61A Lecture 27

Announcements

Dynamic Scope

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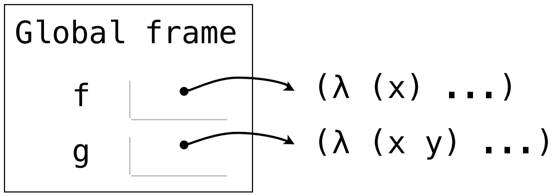


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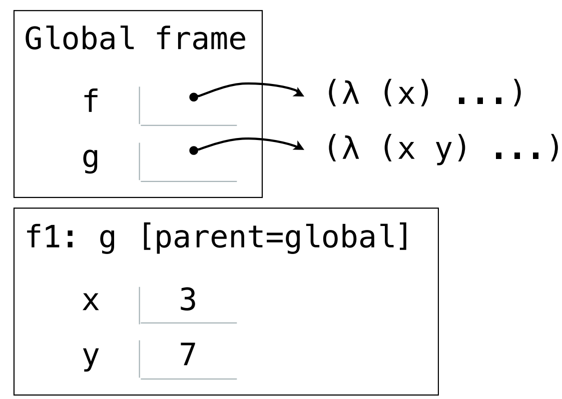


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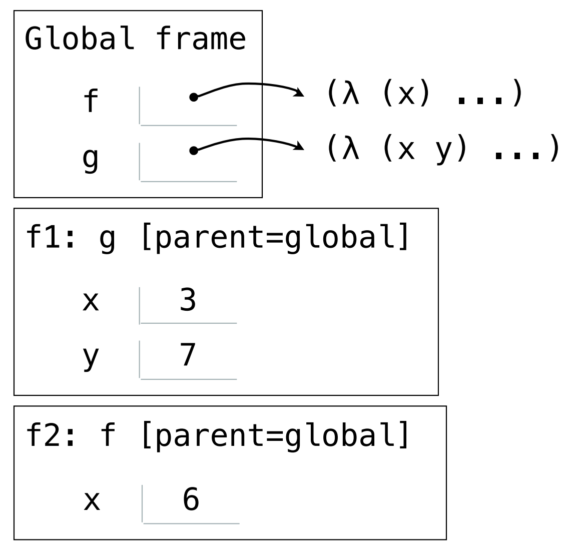

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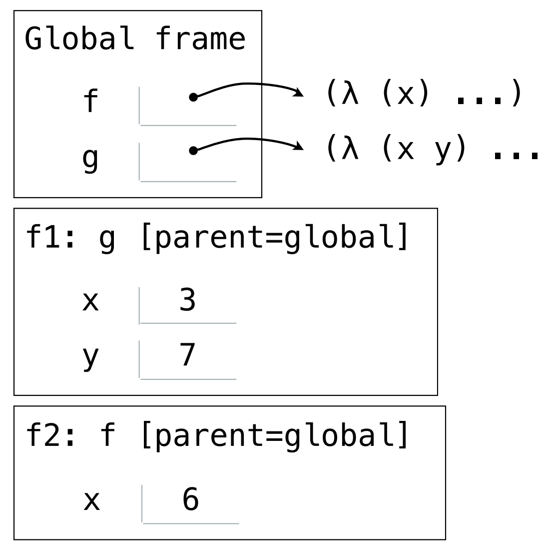

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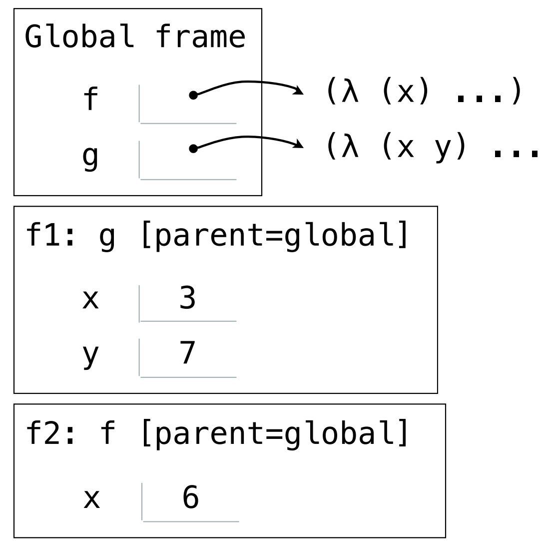

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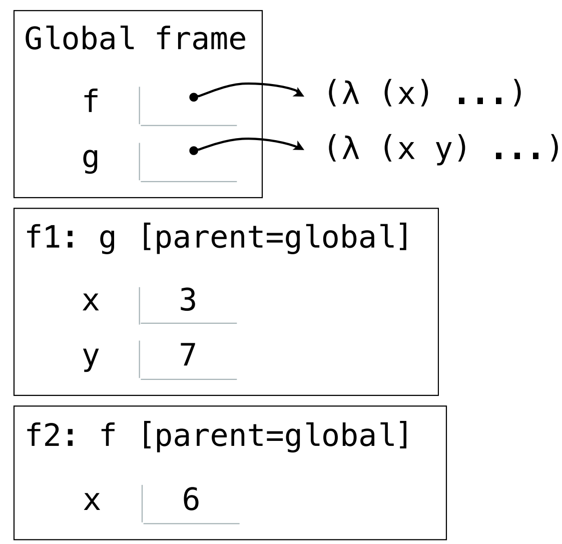

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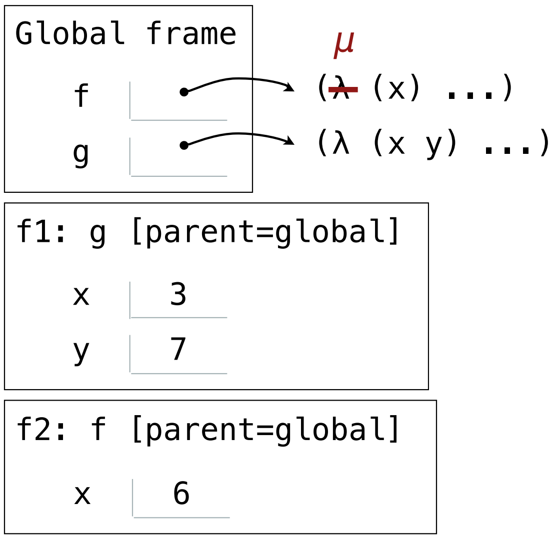

