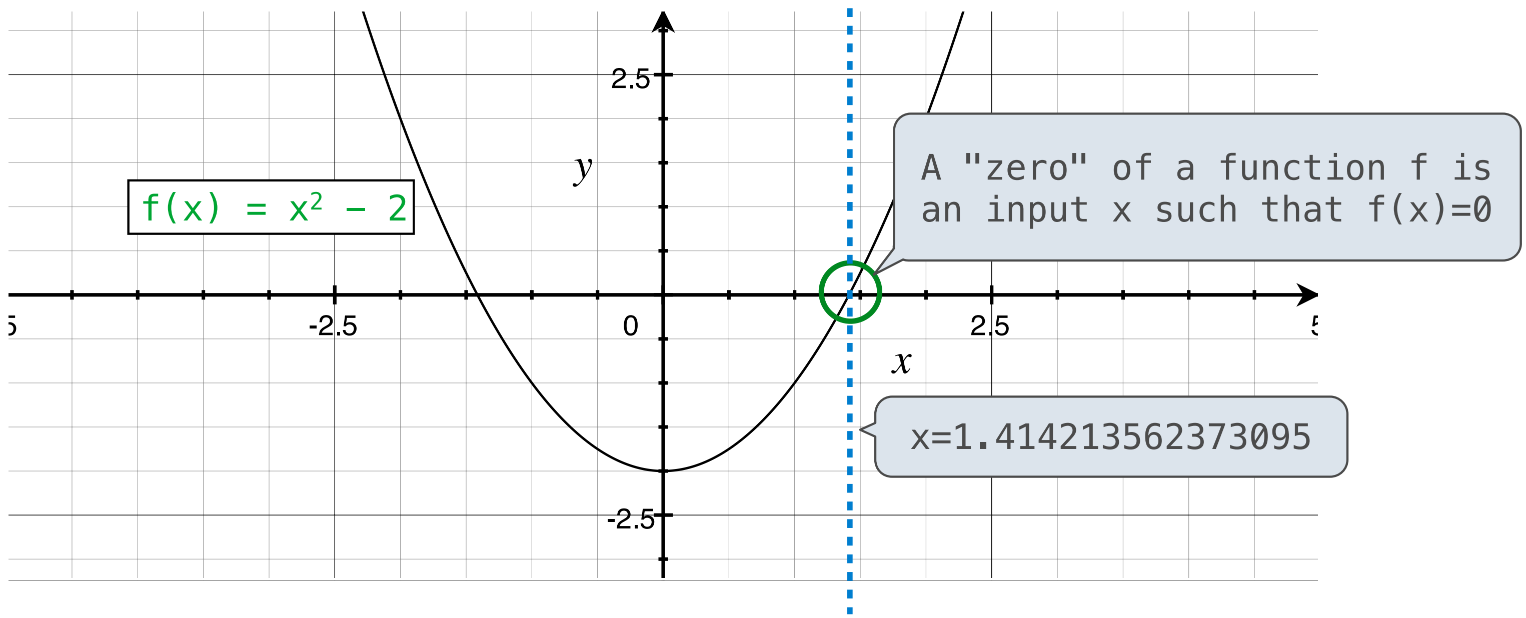
61A Extra Lecture 1

Announcements

Newton's Method

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!



Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

Newton's Method

Given a function f and initial guess x,

Repeatedly improve x:

