61A Extra Lecture 1

Announcements

Newton's Method

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!

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$f(x) = x^2 - 2$

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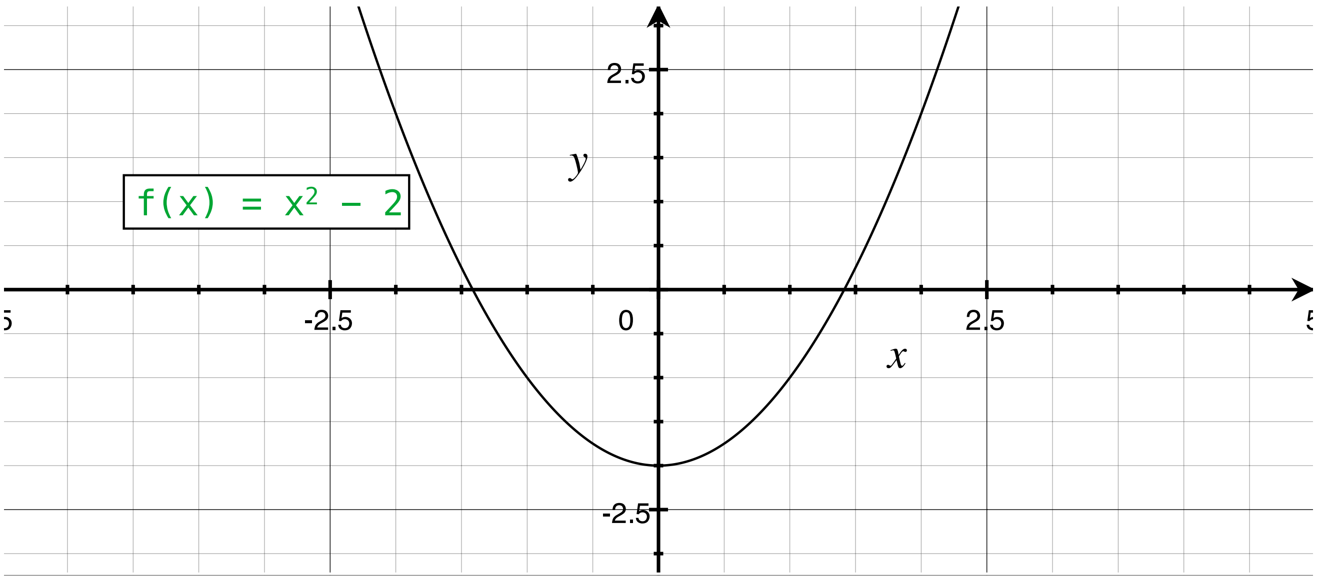

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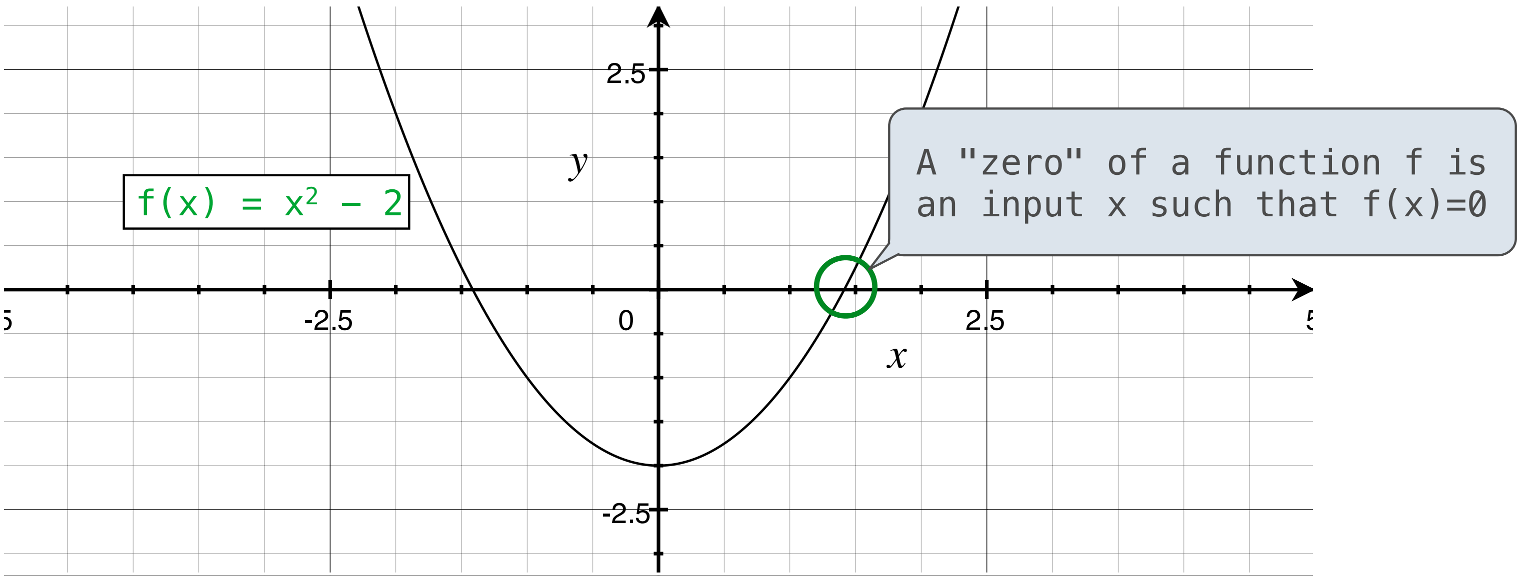

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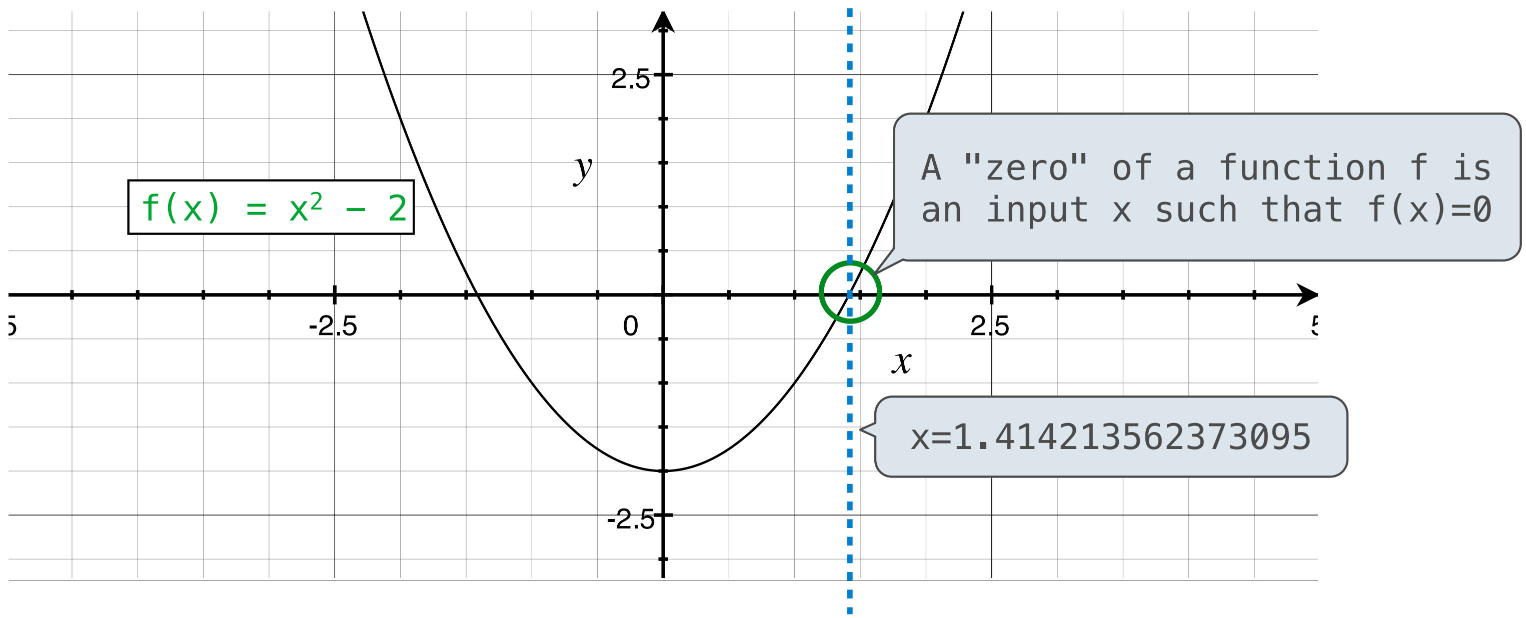

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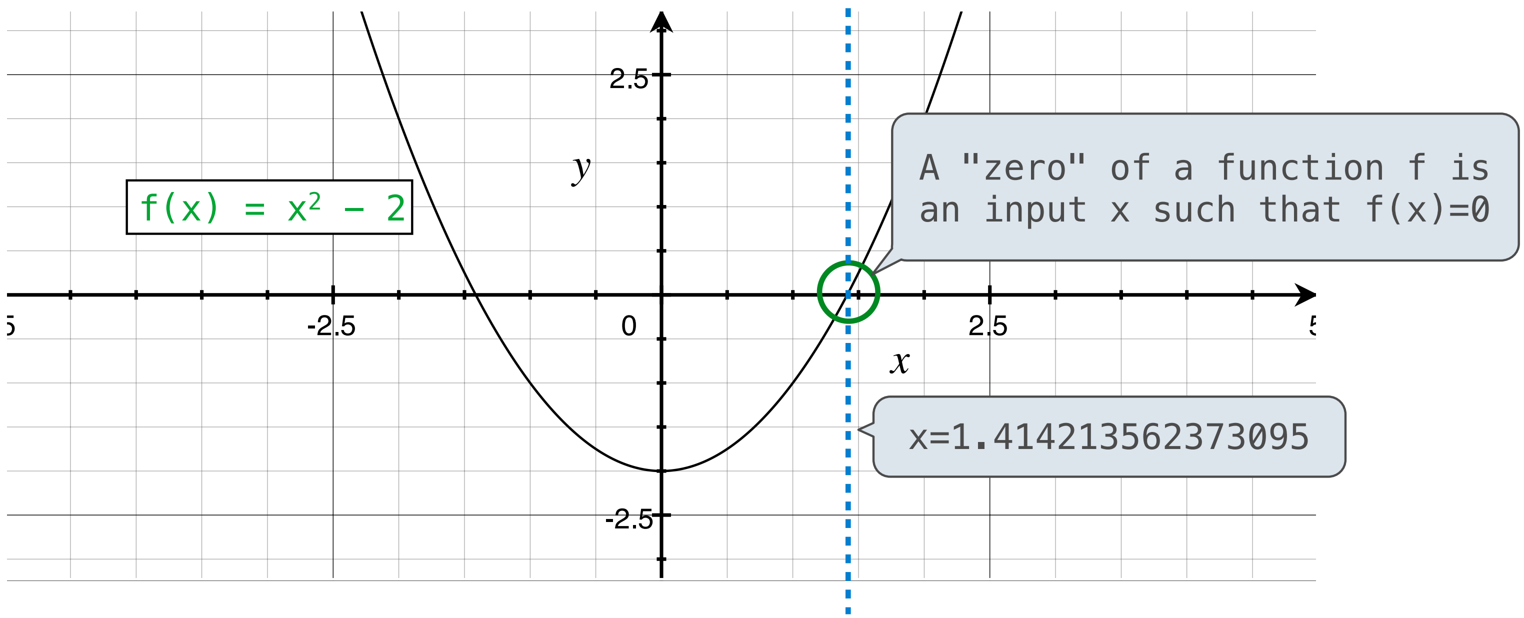

Application: a method for computing square roots, cube roots, etc.