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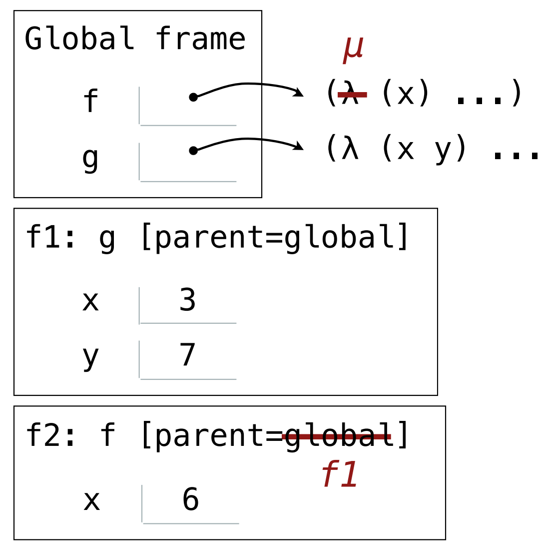

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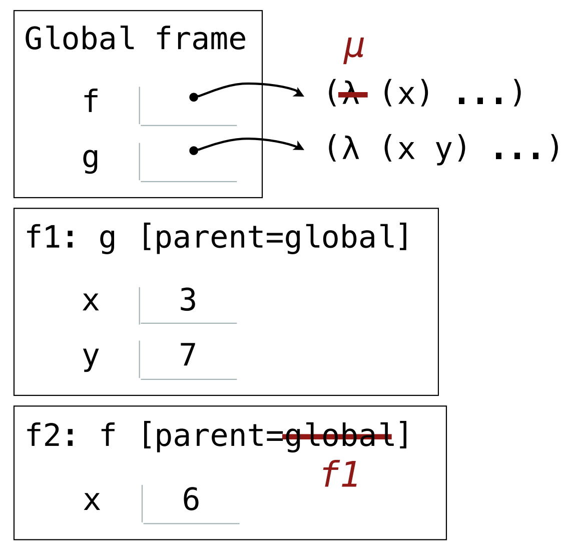

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But... no for/while statements! Can we make basic iteration efficient? Yes!

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(Demo)

Interactive Diagram

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(Demo)

Tail Recursion Examples

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Map and Reduce

Example: Reduce

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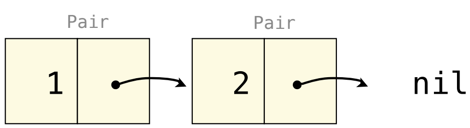

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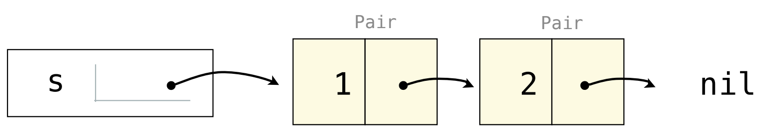

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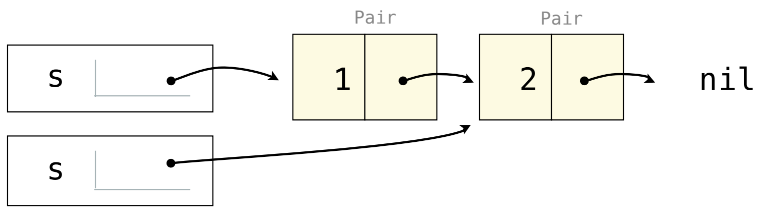

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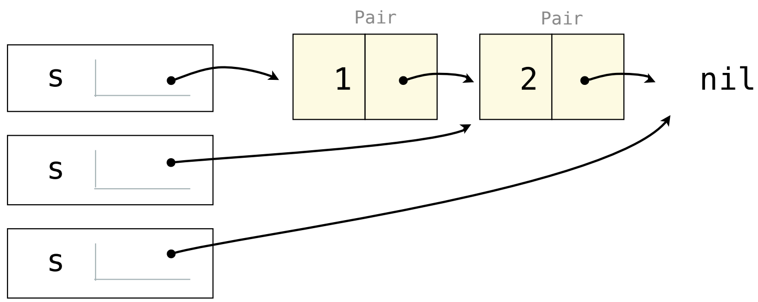

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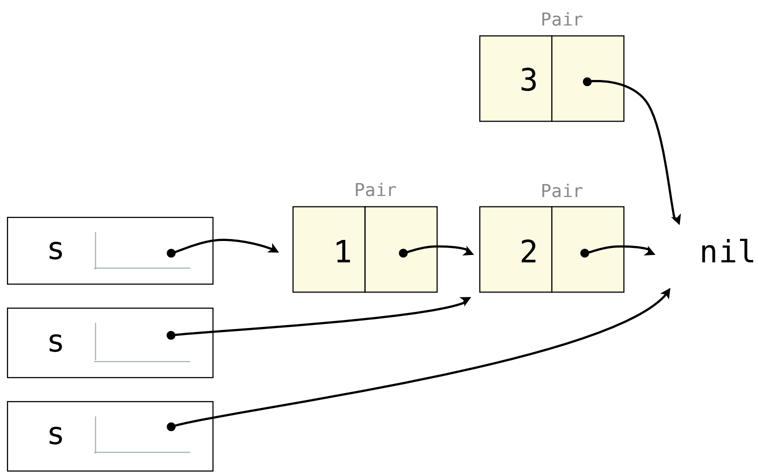