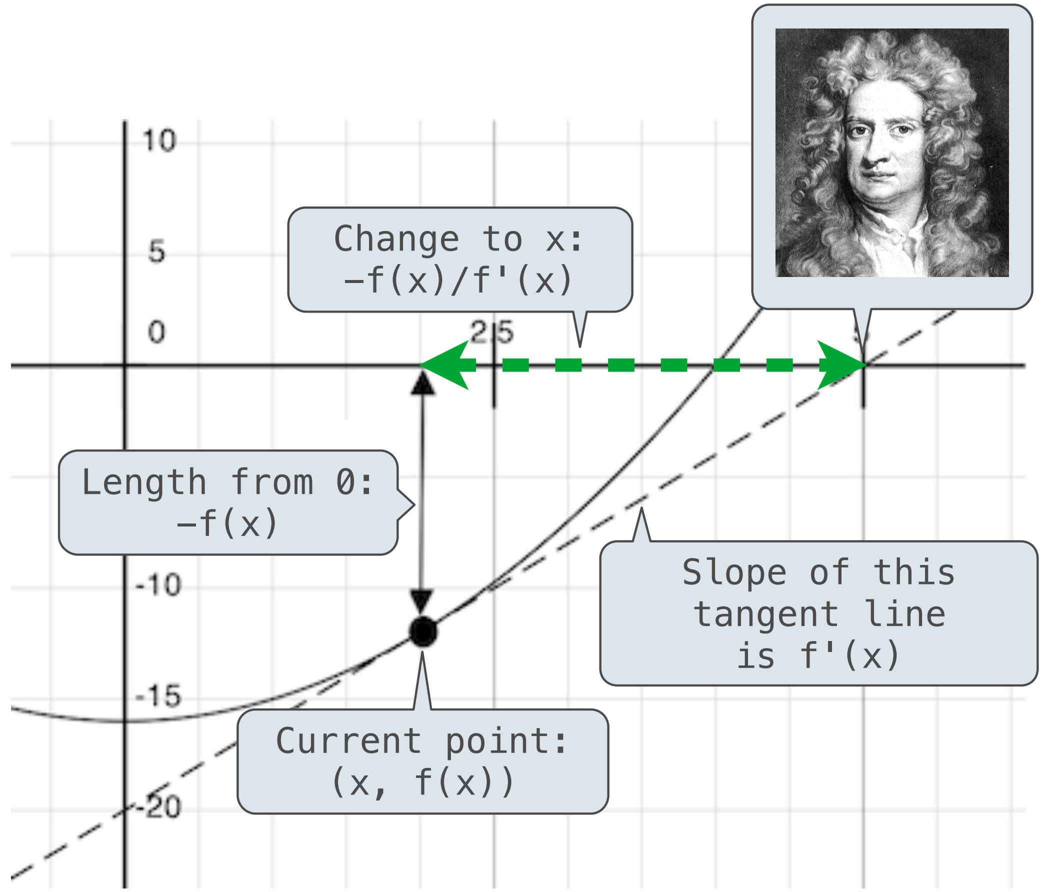
Compute the value of f
at the guess: f(x)

Compute the derivative
of f at the guess: f'(x)

Update guess x to be:

$$x - \frac{f(x)}{f'(x)}$$

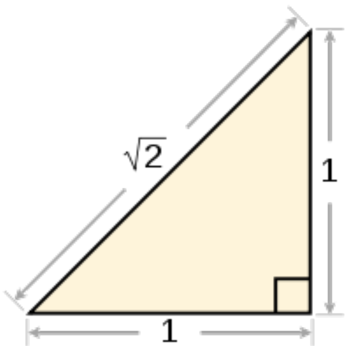
Finish when f(x) = 0 (or close enough)



http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

How to find the square root of 2?

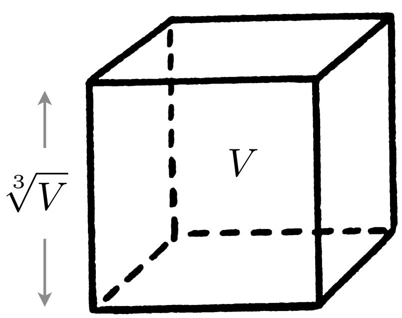



```
>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

$f(x) = x^2 - 2$
$f'(x) = 2x$

Applies Newton's method

How to find the cube root of 729?




```
>>> g  = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

$g(x) = x^3 - 729$
$g'(x) = 3x^2$

Iterative Improvement

Special Case: Square Roots

How to compute square_root(a)

Idea: Iteratively refine a guess x about the square root of a

Update: $$x = \frac{x + \frac{a}{x}}{2}$$ (Demo)

Babylonian Method

Implementation questions:

What guess should start the computation?

How do we know when we are finished?

Special Case: Cube Roots

How to compute cube_root(a)

Idea: Iteratively refine a guess x about the cube root of a

Update:

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

(Demo)

Implementation questions:

What guess should start the computation?