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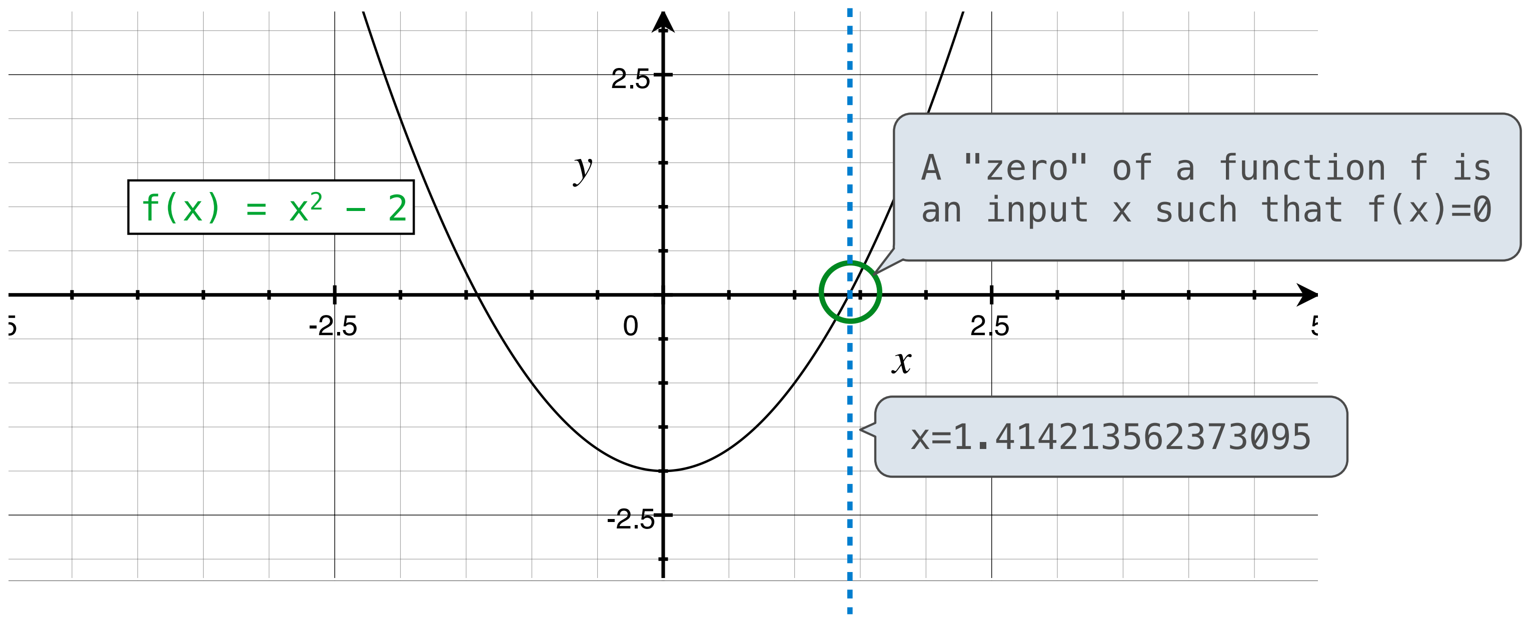

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

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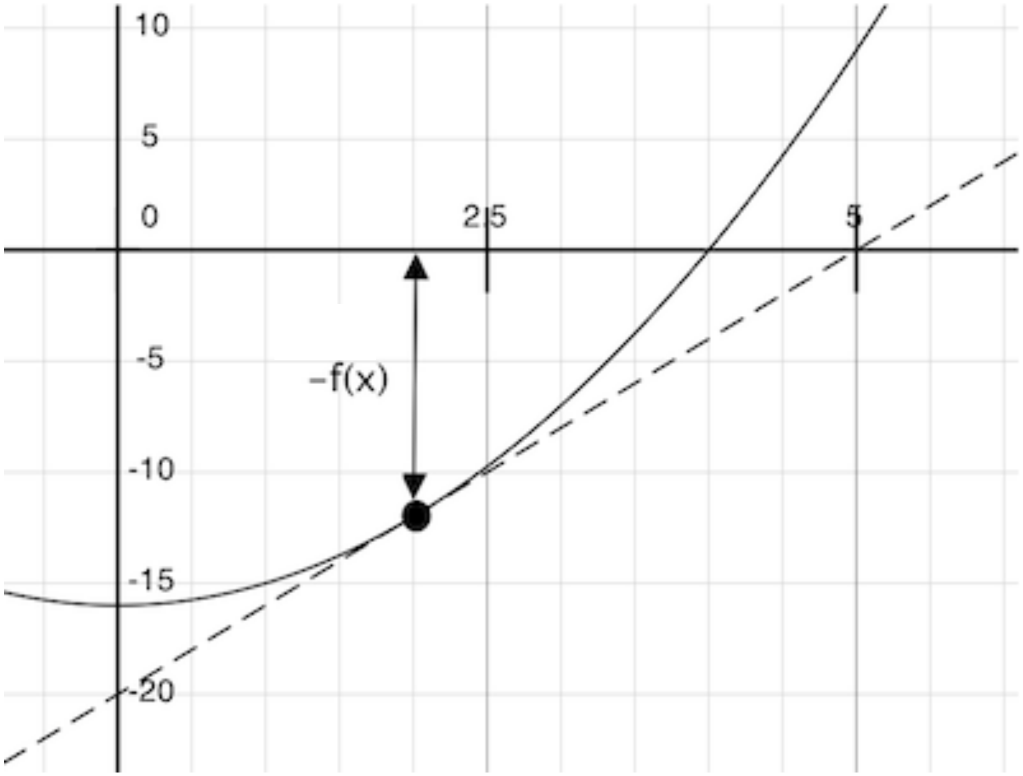

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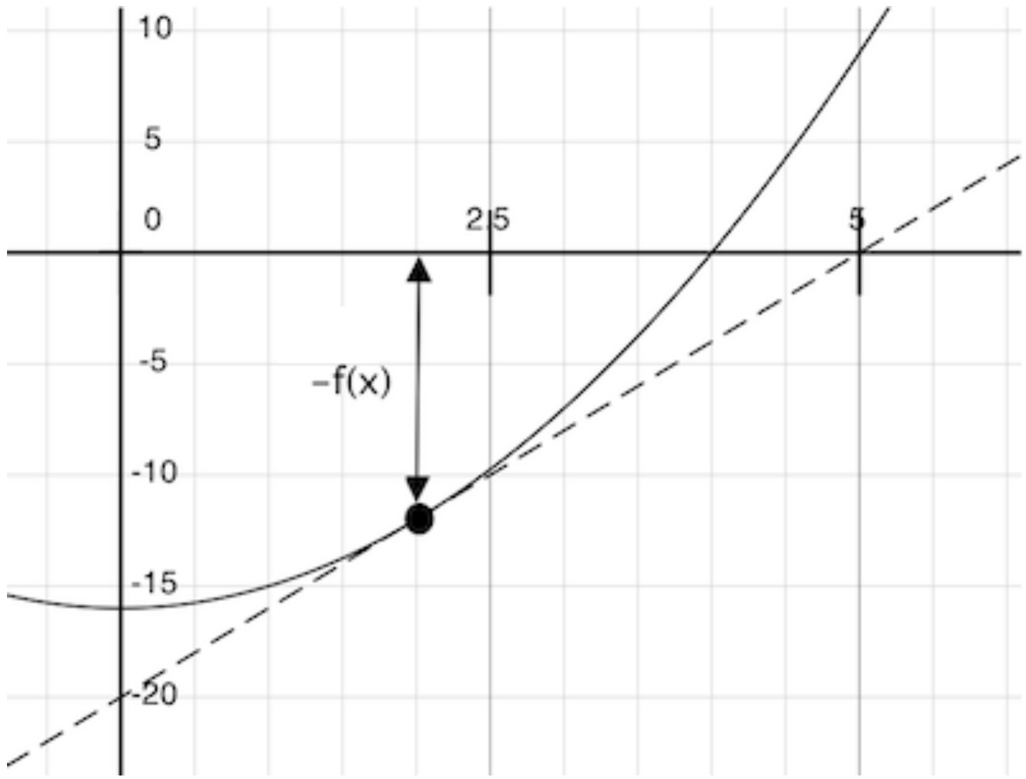

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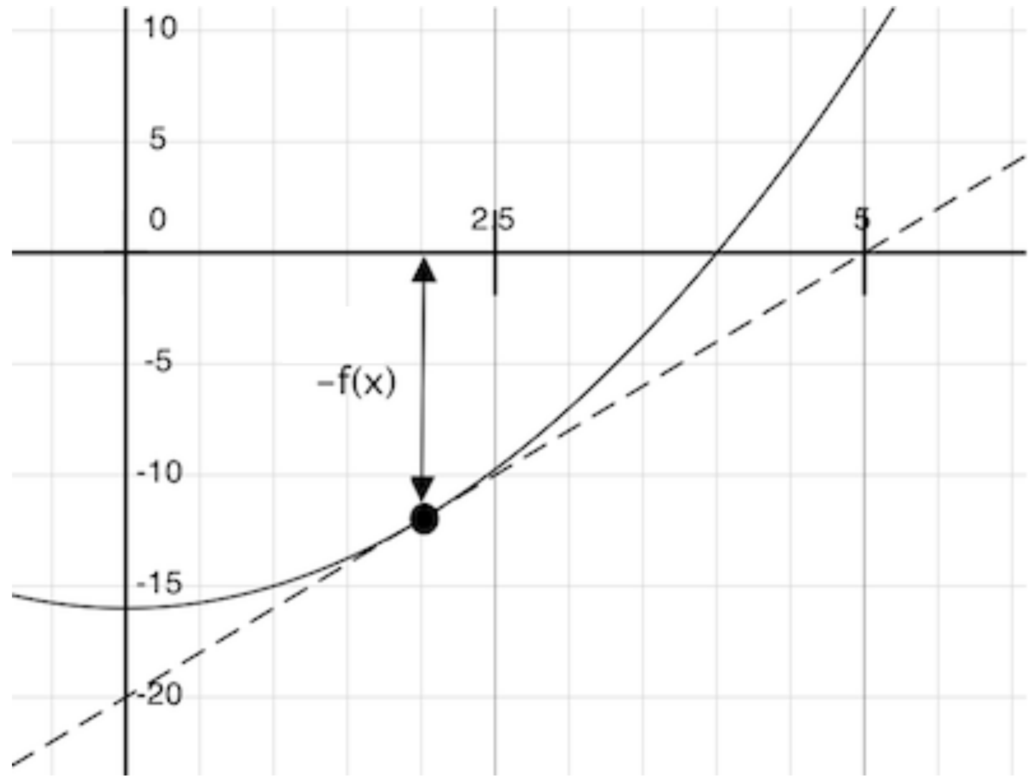

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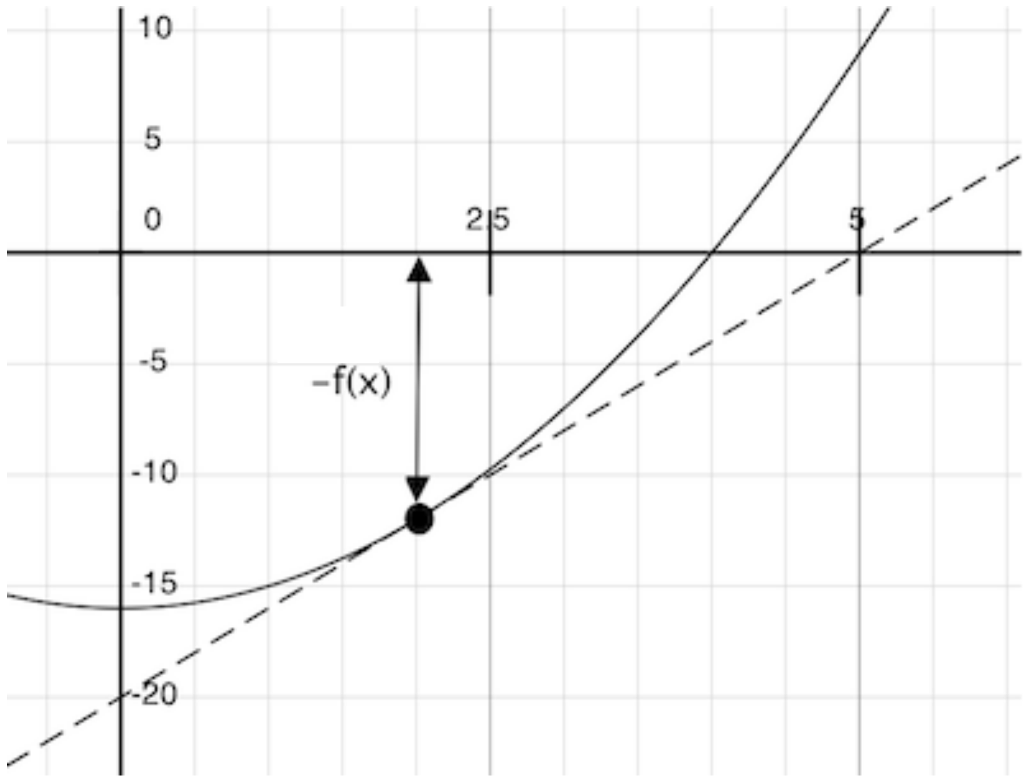
Update guess x to be:

$$x - \frac{f(x)}{f'(x)}$$



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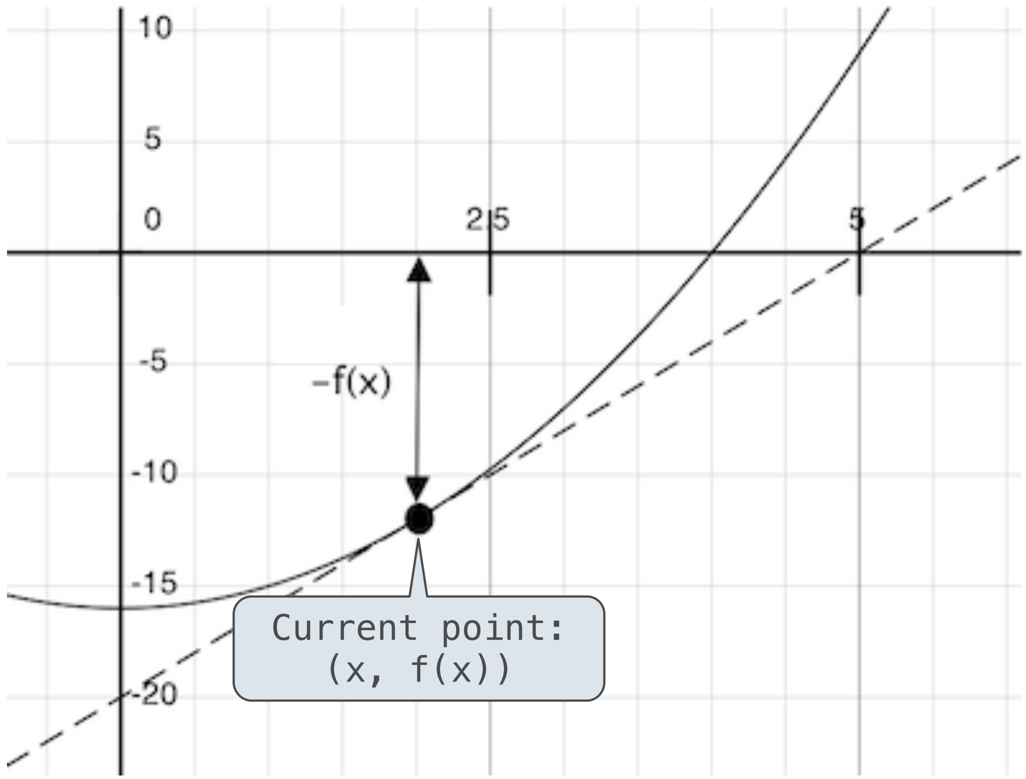

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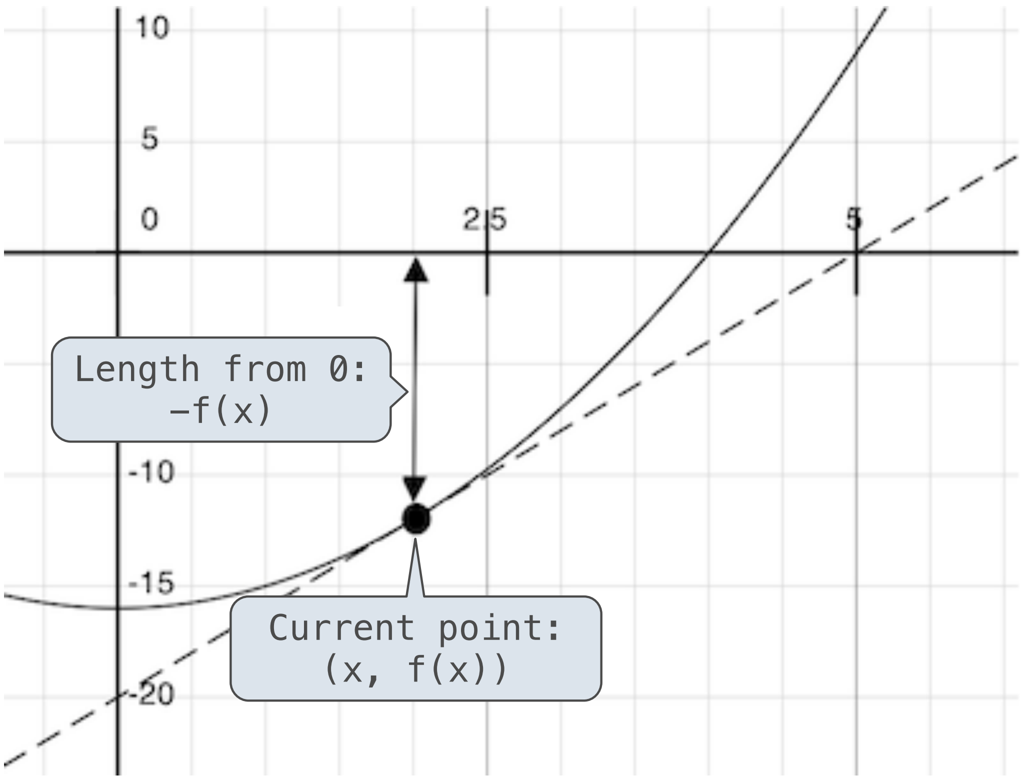

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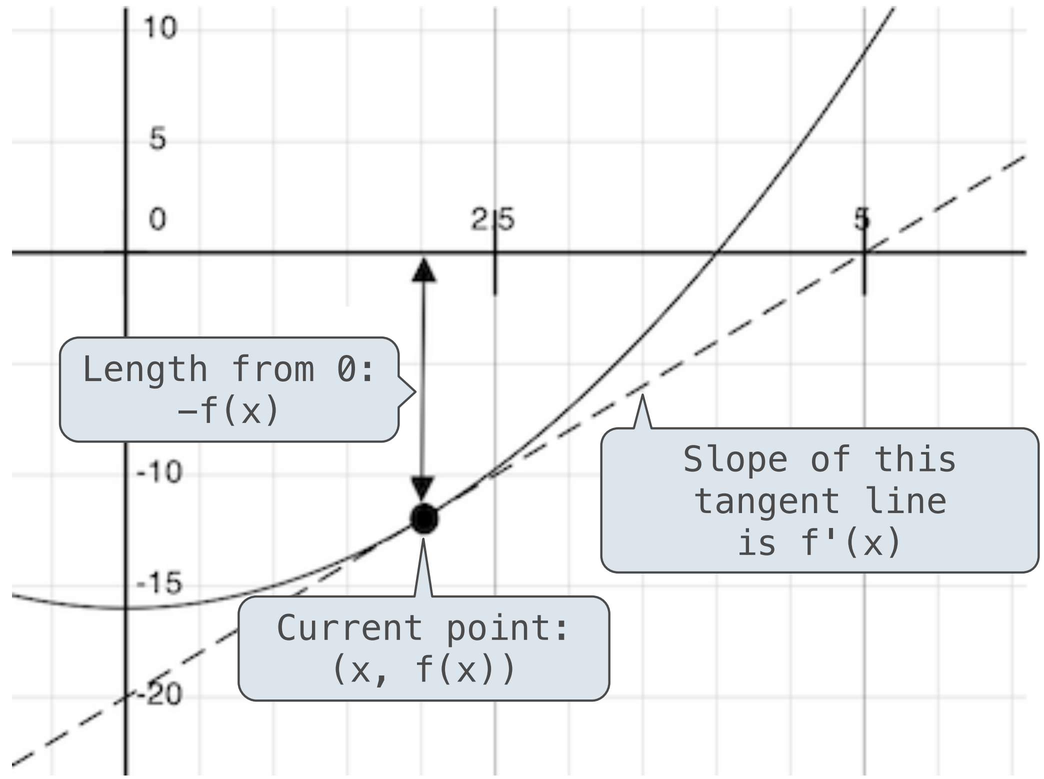

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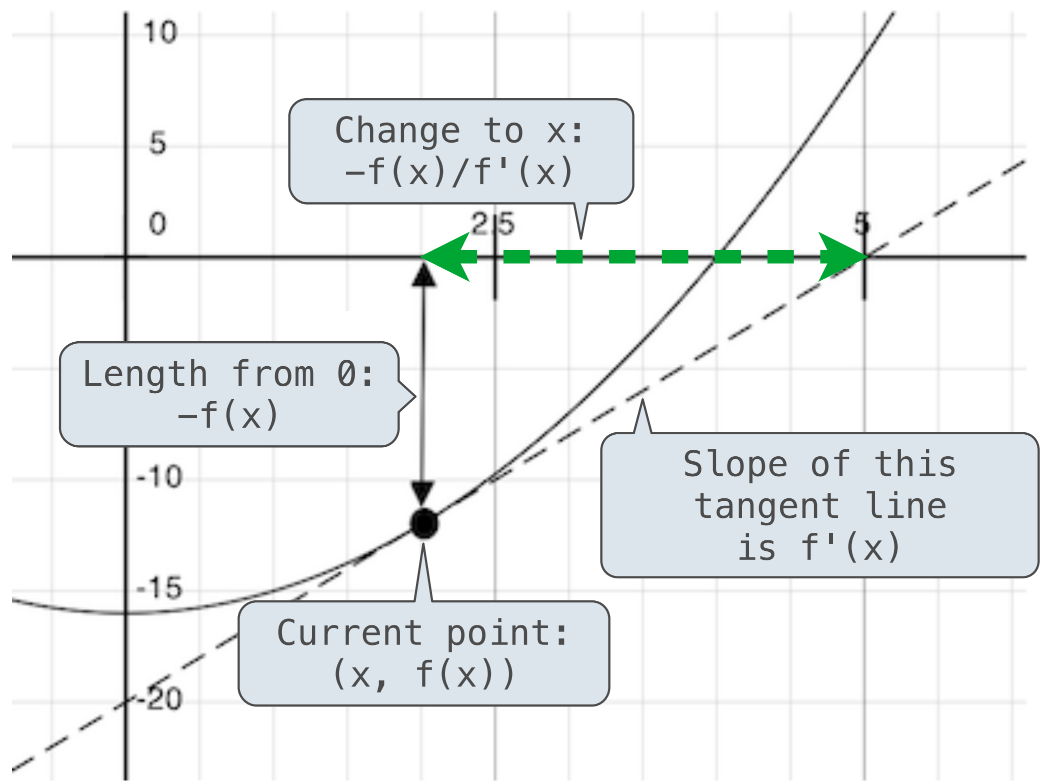

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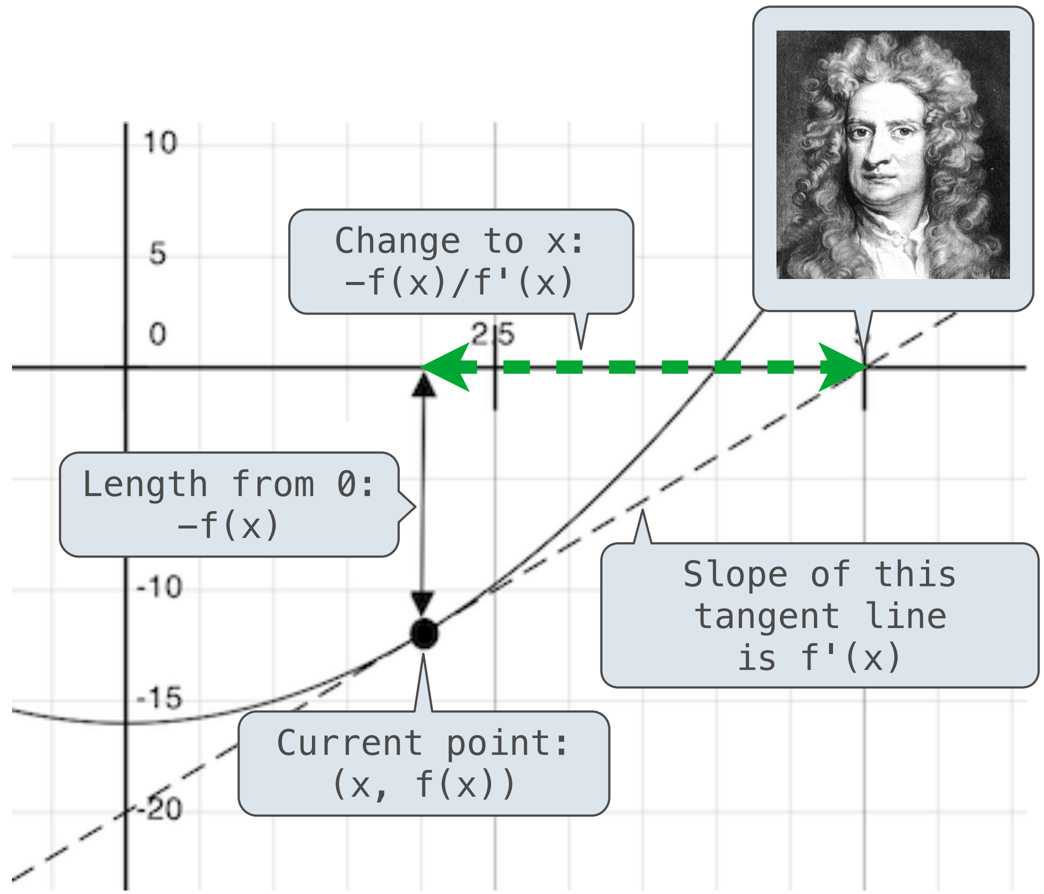

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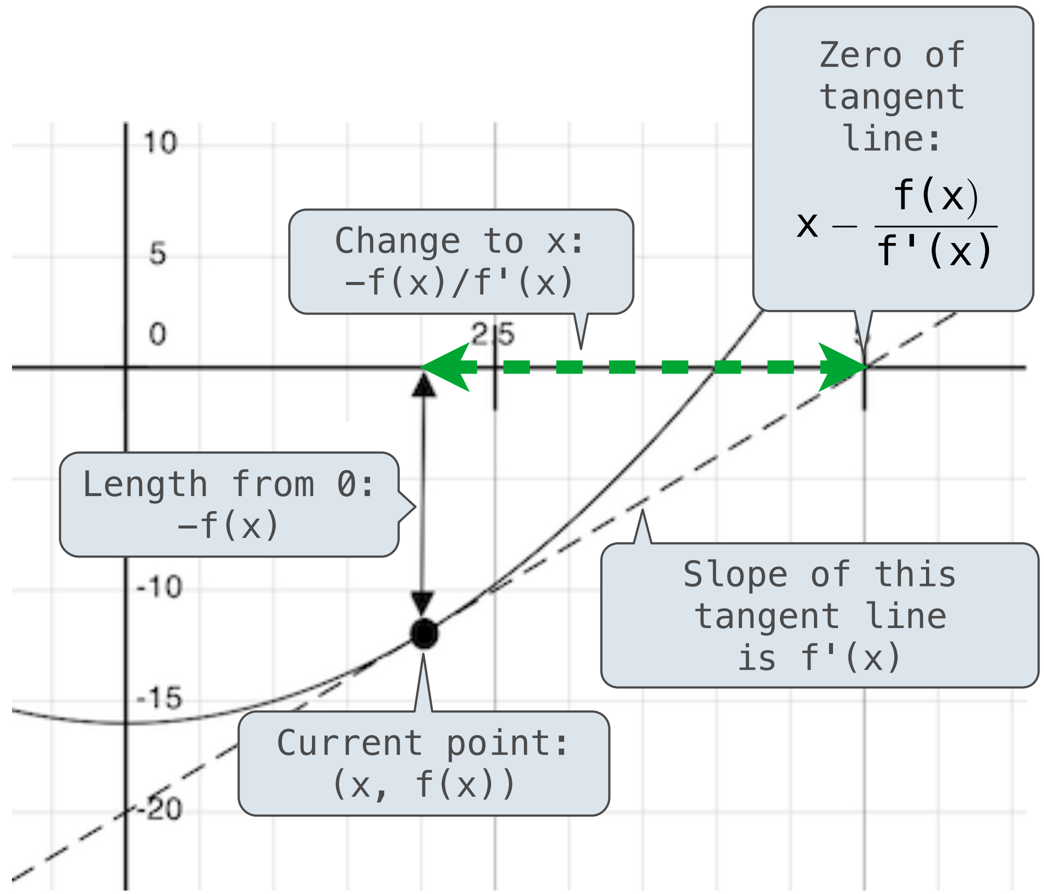

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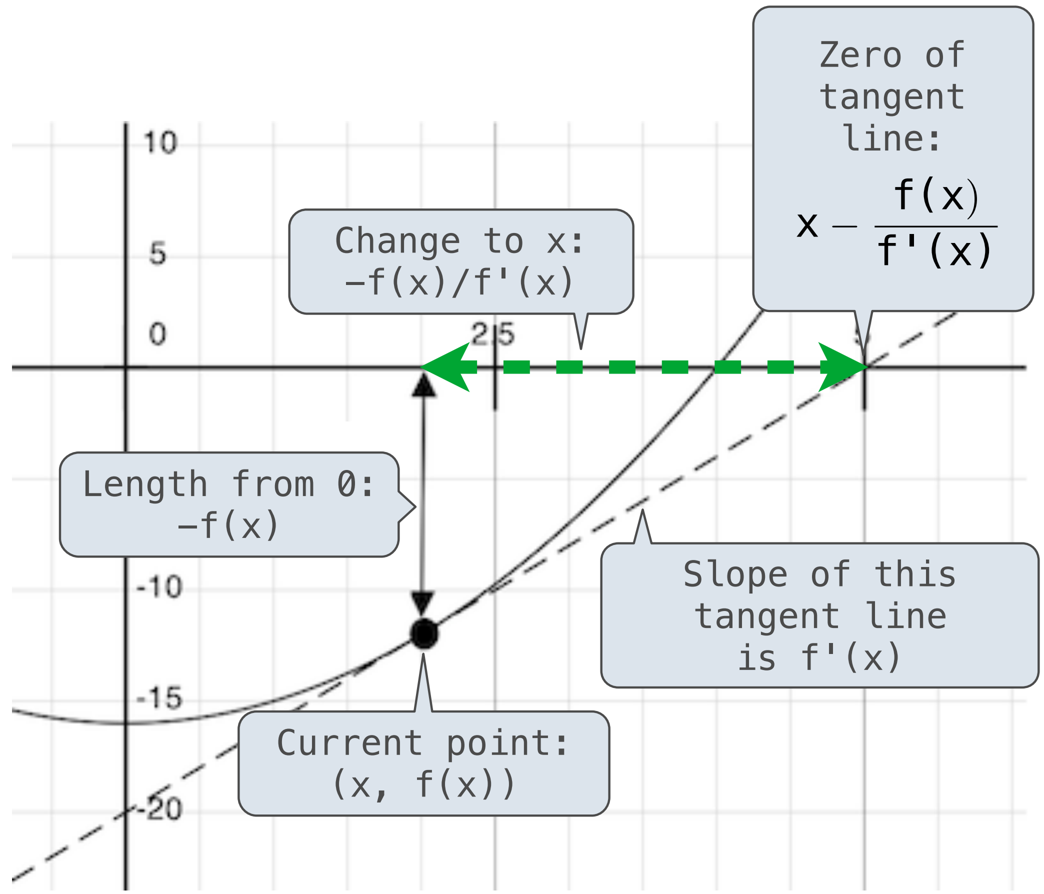

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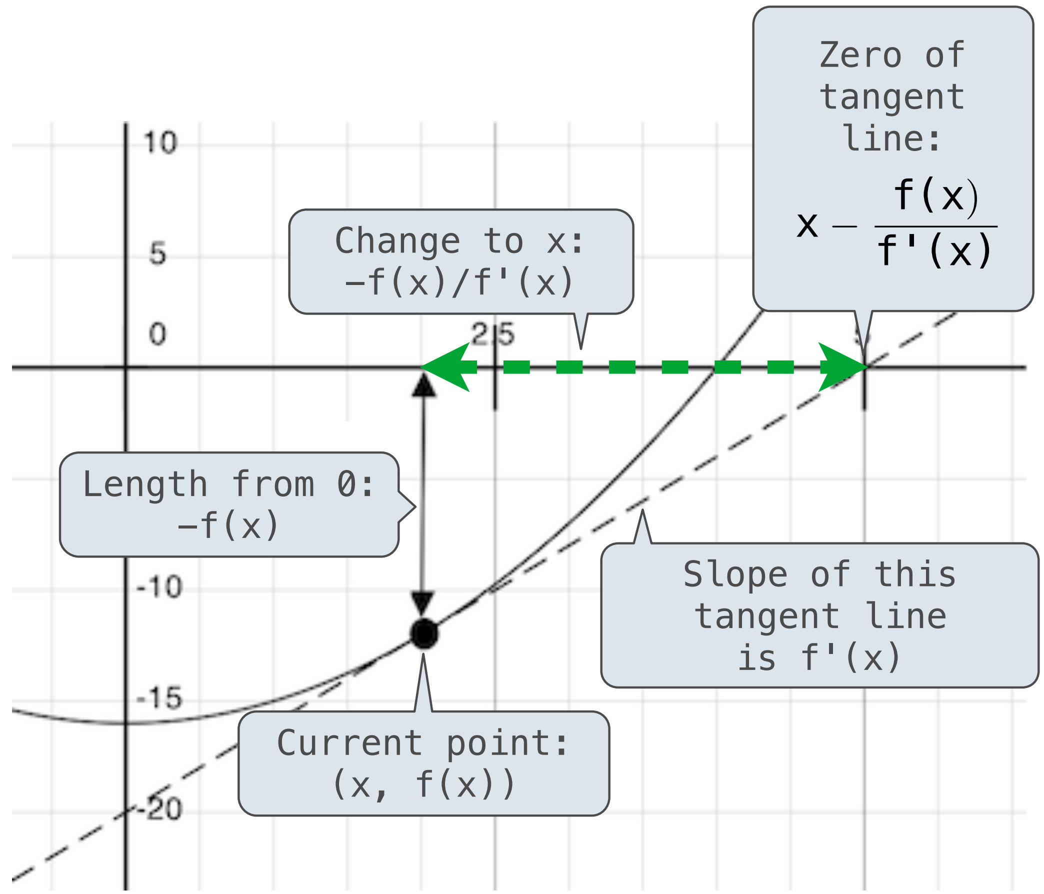

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

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How to find the square root of 2?

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>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
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Using Newton's Method

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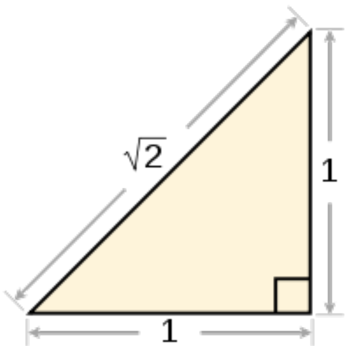


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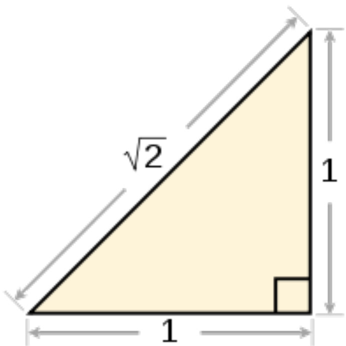


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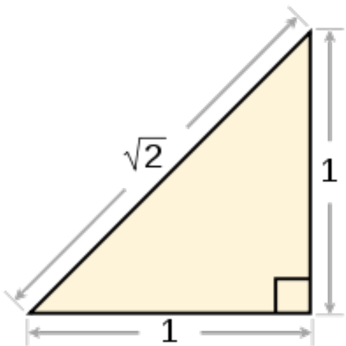


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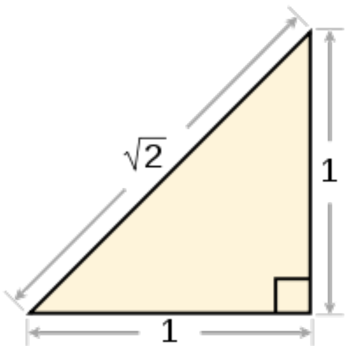


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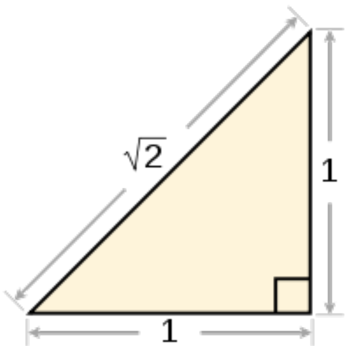


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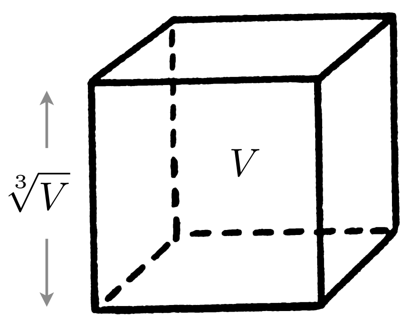

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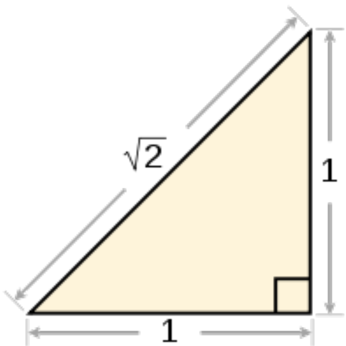


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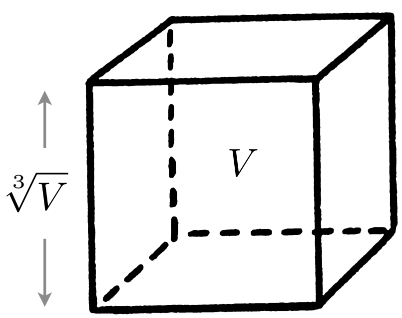


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>>> dg = lambda x: 3*x*x
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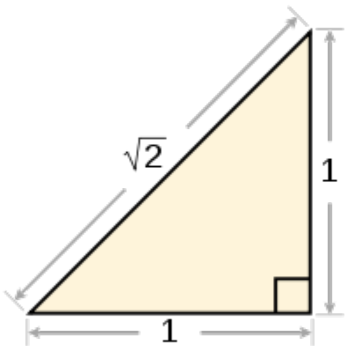


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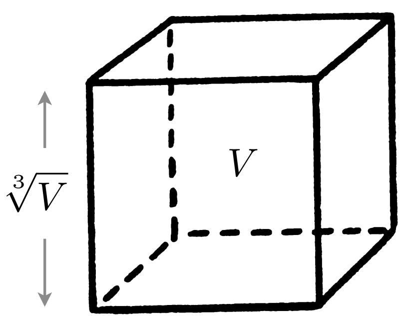


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$g(x) = x^3 - 729$

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Iterative Improvement

Special Case: Square Roots

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Babylonian Method

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Implementing Newton's Method

(Demo)

Extensions

Approximate Differentiation

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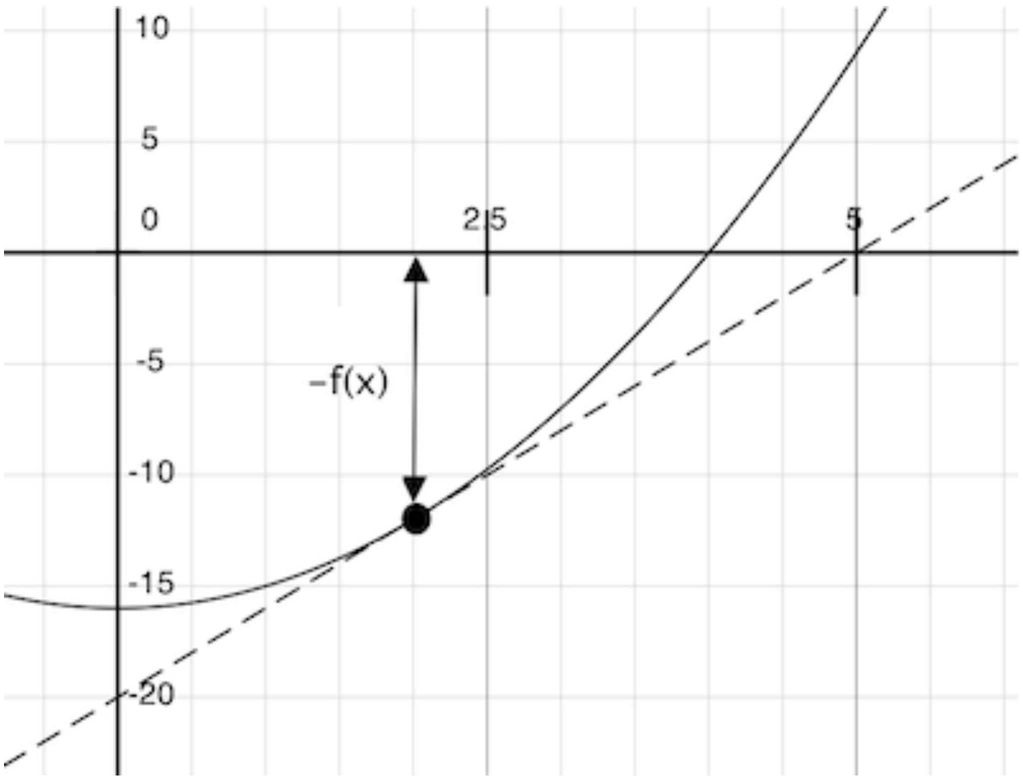
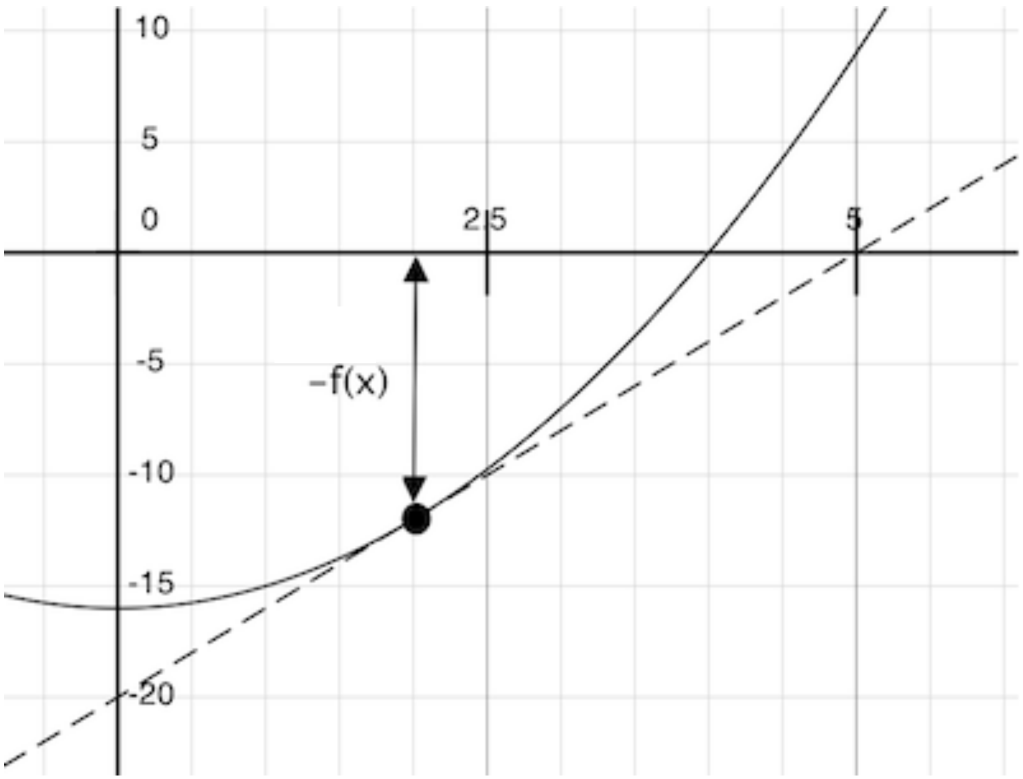


Approximate Differentiation

Differentiation can be performed symbolically or numerically



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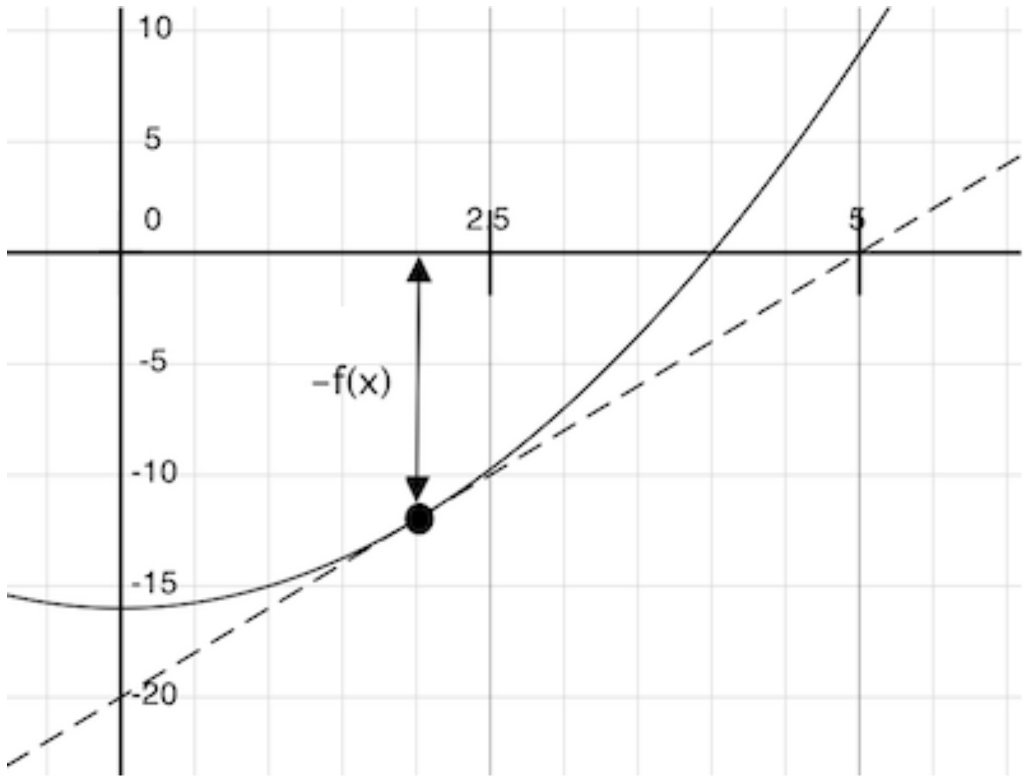
Differentiation can be performed symbolically or numerically

$f(x) = x^2 - 16$



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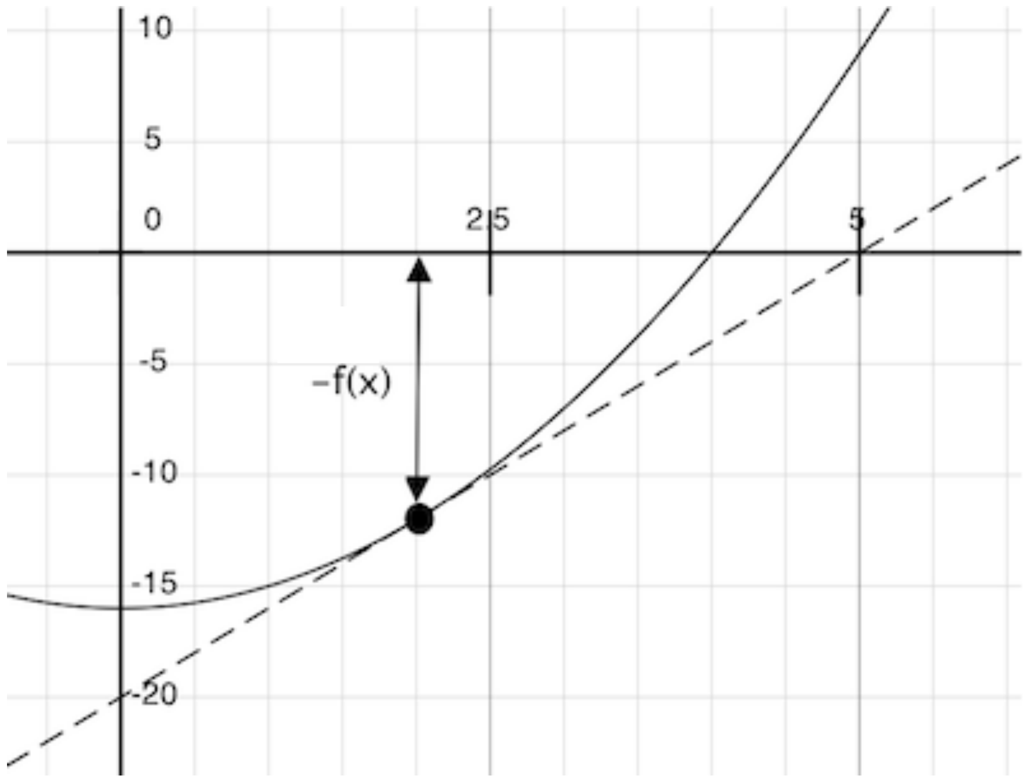

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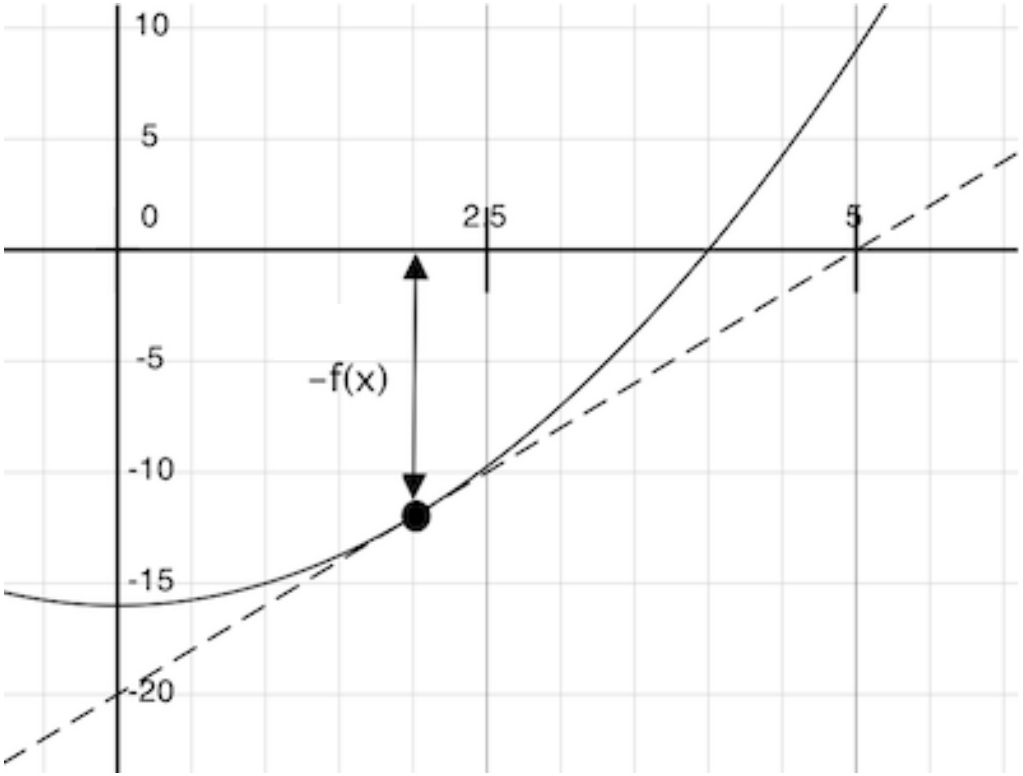

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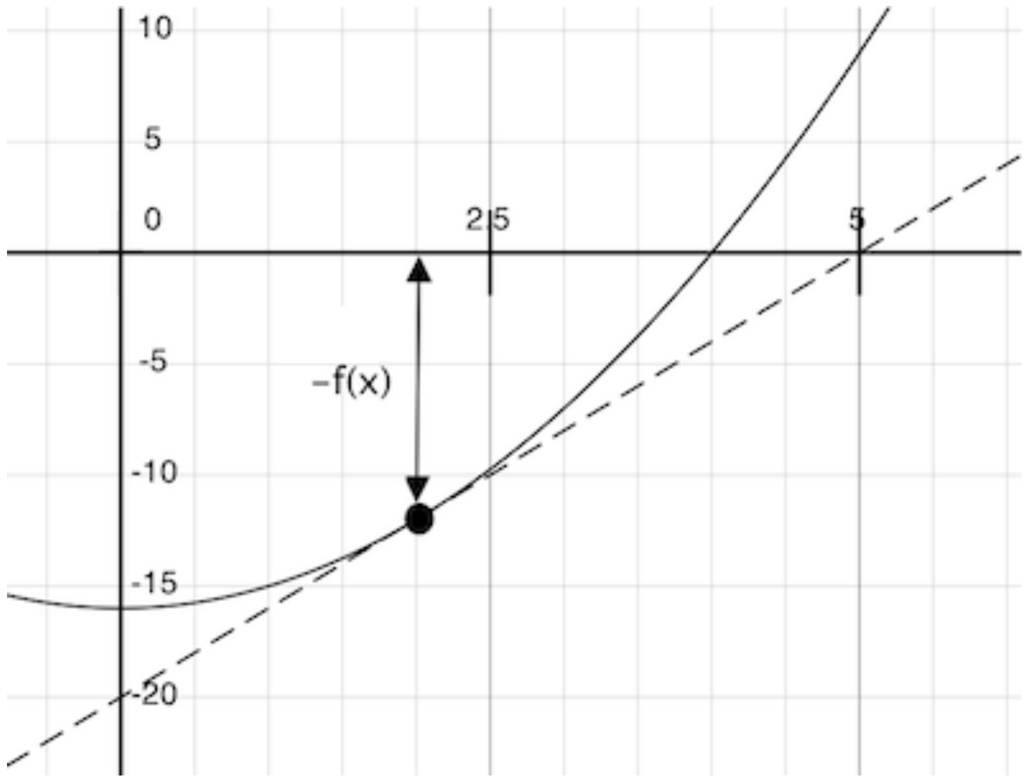

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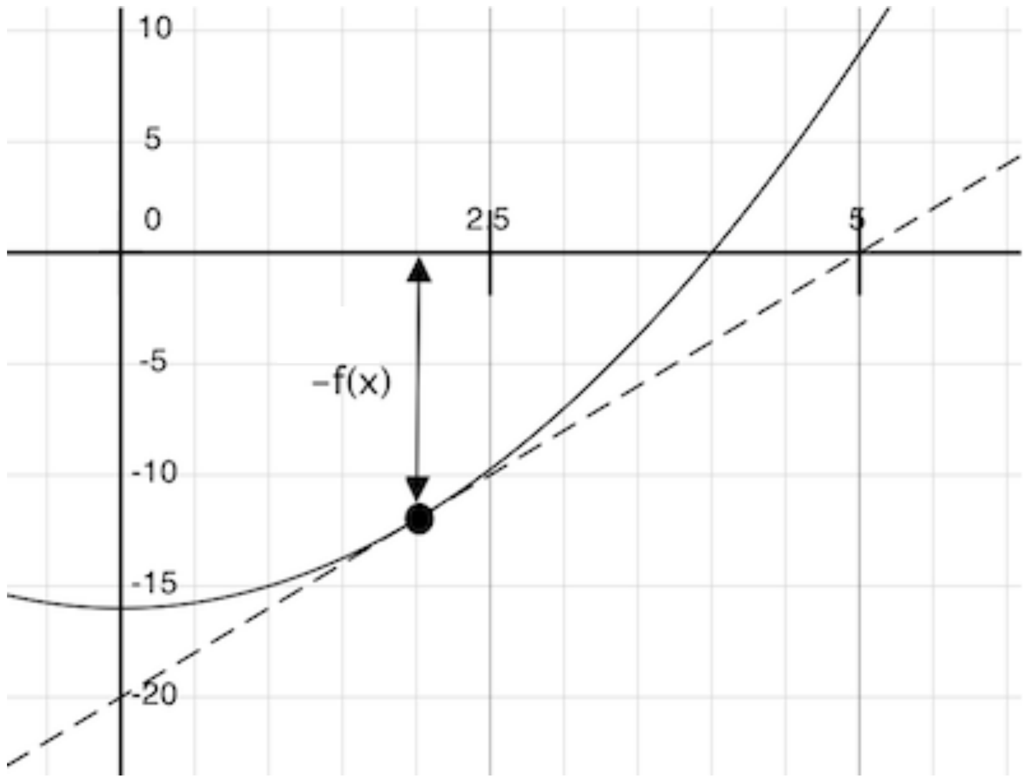

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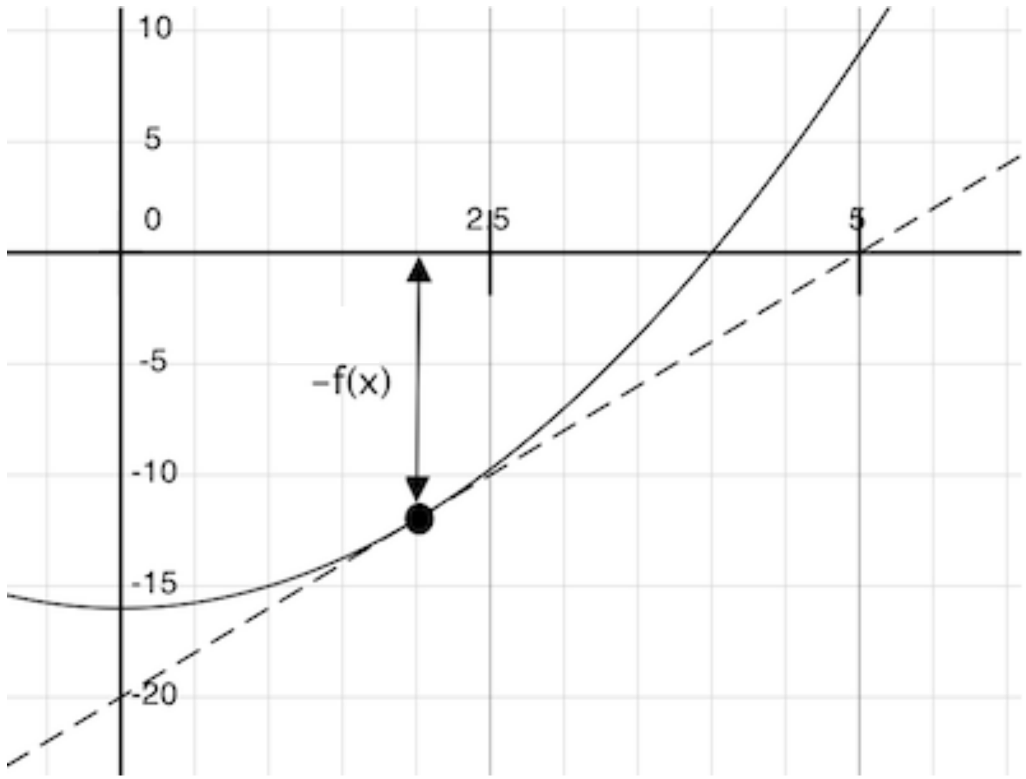

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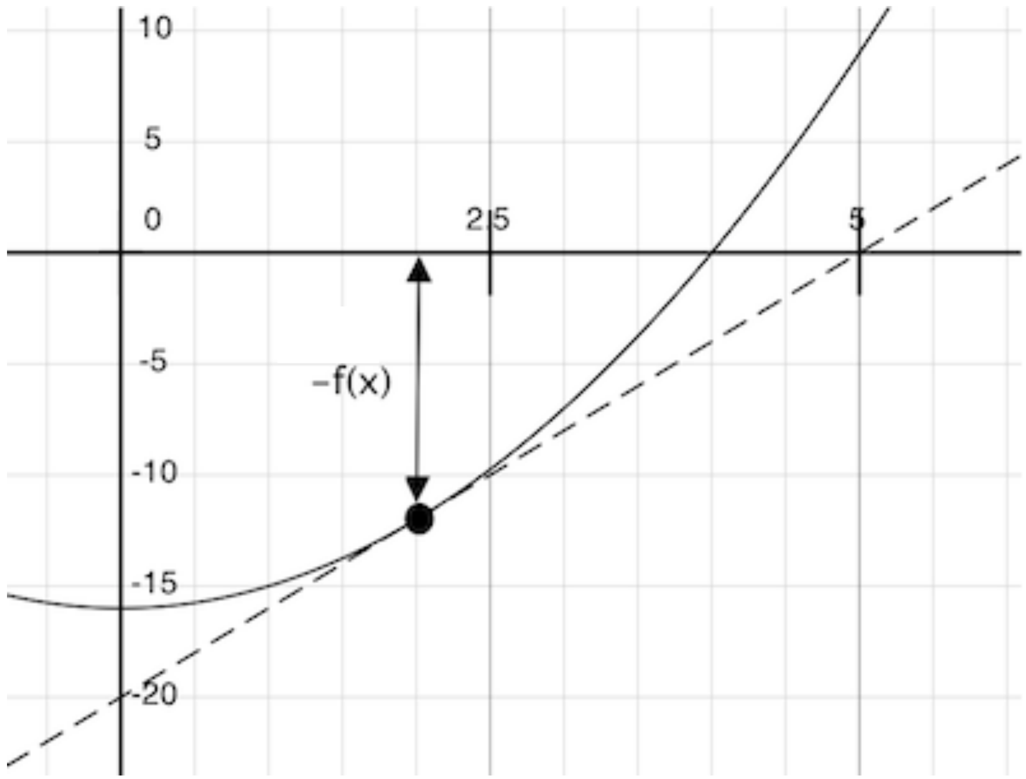

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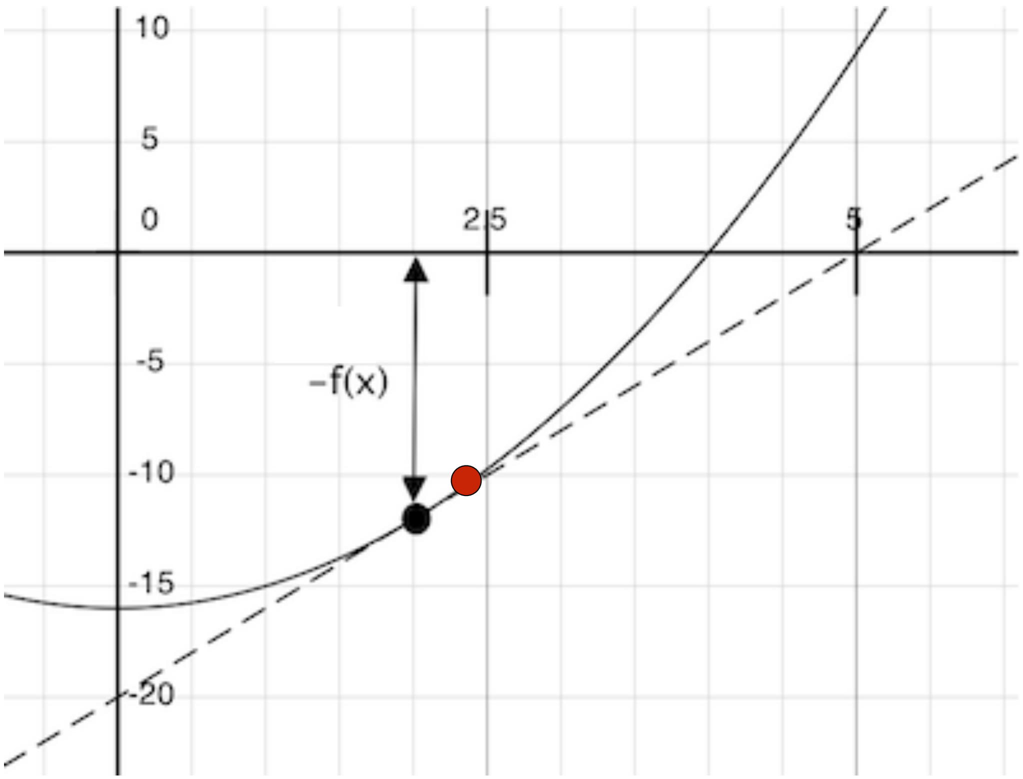

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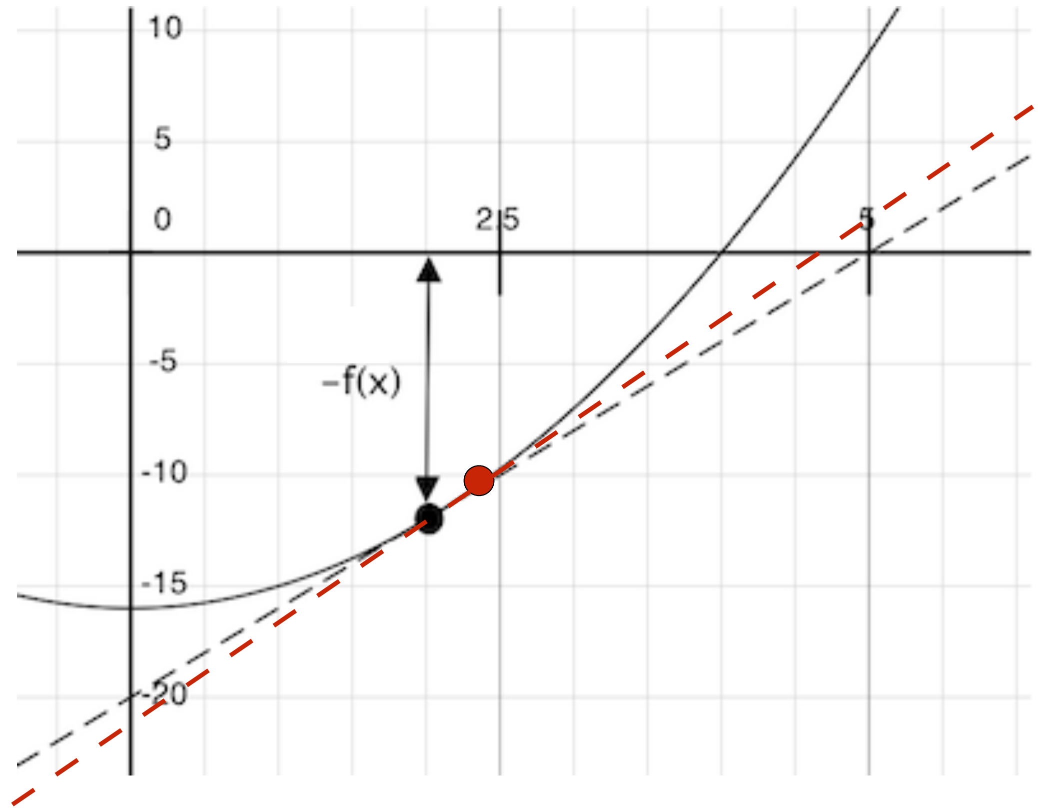

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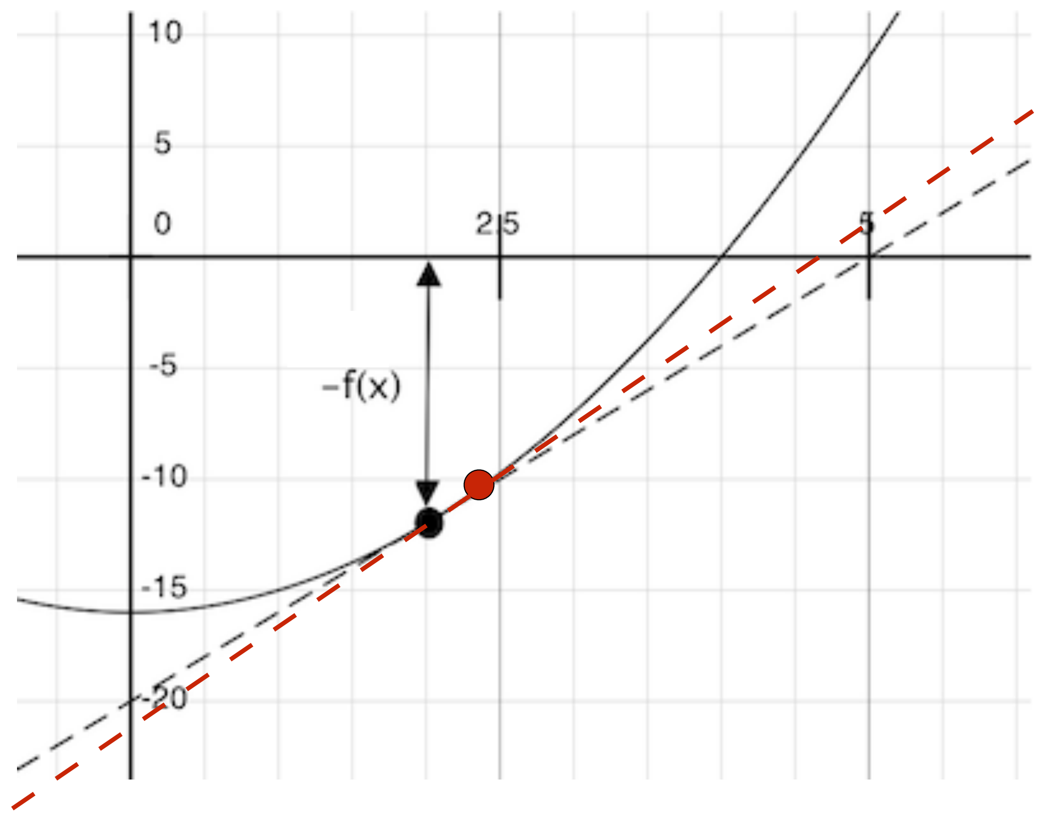

Critical Points and Inverses

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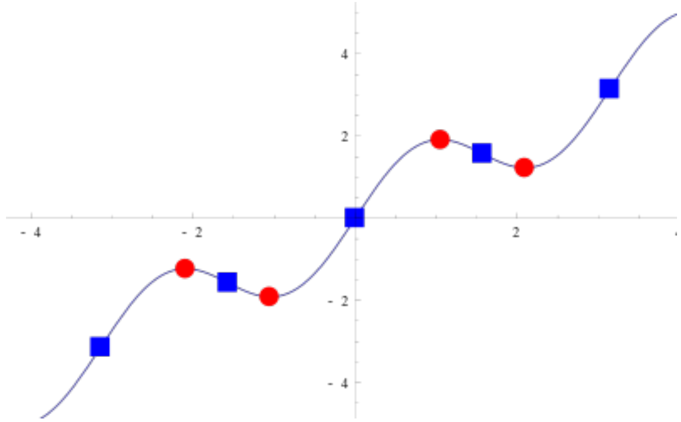

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg

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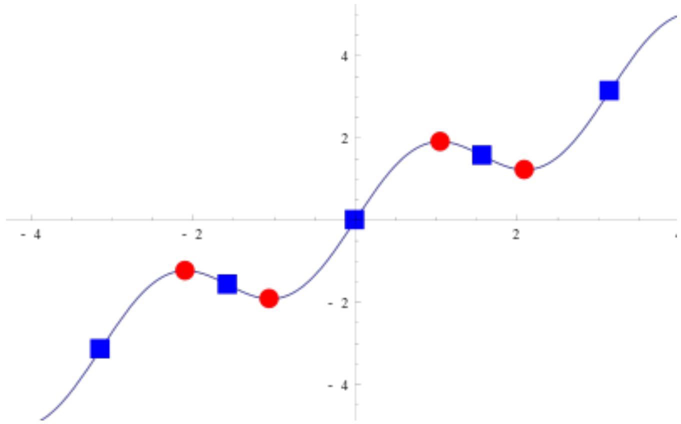

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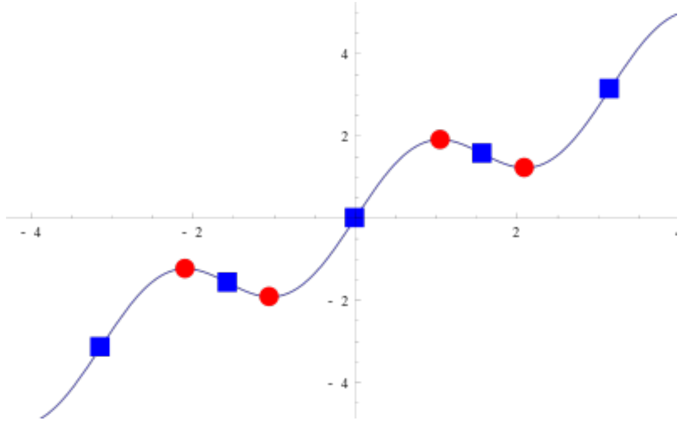

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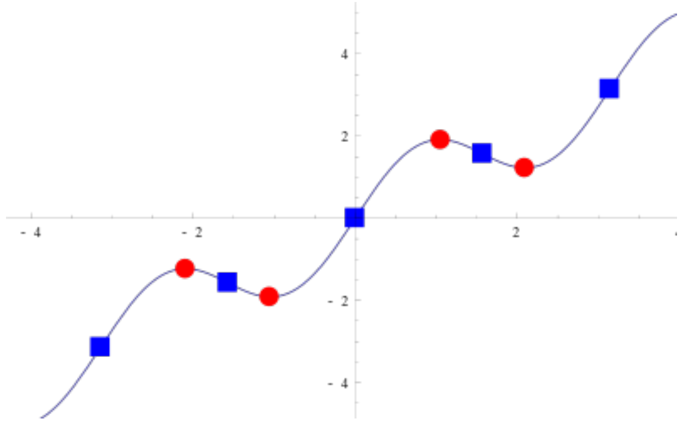
(Demo)



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