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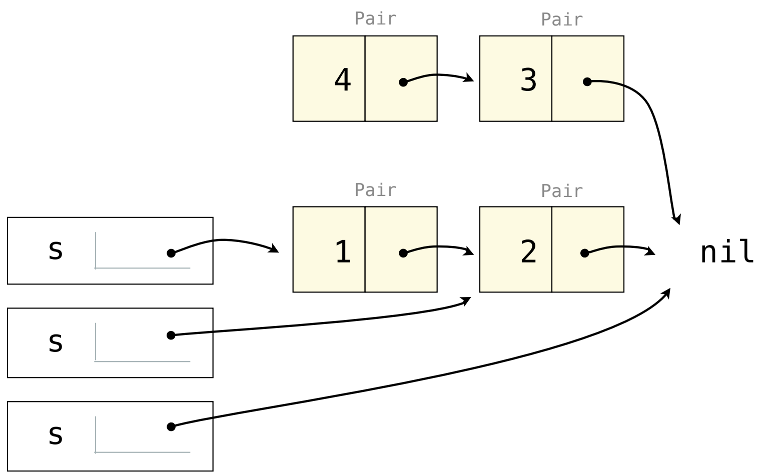

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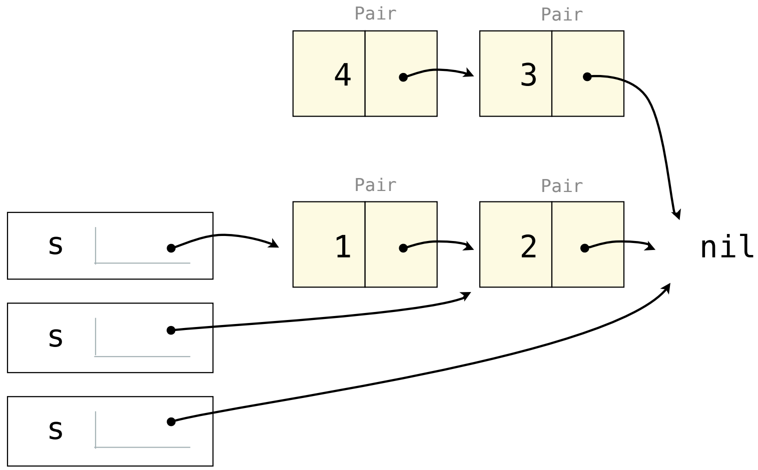

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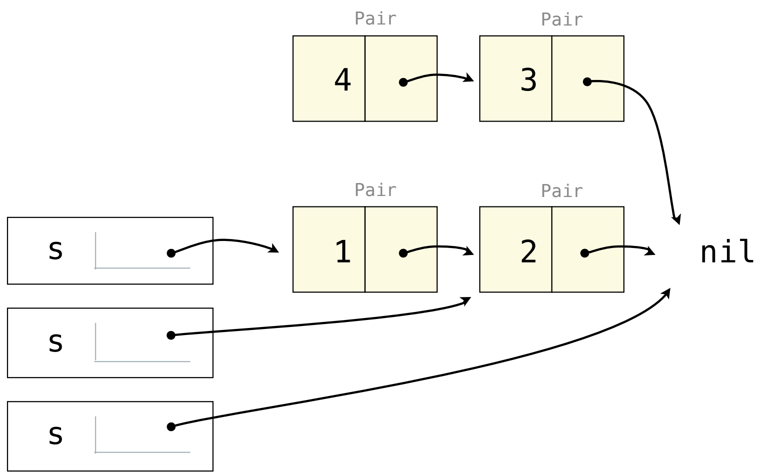

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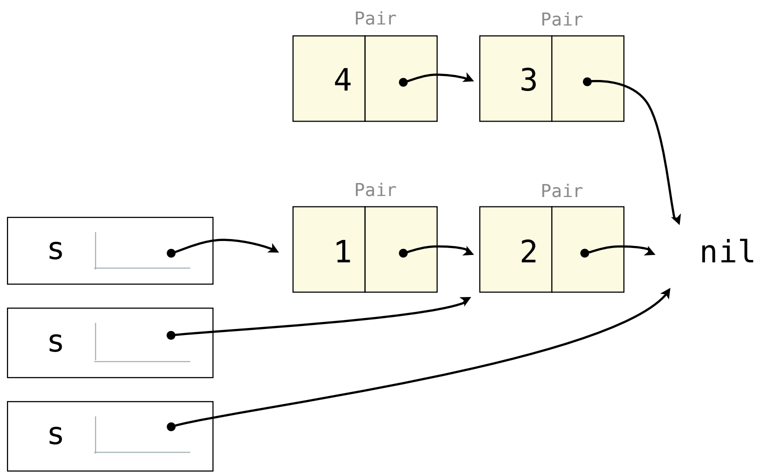

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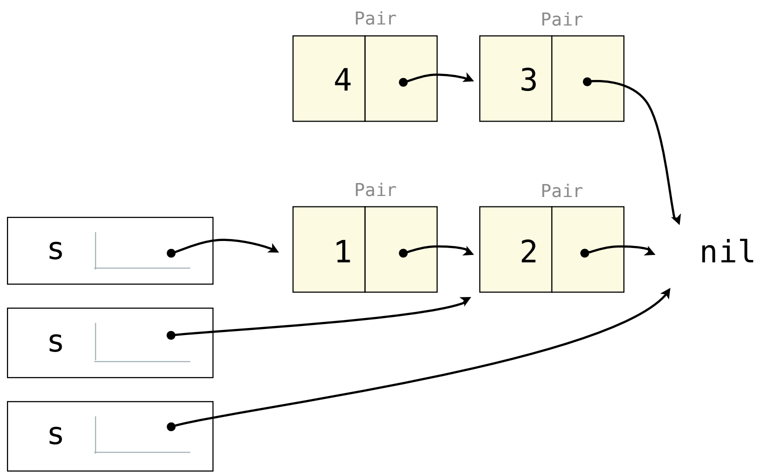

