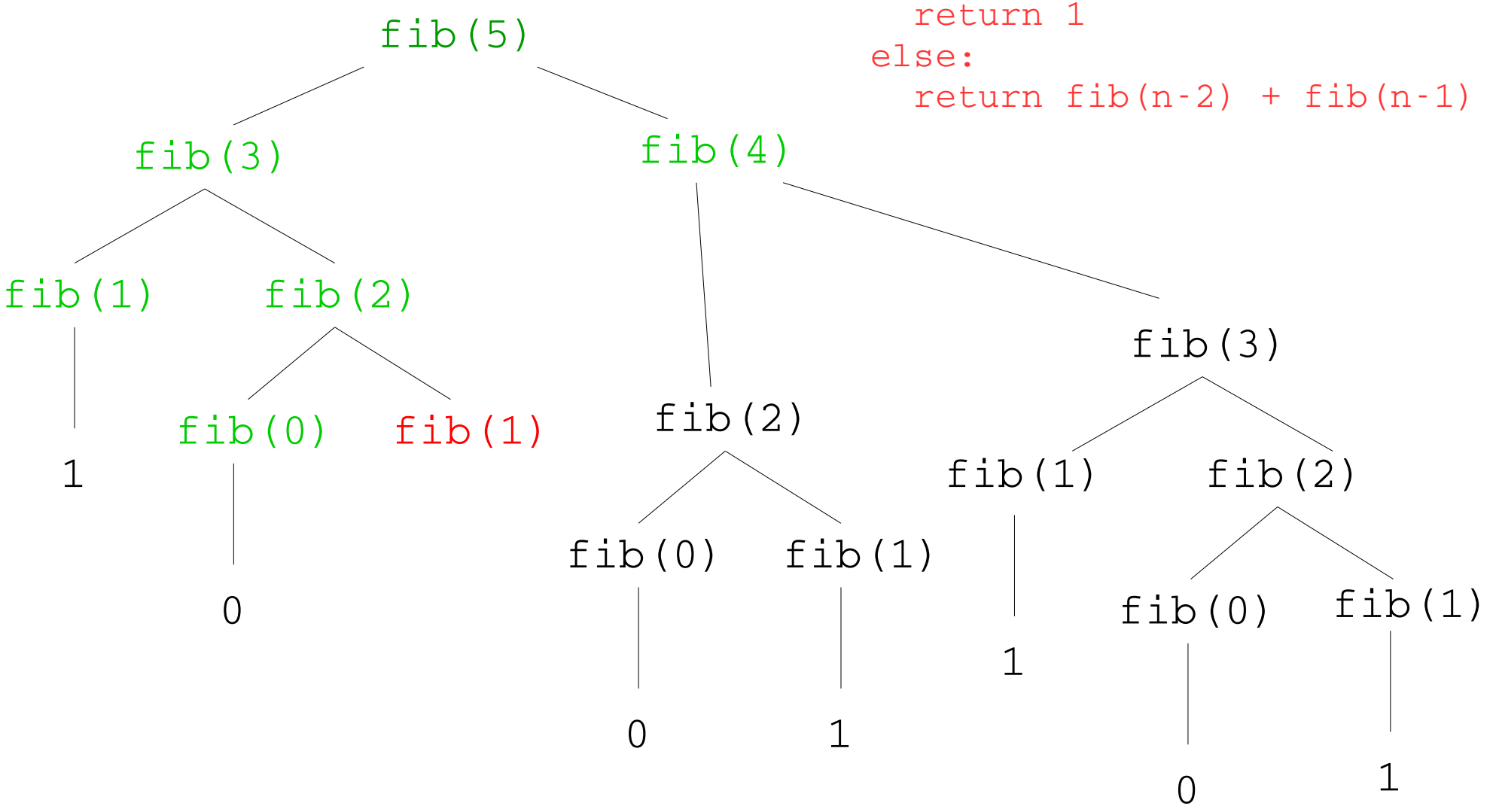

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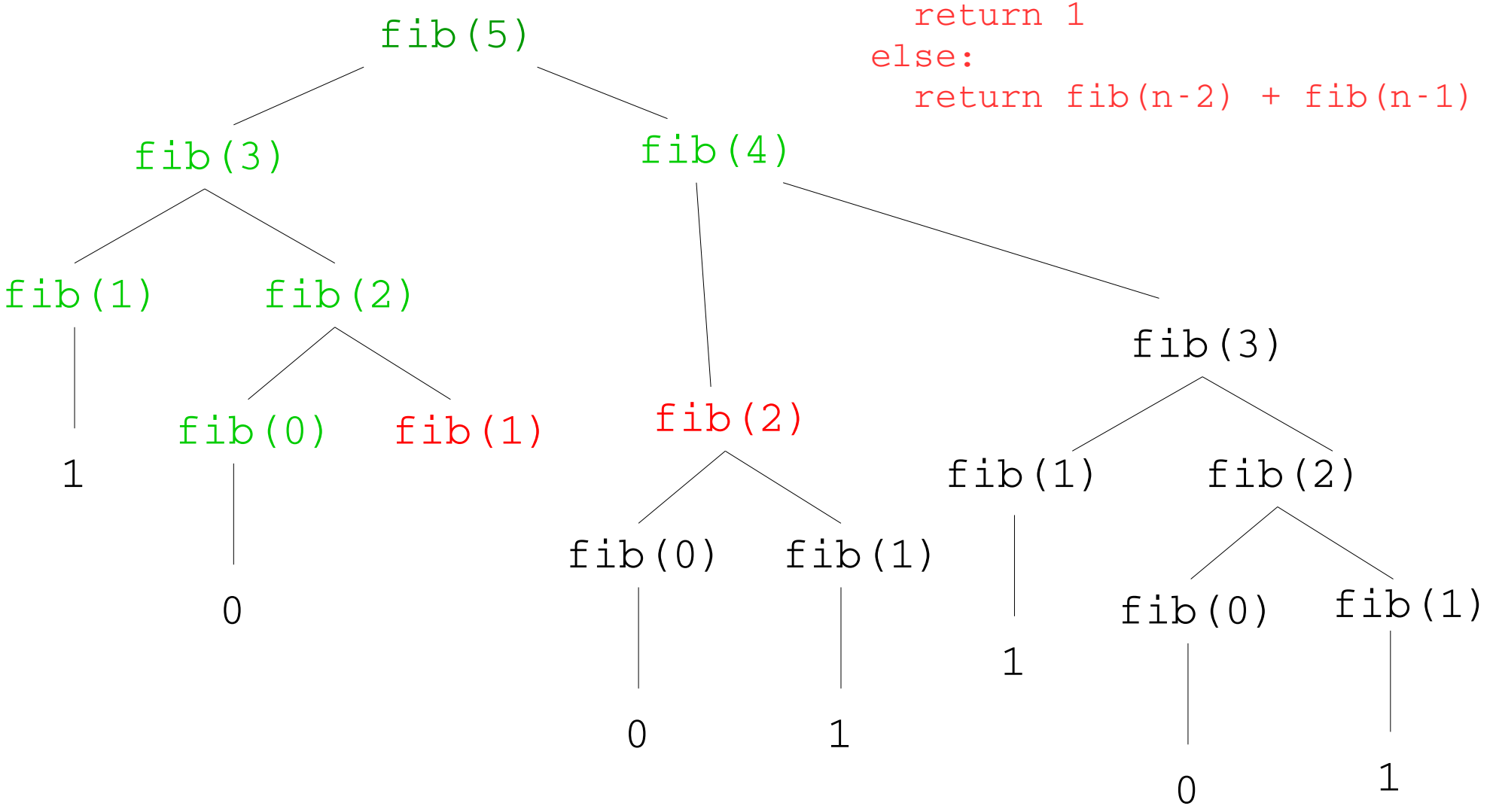

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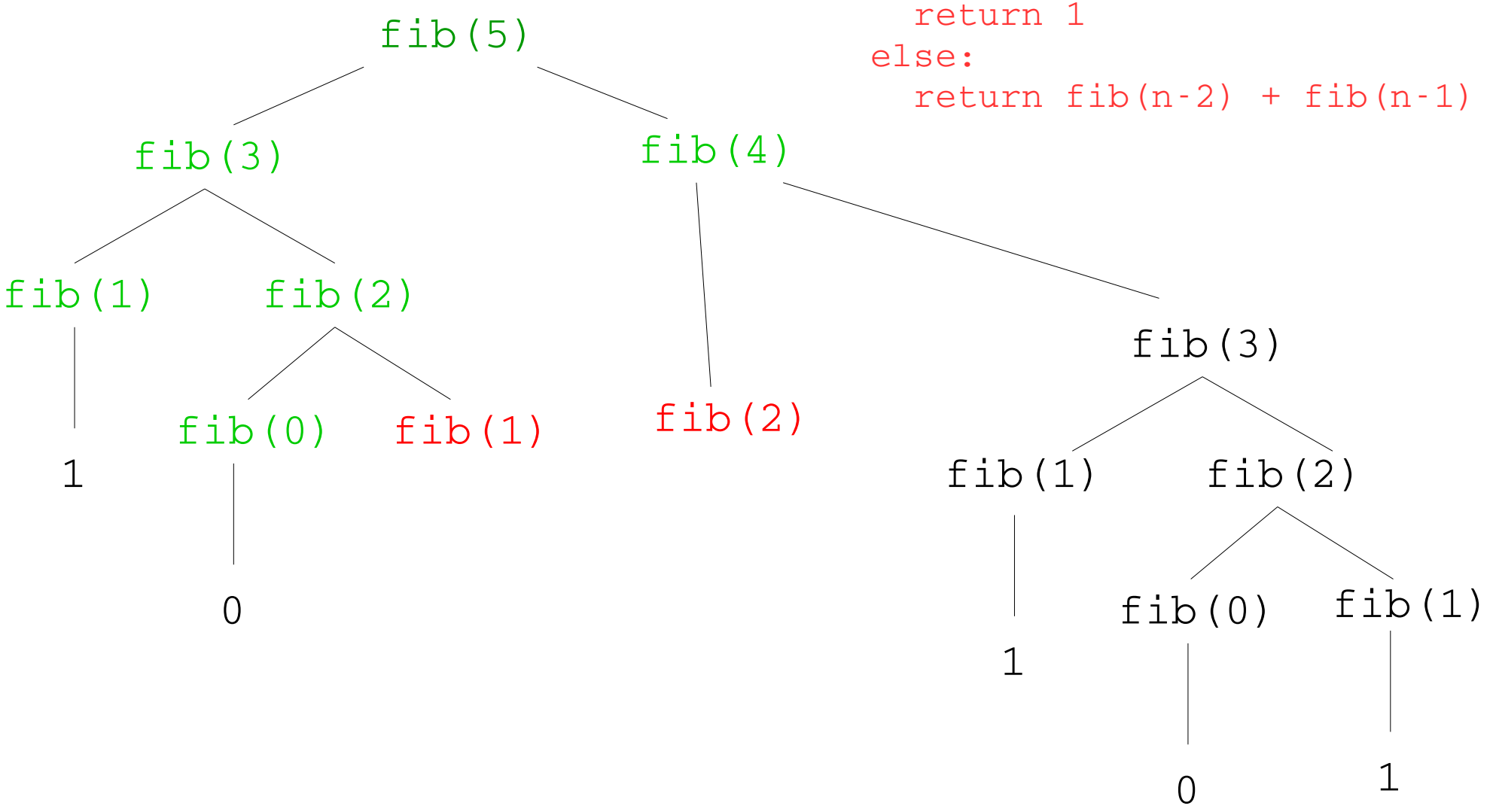

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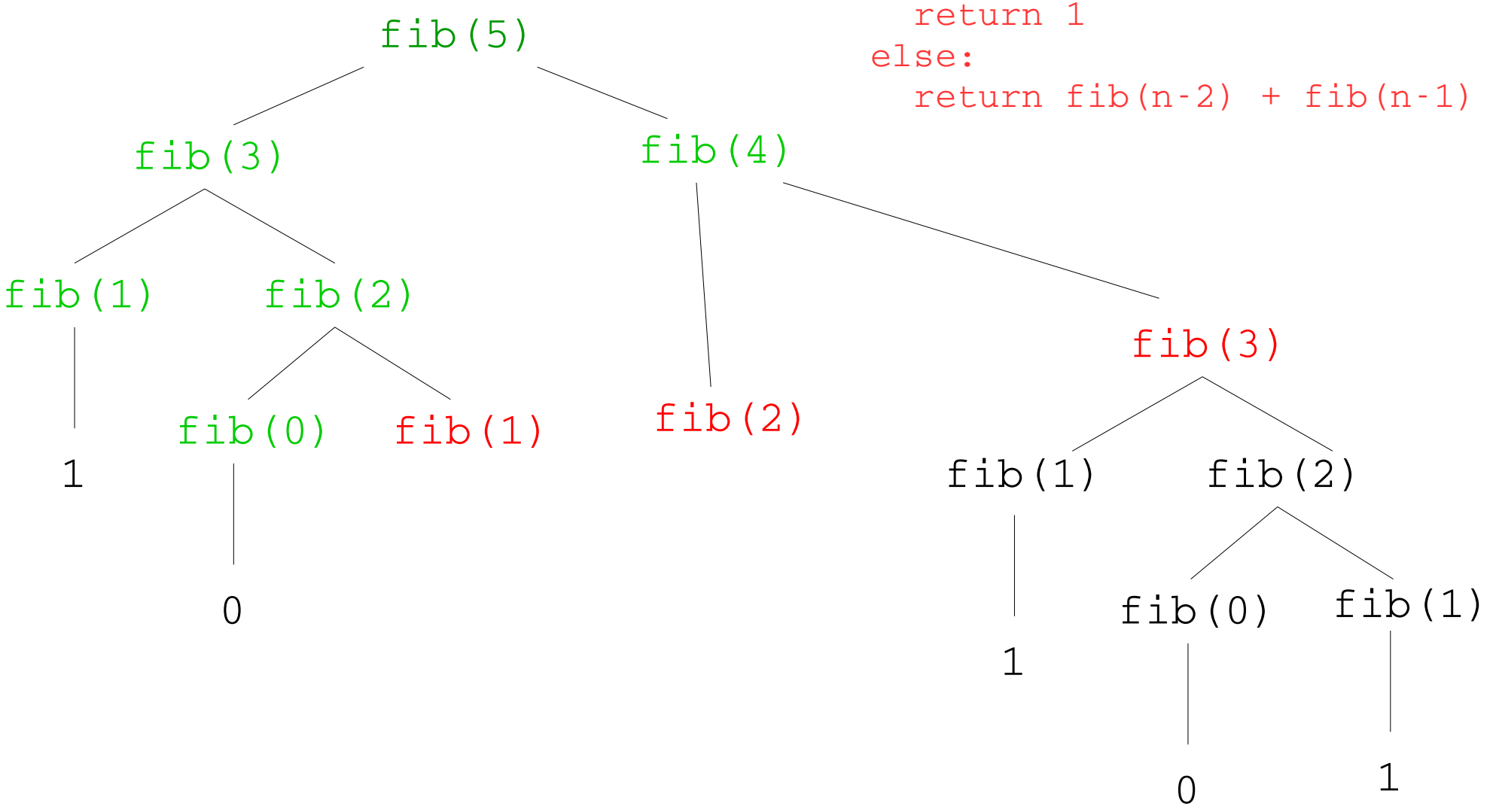

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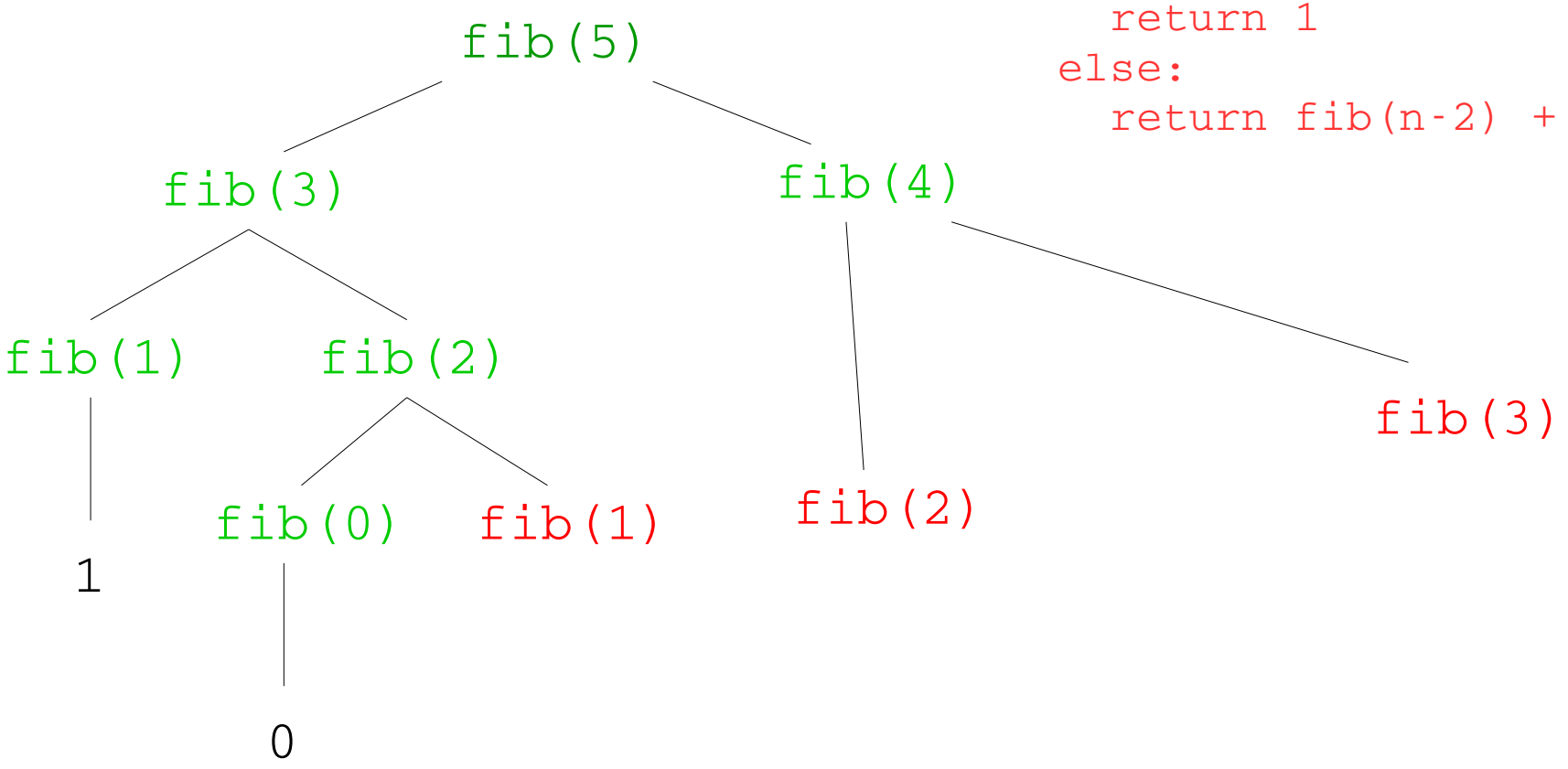
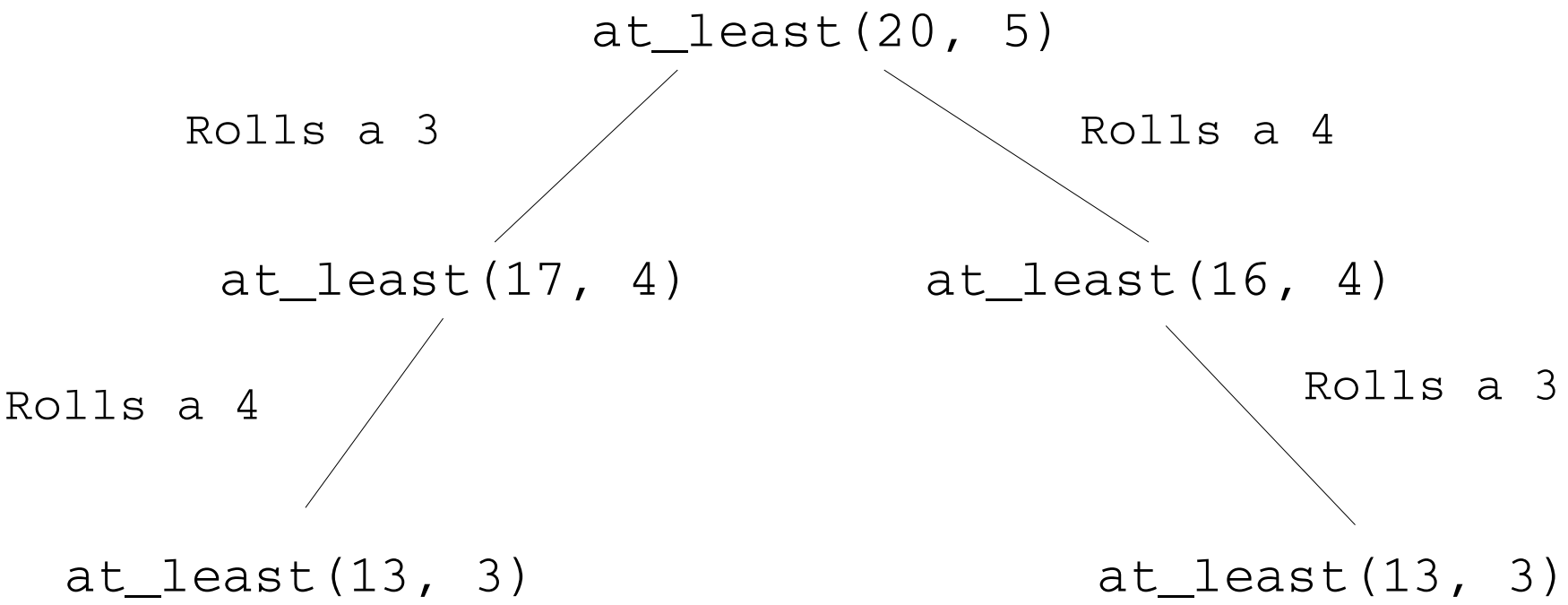


