61A Lecture 21

Announcements

Sets

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One more built-in Python container type

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- Set literals are enclosed in braces

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>>> s = {3, 2, 1, 4, 4}
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>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
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>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
```

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>>> s
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>>> len(s)
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```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
```

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>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Implementing Sets

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What we should be able to do with a set:
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- **Membership testing:** Is a value an element of a set?

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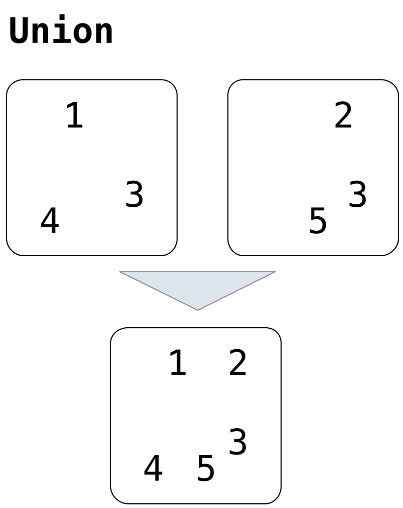

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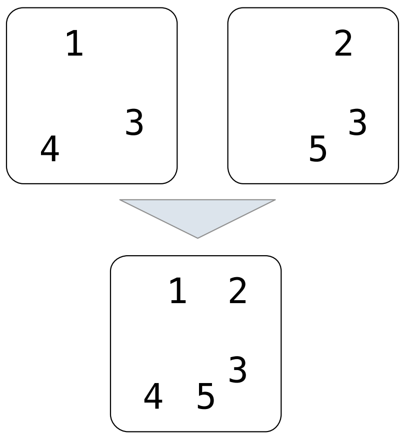
Union



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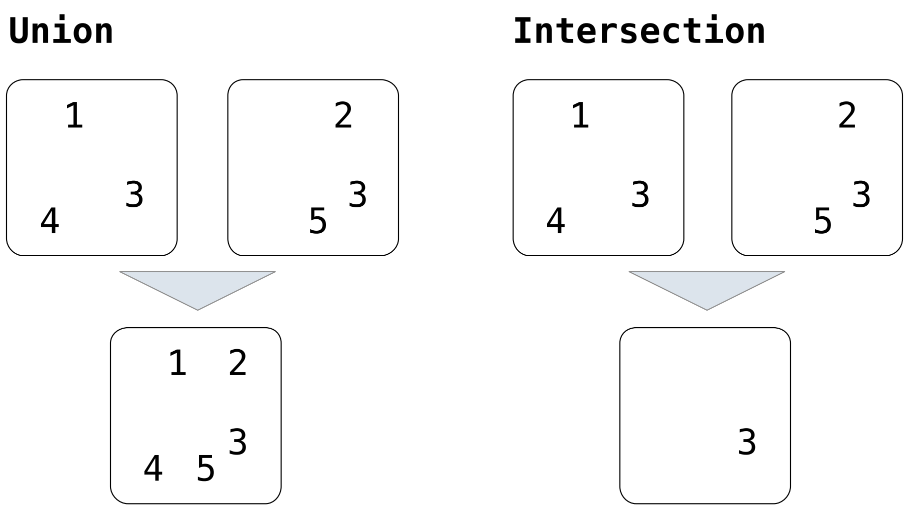

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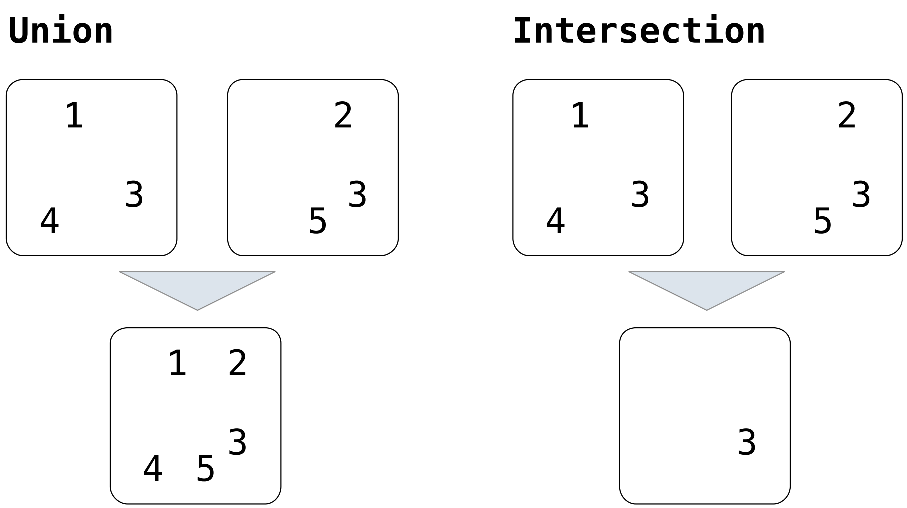