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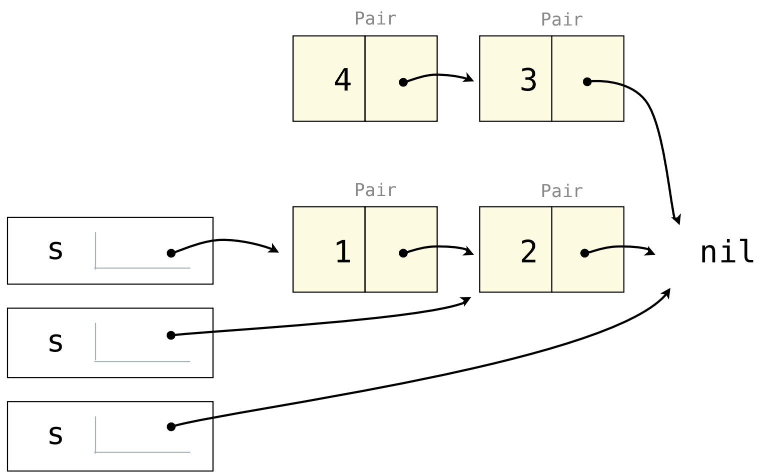

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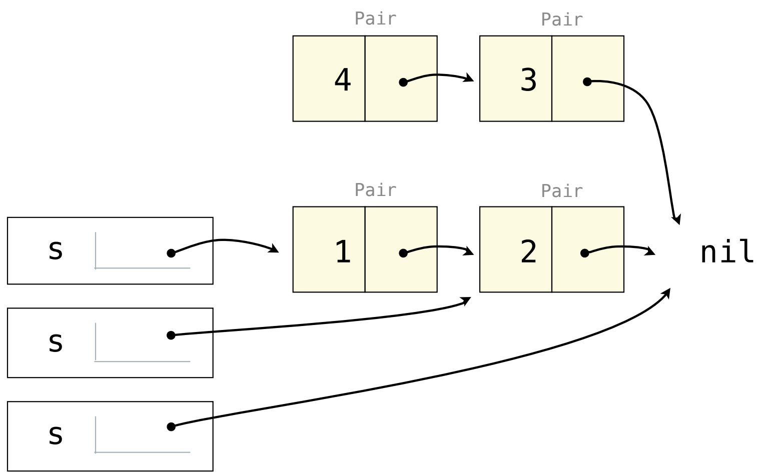

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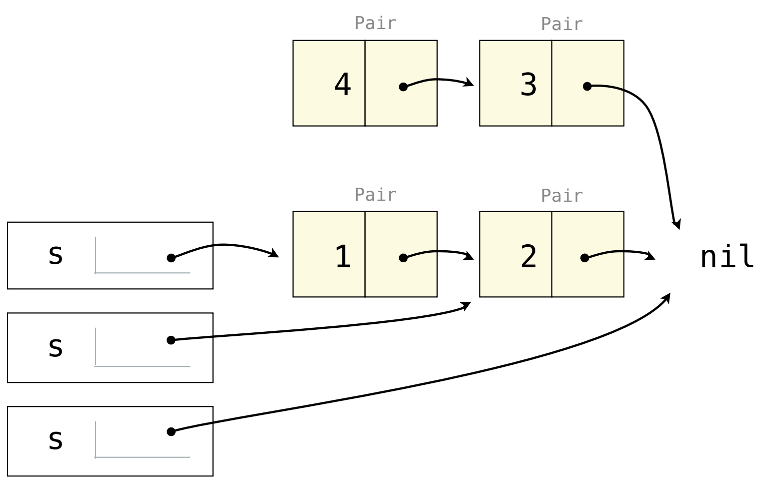

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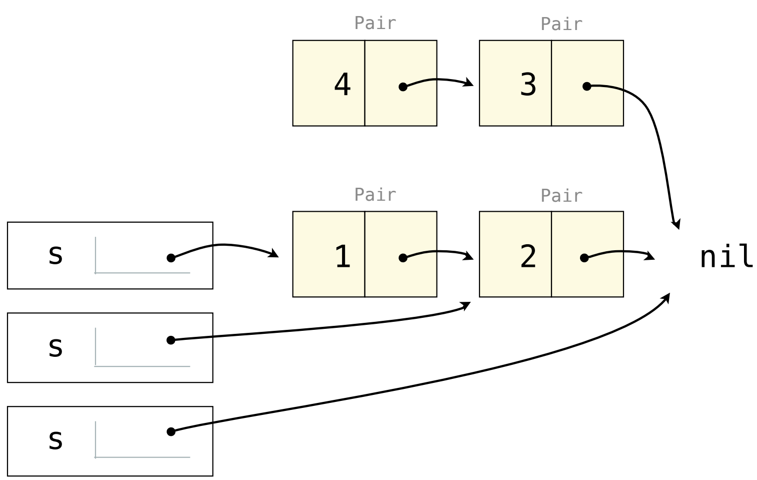


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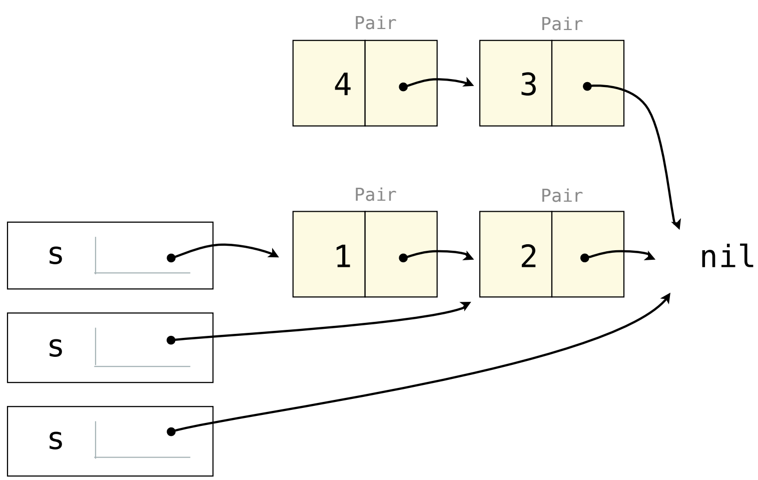


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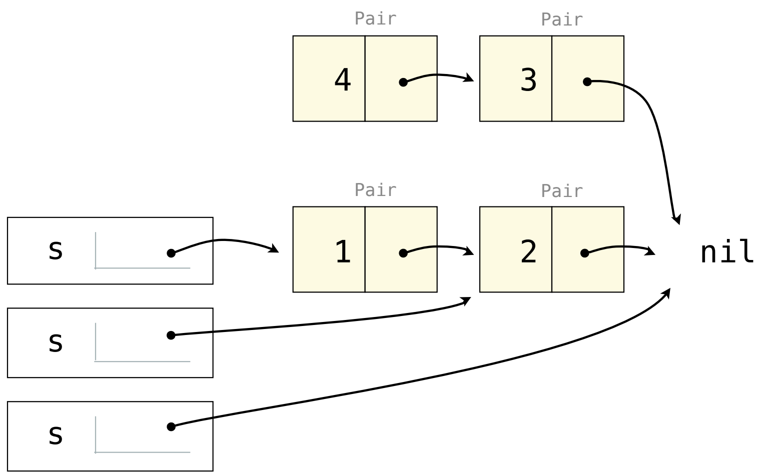


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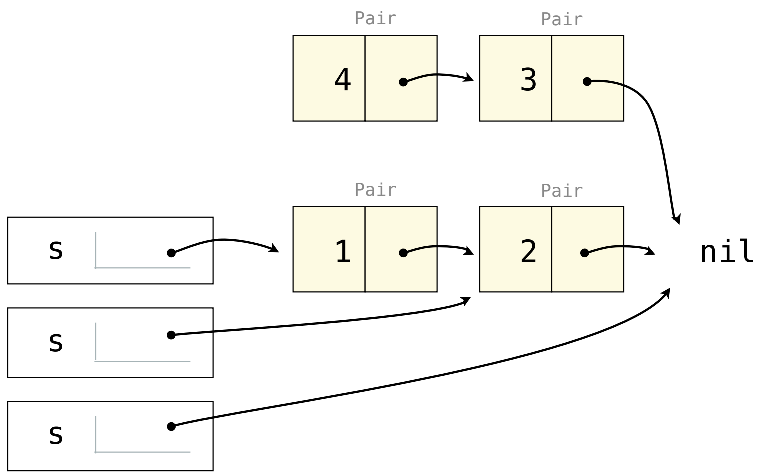


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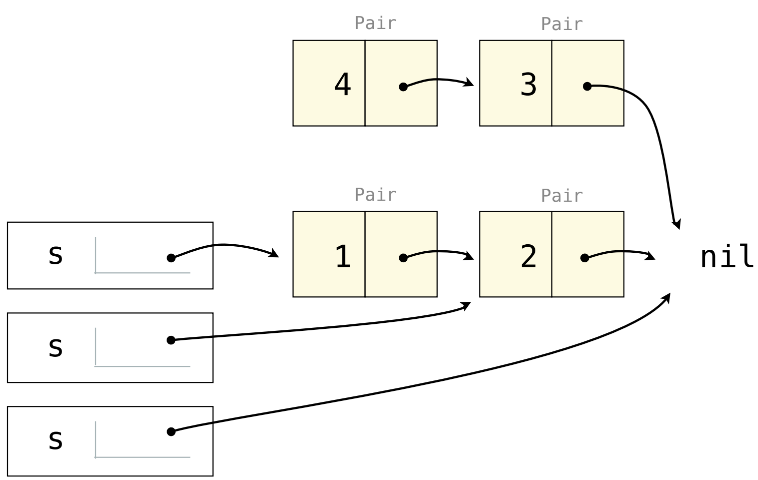


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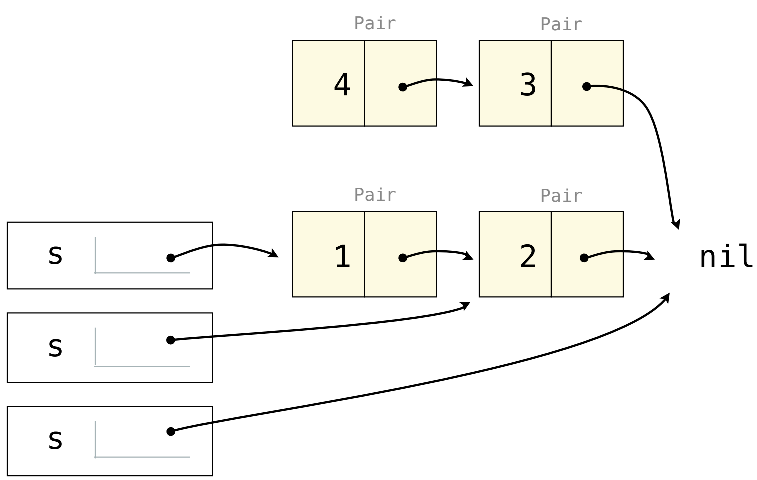


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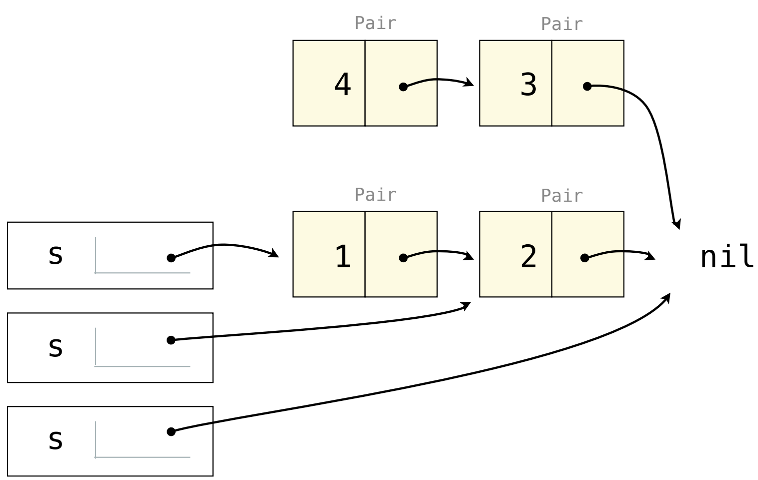


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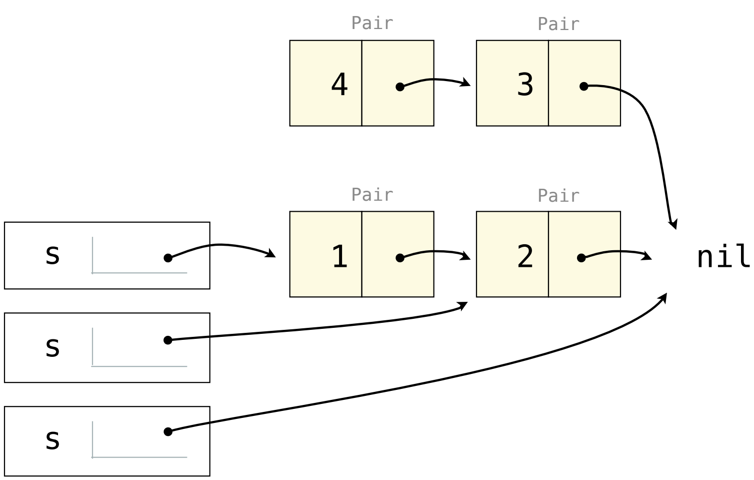


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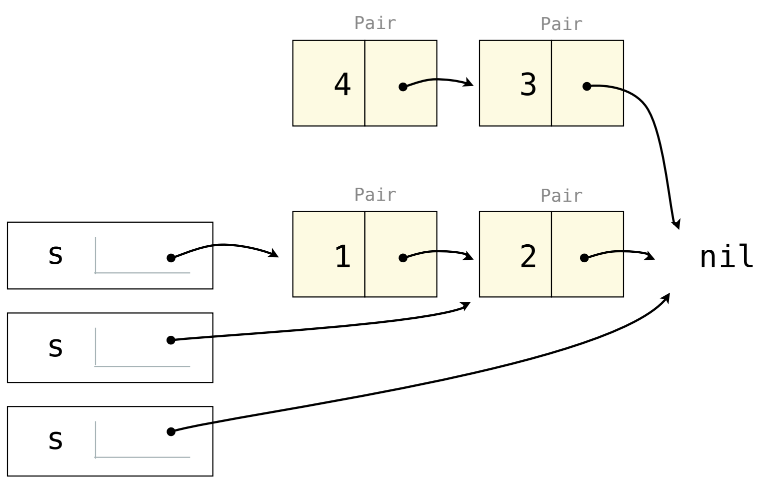


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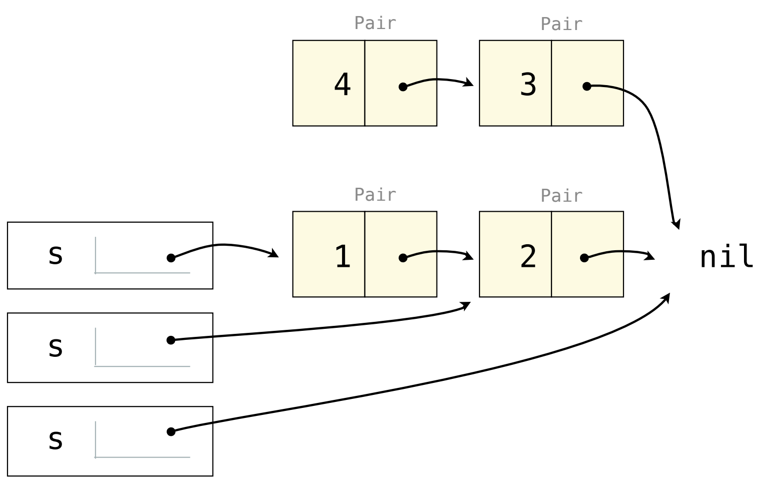


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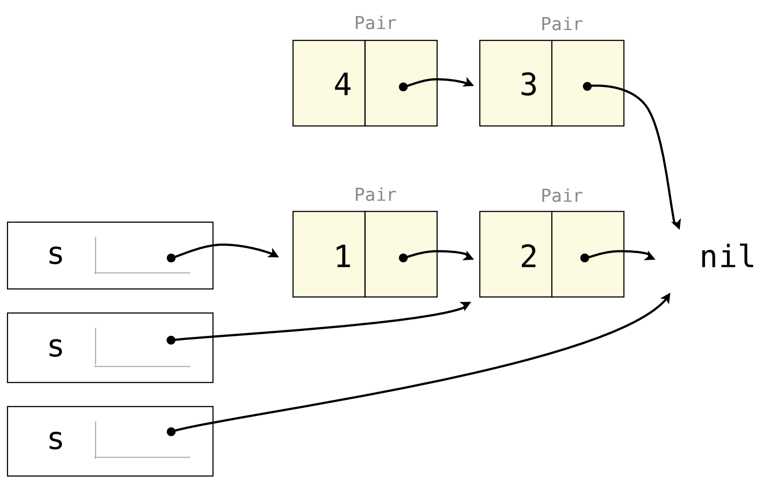


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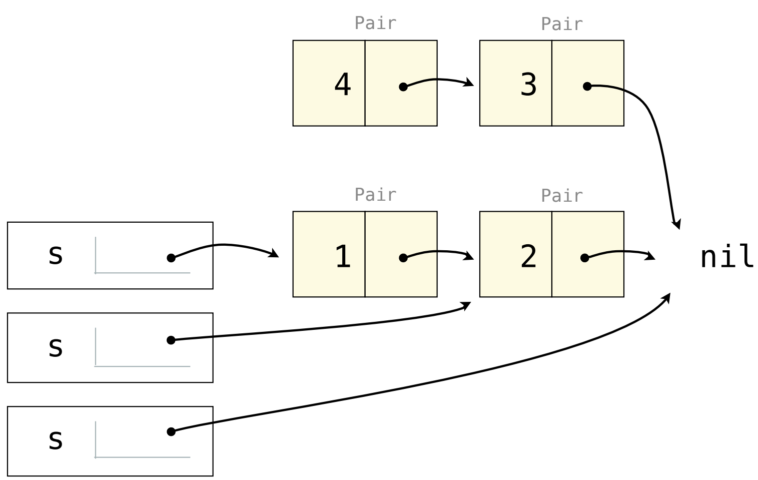