Memoized at_least

- Rolling a 3 then 4 is equivalent to 4 then 3



- We can just use the same calculated value again!

Twenty-One (Nim)

Twenty-One Rules

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Let's play!

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Some states are good; some are bad

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21

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21 ◄—— **20**

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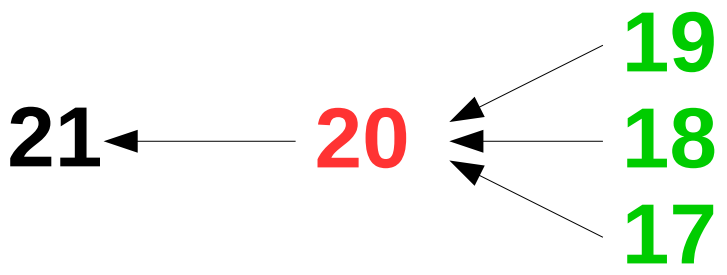

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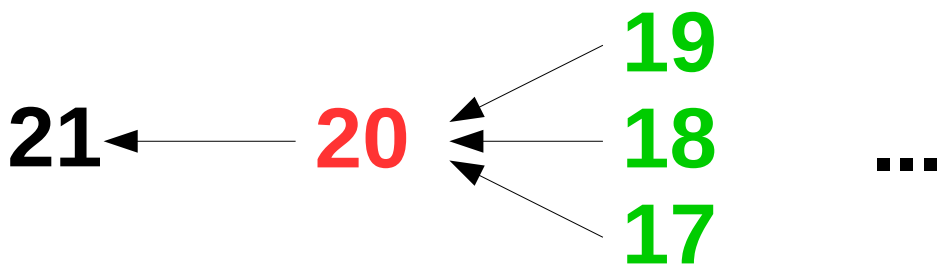


hog

a game of throws

recap

- sequential
- stochastic
- zero-sum
- perfect information

state

- how do we keep track of where we are?

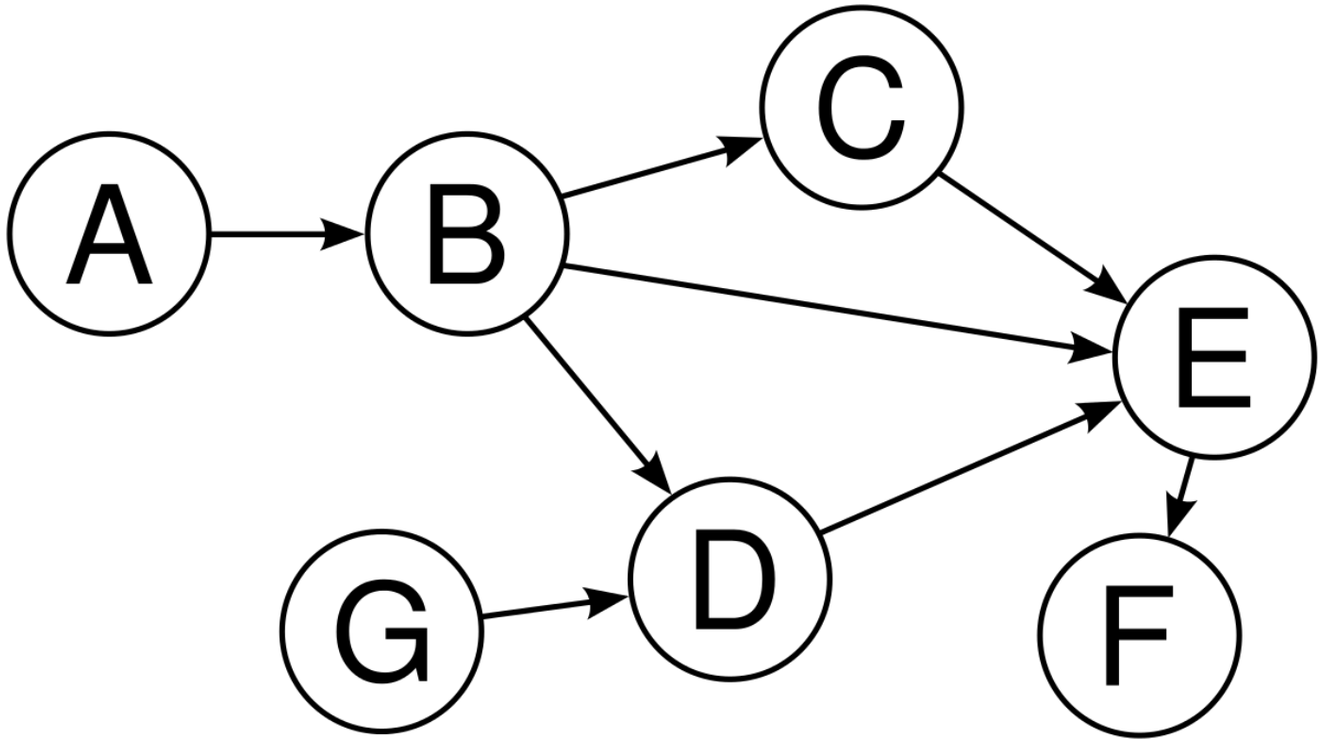
```
(p0_score, p1_score, who)
```

hog strats

- (demo)

graphs

- vertices and edges
- can be directed!



By David W. at de.wikipedia (Original text : David W.) [Public domain], from Wikimedia

hog…is just a graph!

- each state in the game can be expressed as a vertex in the graph

- edges represent transitions between states (whenever a player makes a move)

game trees

- sum of scores is always strictly increasing
- directed acyclic graph (DAG)
  - no cycles
  - sources lead to sinks

probability 101

- expected value

$$\mathbb{E}[X] = \sum_{x} xP(X = x)$$

$$\mathbb{E}[S_n] = \sum_{s=0}^{6n} sP(S_n = s)$$

calculating winrates

- run thousands of experiments (empirical)
- use the power of expectation! (analytic)

calculating winrates

- Let $P(s0, s1)$ be the probability of winning when it's player 0's turn, and the scores are s0 to s1

$$P(s0, s1) = \sum_{s} P(S_n = s)(1 - P(s1, s0 + s))$$

$$P(s0 \geq 100, s1) = 1 \qquad P(s0, s1 \geq 100) = 0$$

probabilistic algorithms

- Nim (minimax)
  - *minimize* the loss from the worst-case scenario
- Hog (expectiminimax)
  - now there's randomness!
  - minimize the *expectation* of the worst-case scenario

countering strategies

- (demo)

contest rules

- *Pork Chop*: You get one chance to force a switch with your opponent

```
(p0_score, p1_score, who,
```

church numerals

electric boogaloo

church numerals

- what do numbers actually represent?

$$f^n(x)$$

successor

$$f^{n+1}(x) = f(f^n(x))$$

add

$$f^{m+n}(x) = f^m(f^n(x))$$

multiply

$$f^{mn}(x) = (f^m)^n(x)$$

exponentiation

$$n(m)(f) = m^n(f)$$

other operations?

- *predecessor* operation is opposite of *successor*
- subtraction, division $pred(f^n(x)) = f^{n-1}x$
- signed numbers!
- boolean logic!
- rationals, reals, and complex numbers???!!!11

that's all, folks

hog contest > midterm1