How do we know when we are finished?

Implementing Newton's Method

(Demo)

Extensions

Approximate Differentiation

```
Differentiation can be performed
symbolically or numerically
```

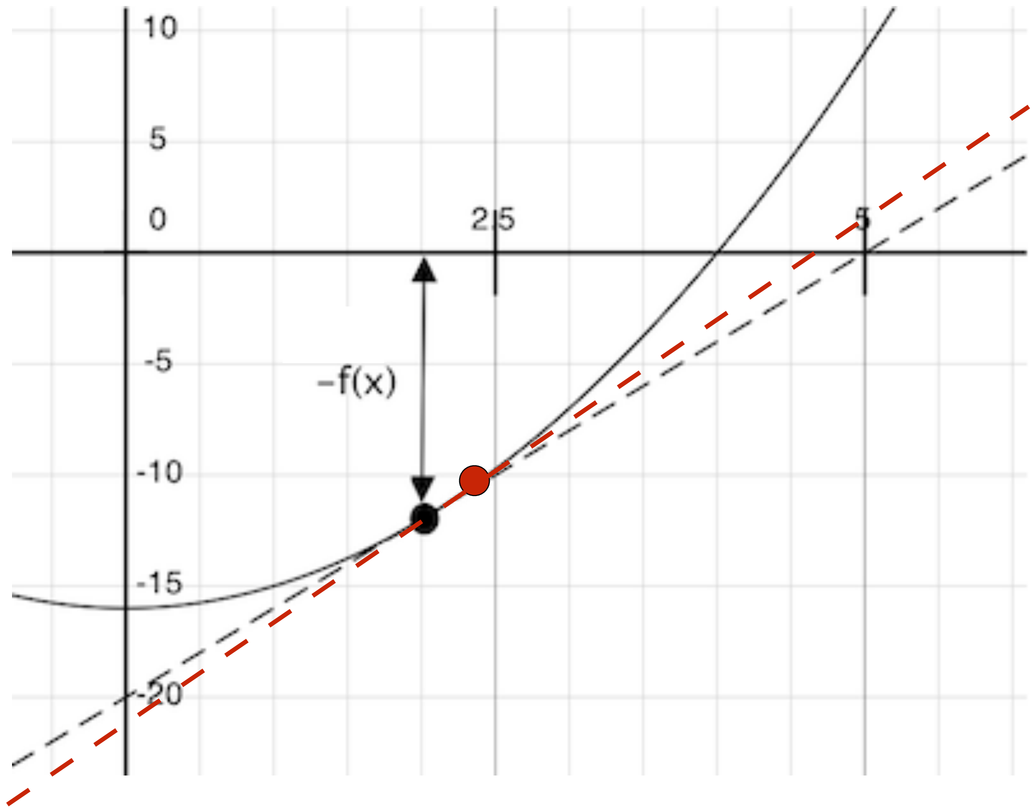
f(x) = x^2 – 16

f'(x) = 2x

f'(2) = 4

$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$ (if a is small)



(Demo)

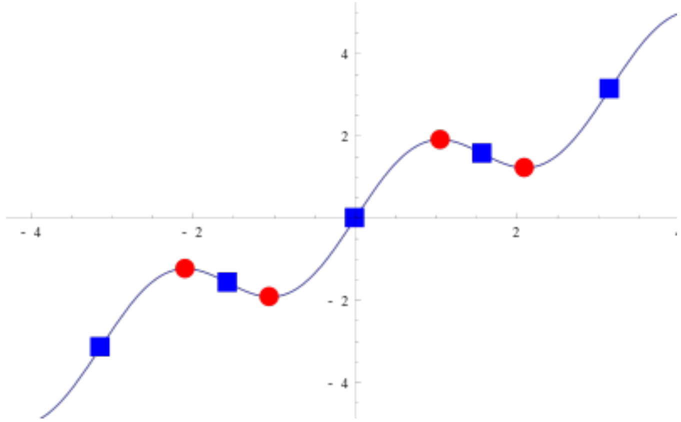
Critical Points and Inverses

Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0

```
derive = lambda f: lambda x: slope(f, x)
```

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value x such that f(x) = y

(Demo)



http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg