61A Lecture 29

Announcements

Efficient Sequence Processing

Sequence Operations

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Map, filter, and reduce express sequence manipulation using compact expressions

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range iterator

next: 1
end: 6

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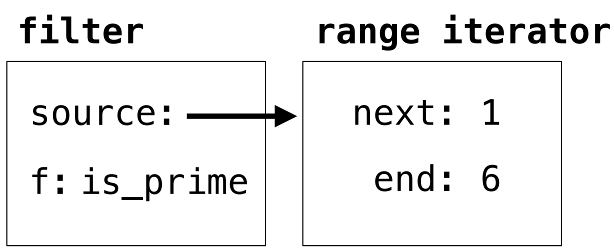

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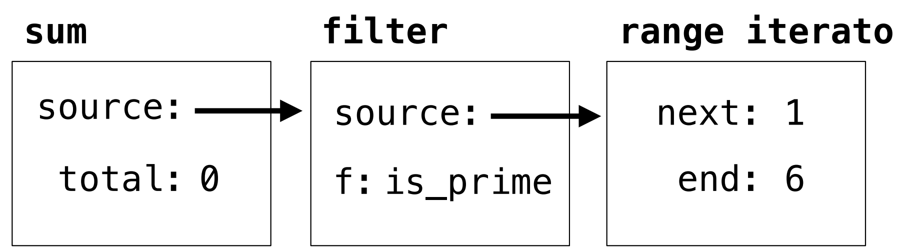

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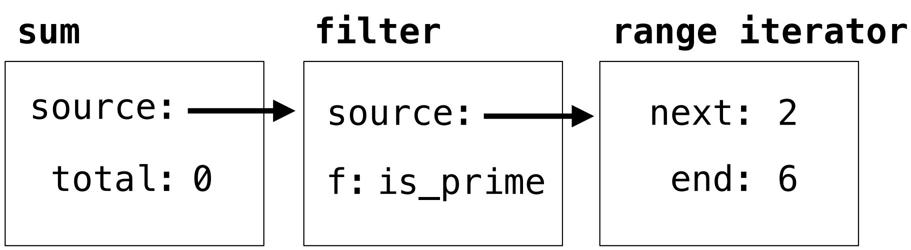

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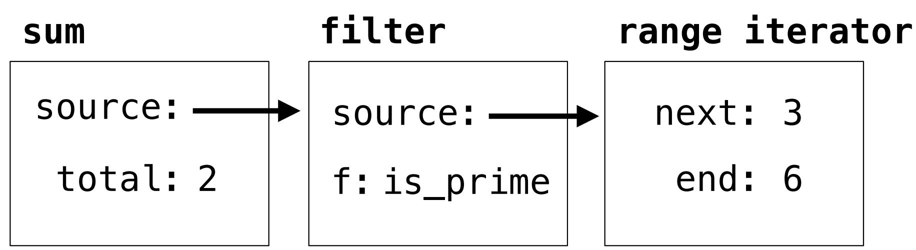

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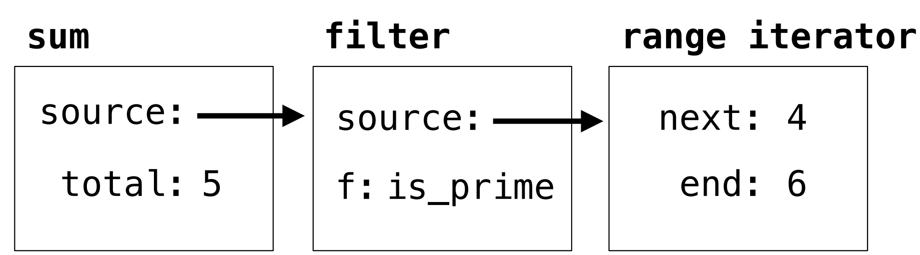

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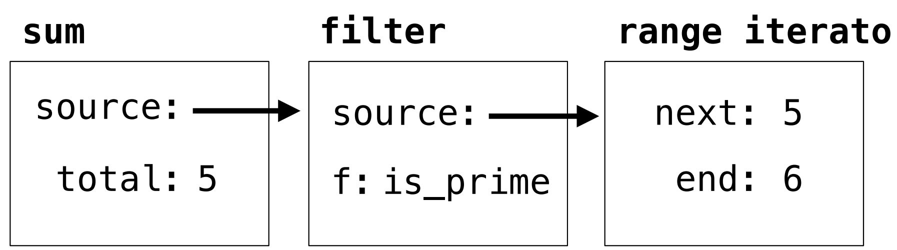

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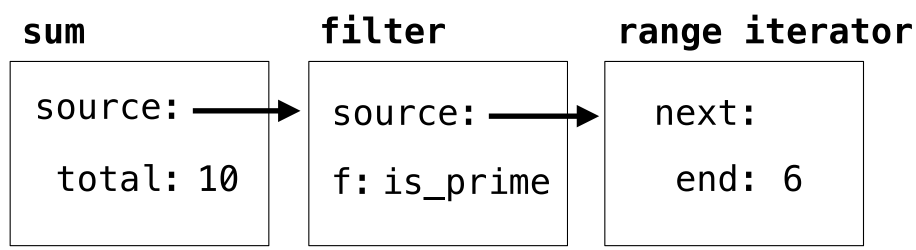

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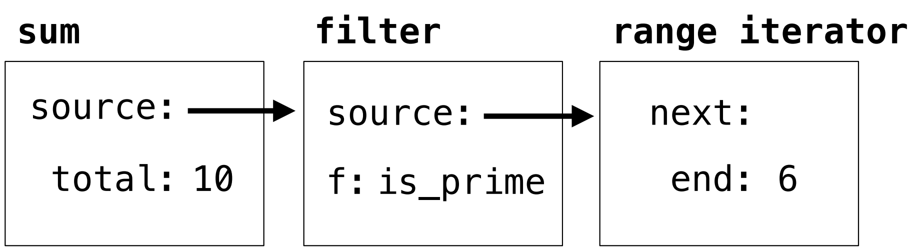

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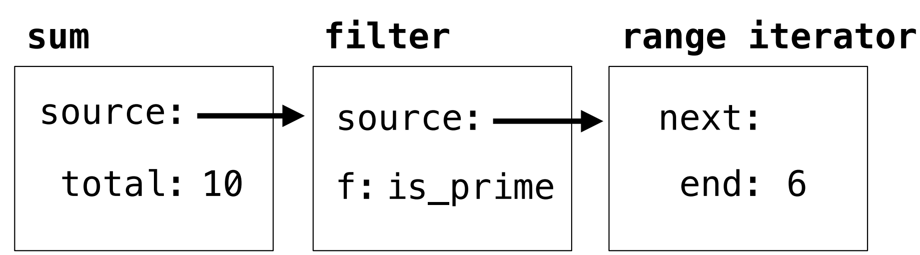

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(Demo)

Streams

Streams are Lazy Scheme Lists

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(Demo)

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scm> (car (cdr-stream (cd-stream lots)))
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Infinite Streams

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(Demo)

Stream Processing

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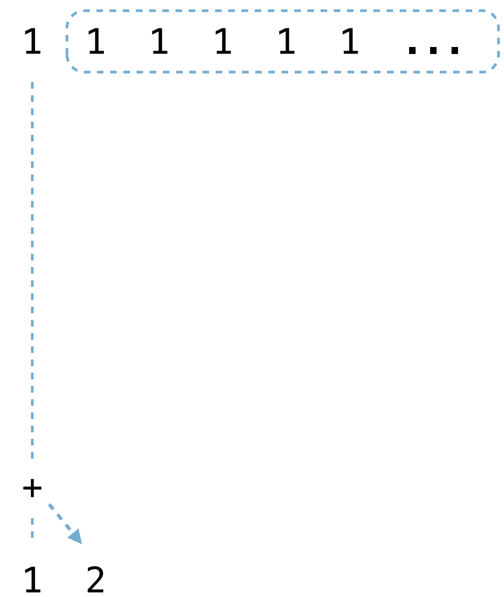

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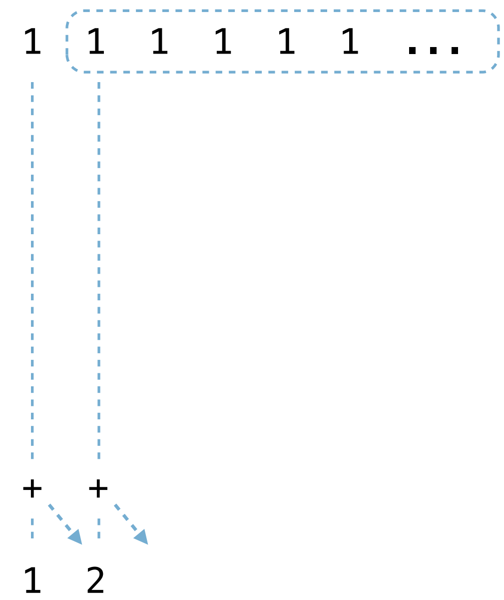

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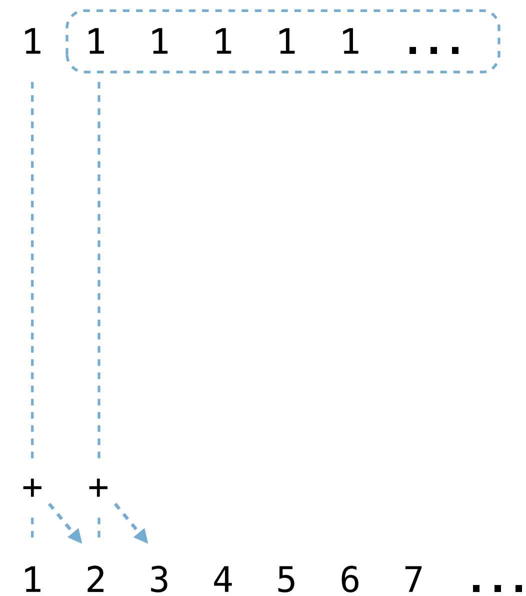

Higher-Order Stream Functions

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but change cons to cons-stream
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```
(define (map f s)
  (if (null? s)
      nil
      (cons (f (car s))
            (map f
                 (cdr s)))))

(define (filter f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons (car s)
                (filter f (cdr s)))
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(define (reduce f s start)
  (if (null? s)
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              (cdr s)
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Higher-Order Functions on Streams

Implementations are identical,
but change cons to cons–stream
and change cdr to cdr–stream

```
(define (map-stream f s)
  (if (null? s)
      nil
      (cons-stream (f (car s))
            (map-stream f
                 (cdr-stream s)))))

(define (filter-stream f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons-stream (car s)
                (filter-stream f (cdr-stream s)))
          (filter-stream f (cdr-stream s)))))

(define (reduce-stream f s start)
  (if (null? s)
      start
      (reduce-stream f
              (cdr-stream s)
              (f start (car s)))))
```

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(Demo)