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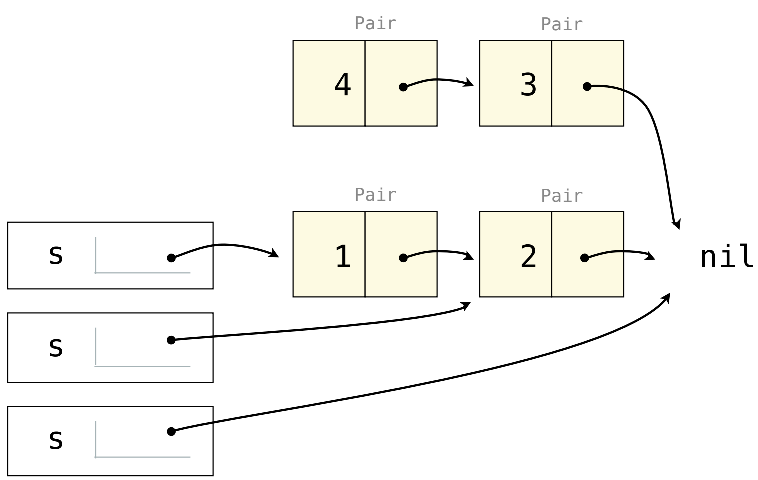


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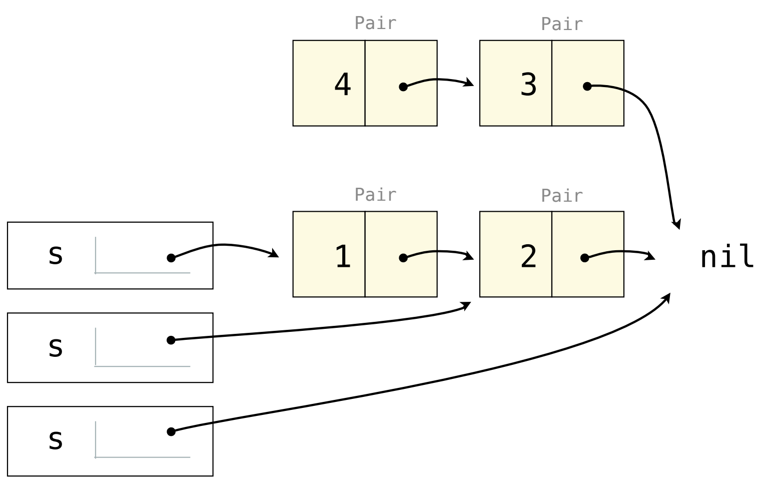


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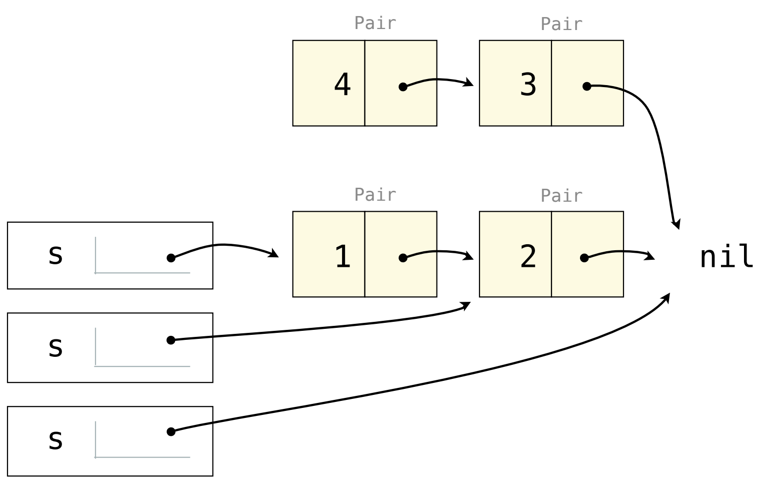


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General Computing Machines

An Analogy: Programs Define Machines

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Programs specify the logic of a computational device

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Programs specify the logic of a computational device

factorial

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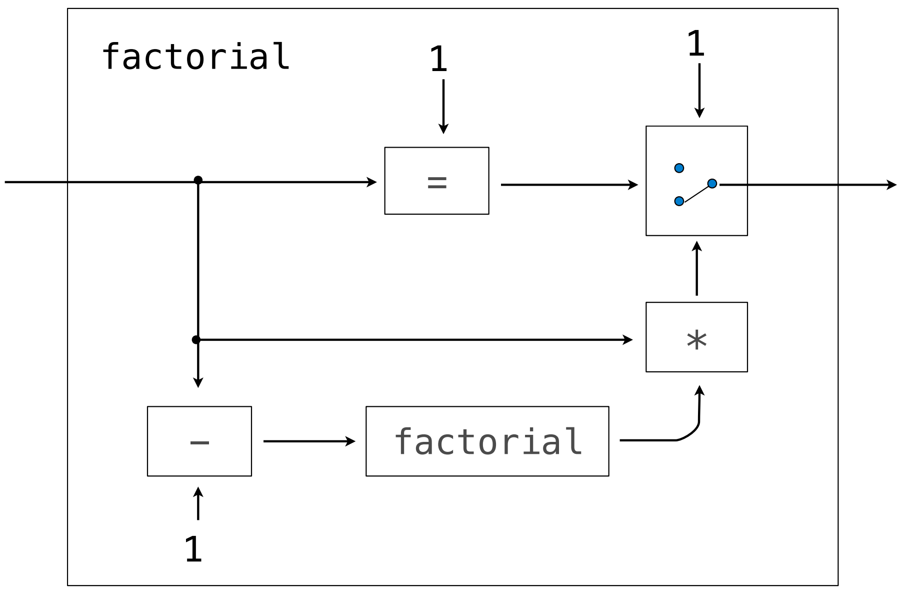

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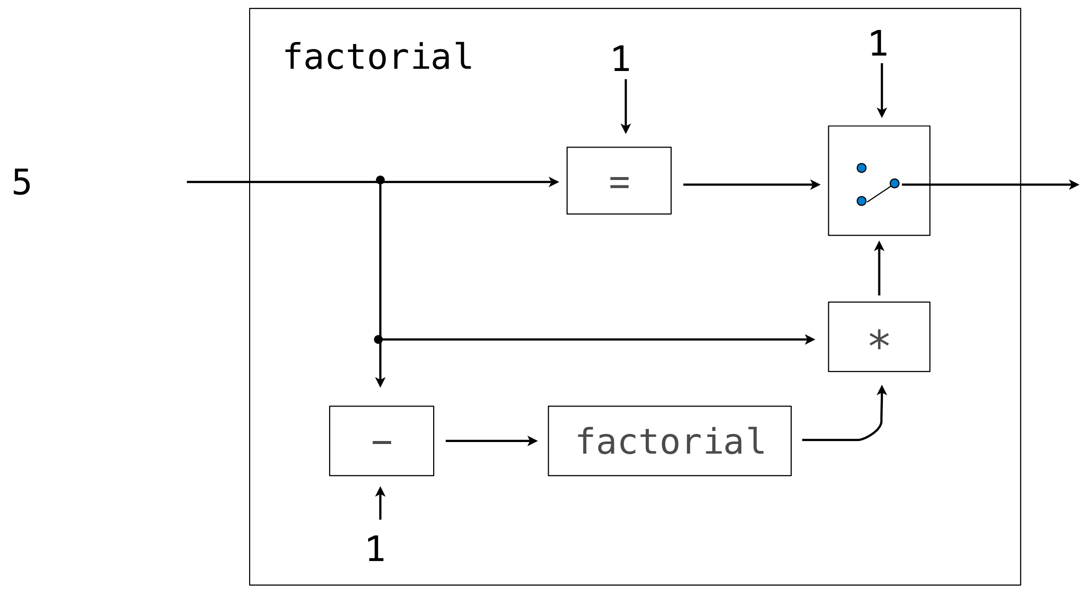

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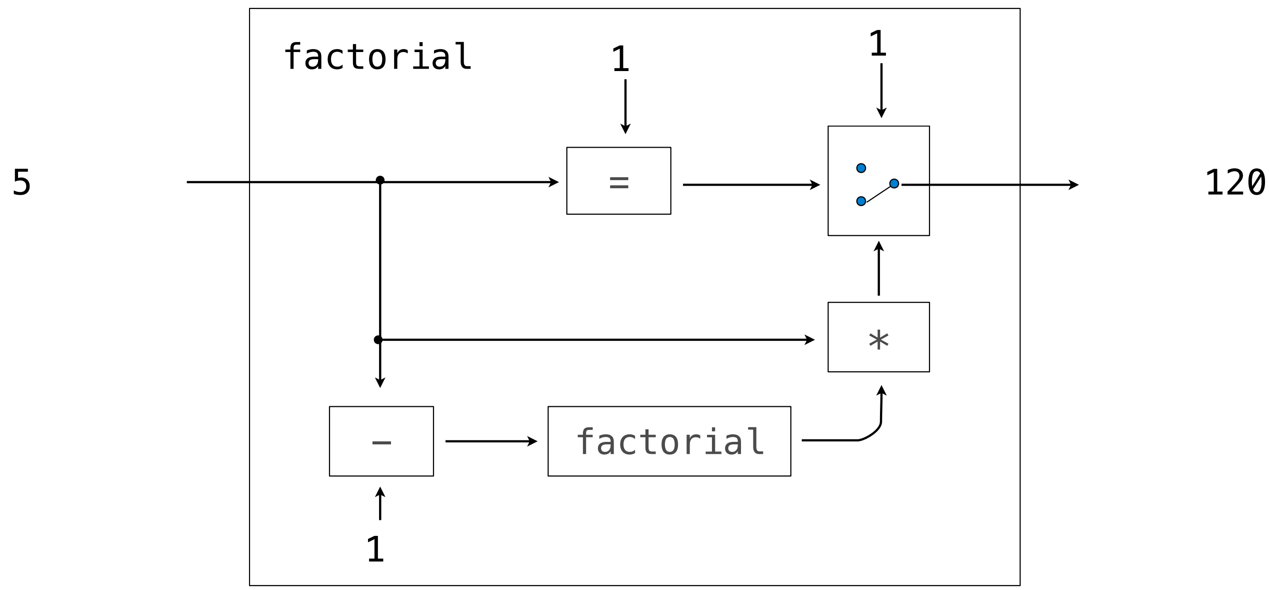

Interpreters are General Computing Machine

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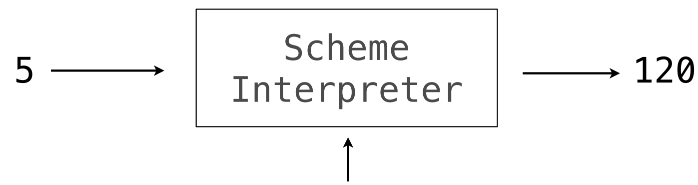


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(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
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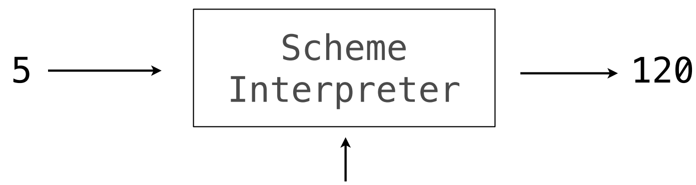


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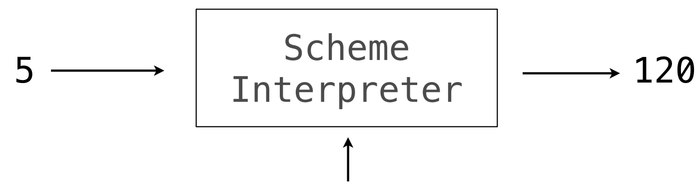


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A bridge between the data objects that are manipulated by our programming language and the programming language itself

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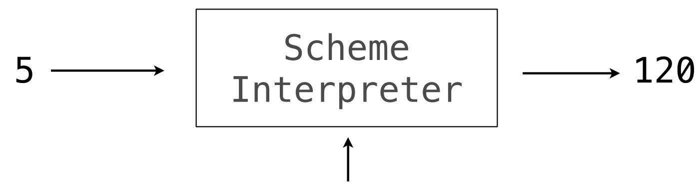


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Internally, it is just a set of evaluation rules