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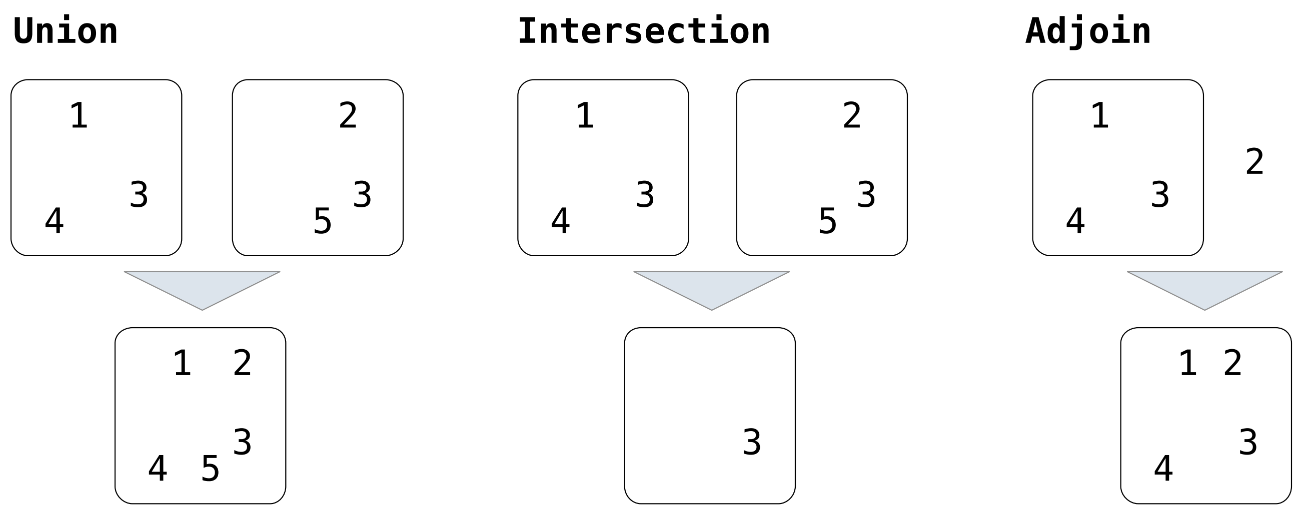

Sets as Unordered Sequences

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    >>> s = Link(1, Link(2, Link(3)))
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Assuming v either does not appear in s ***or*** *appears in a uniformly distributed random location*

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(Demo)

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def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    else:
        return Link(v, s)
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Sets as Unordered Sequences

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The size of the set

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def adjoin_set(s, v):
    if set_contains(s, v):
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def intersect_set(set1, set2):
    in_set2 = lambda v: set_contains(set2, v)
    return filter_link(in_set2, set1)
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```
def union_set(set1, set2):
    not_in_set2 = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_link(not_in_set2, set1)
    return extend_link(set1_not_set2, set2)
```

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Use sets to contain values		

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Parts of the program that...	Assume that sets are...	Using...
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Sets as Ordered Sequences

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, set_contains, adjoin_set, intersect_set, union_set

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Implement set operations	Ordered linked lists	

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Different parts of a program may make different assumptions about data

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        if e1 == e2:
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        elif e1 < e2:
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Order of growth?

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Sets as Binary Search Trees

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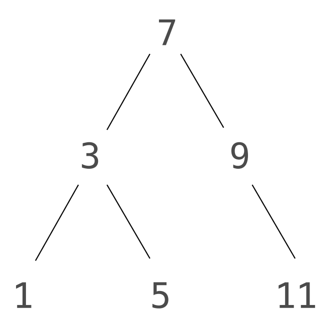

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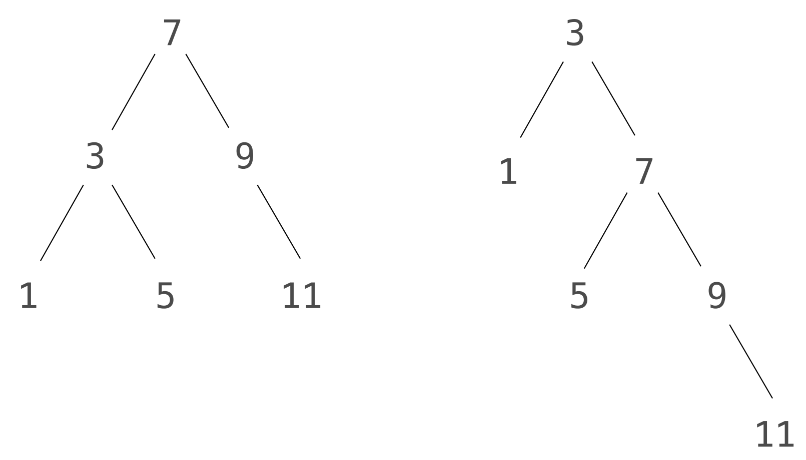

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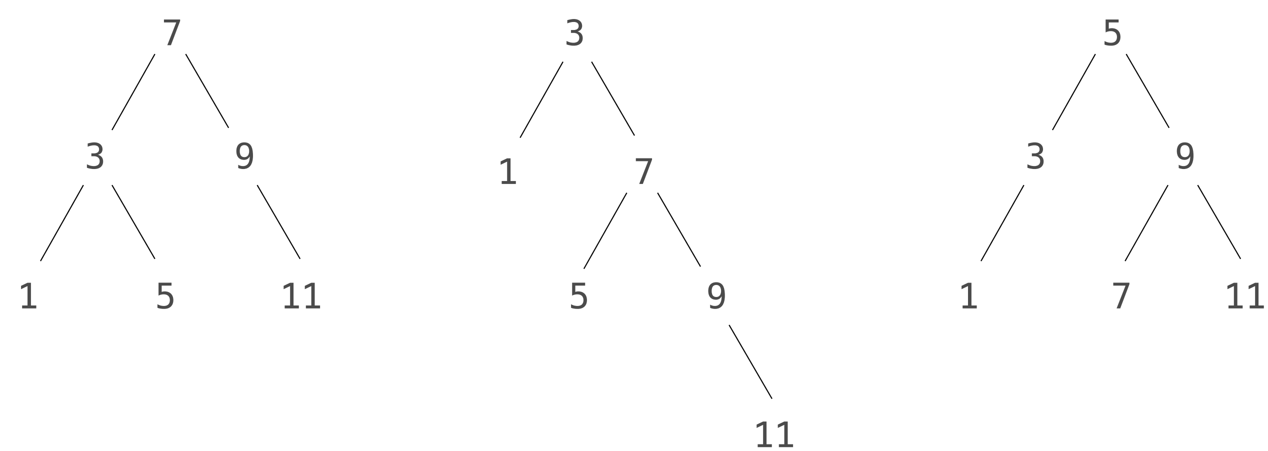

Binary Tree Class

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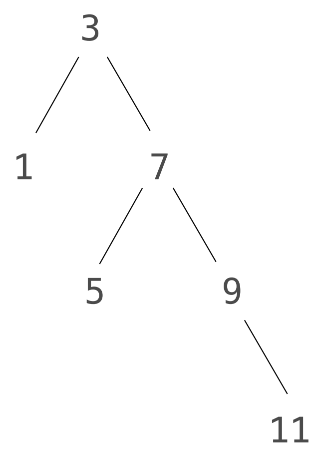

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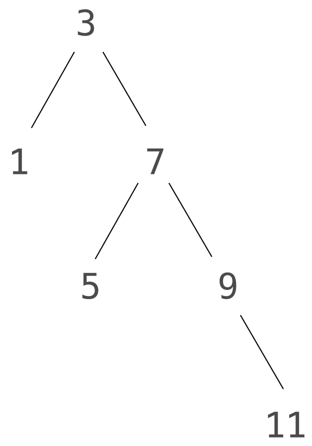
Idea: Fill the place of a missing left branch with an empty tree



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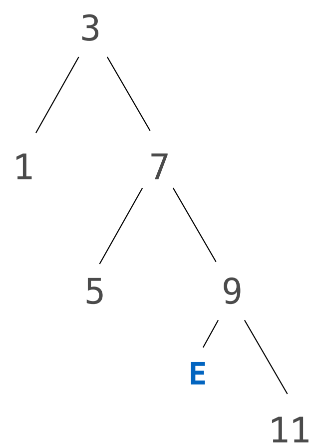

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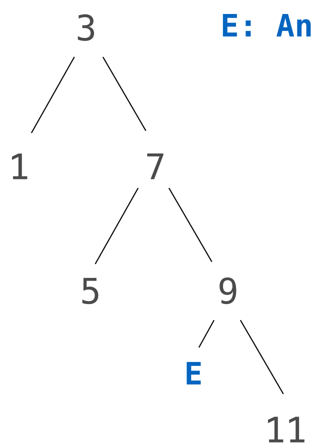


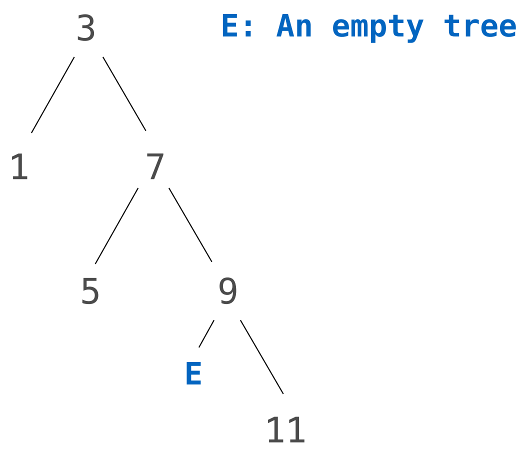
E: An empty tree

Binary Tree Class

A binary tree is a tree that has
a left branch and a right branch

Idea: Fill the place of a missing
left branch with an empty tree

Idea 2: An instance of BinaryTree
always has *exactly* two branches



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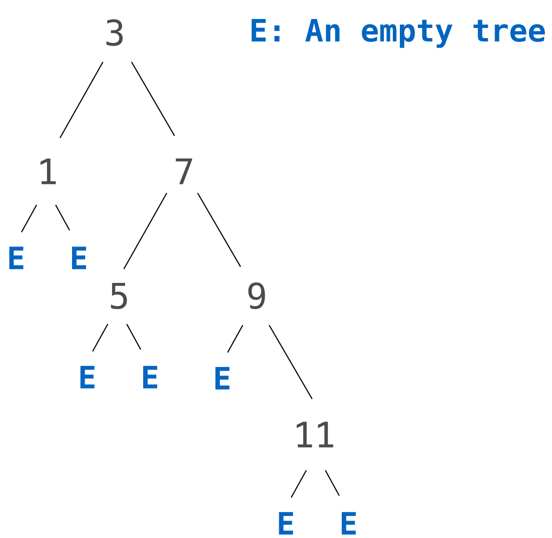

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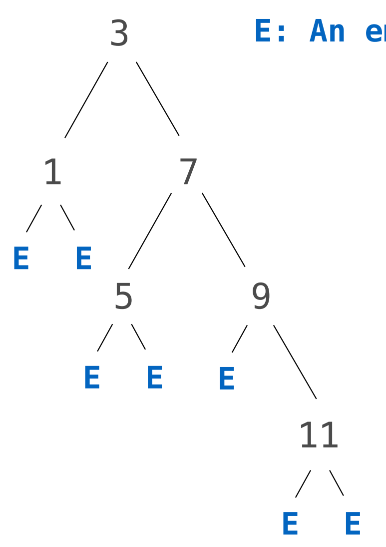

E: An empty tree

```python
class BinaryTree(Tree):
    empty = Tree(None)
    empty.is_empty = True

    def __init__(self, entry, left=empty, right=empty):
        Tree.__init__(self, entry, (left, right))
        self.is_empty = False

    @property
    def left(self):
        return self.branches[0]

    @property
    def right(self):
        return self.branches[1]
```

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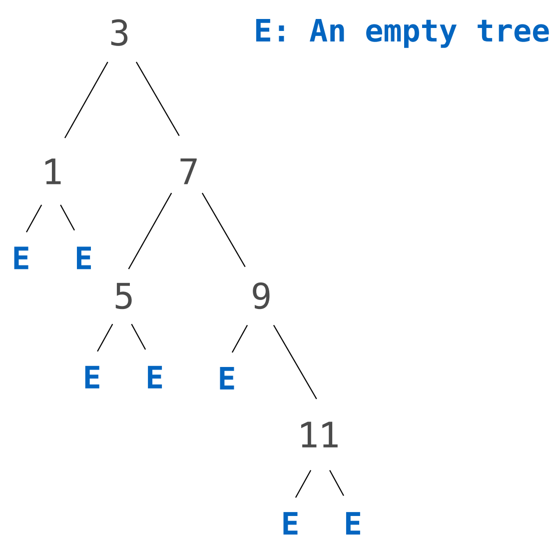


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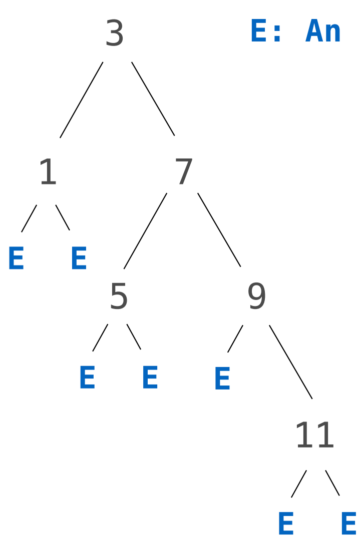

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Bin = BinaryTree
t = Bin(3, Bin(1),
           Bin(7, Bin(5),
                  Bin(9, Bin.empty,
                         Bin(11))))
```

Membership in Binary Search Trees

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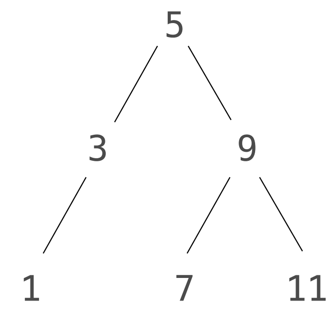

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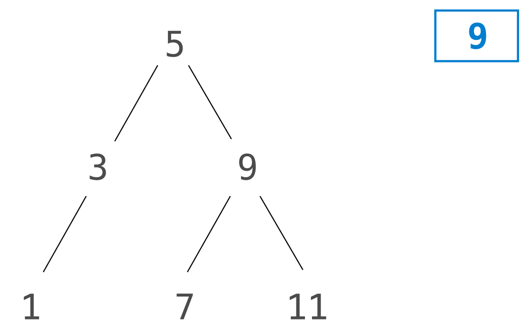

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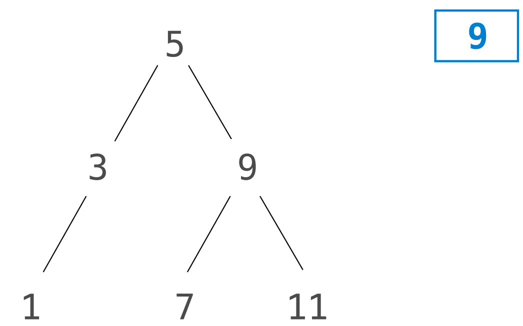

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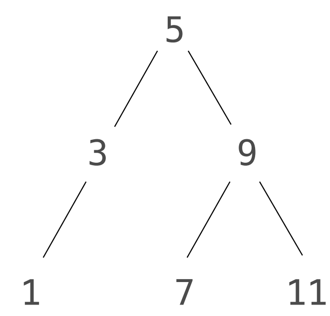

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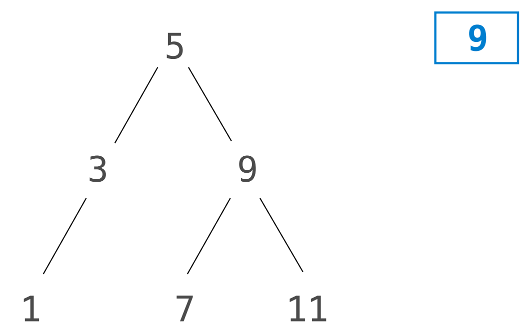

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```
def set_contains(s, v):
    if s.is_empty:
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        return set_contains(s.right, v)
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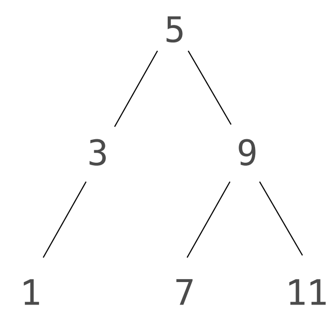

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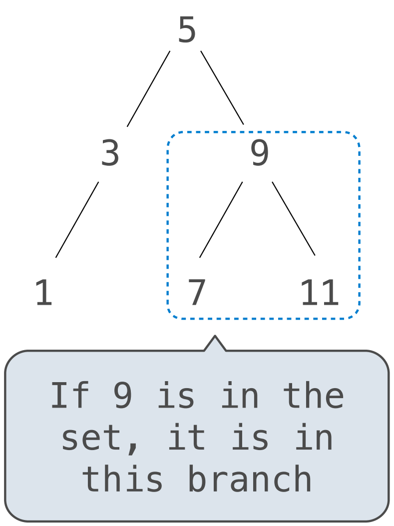

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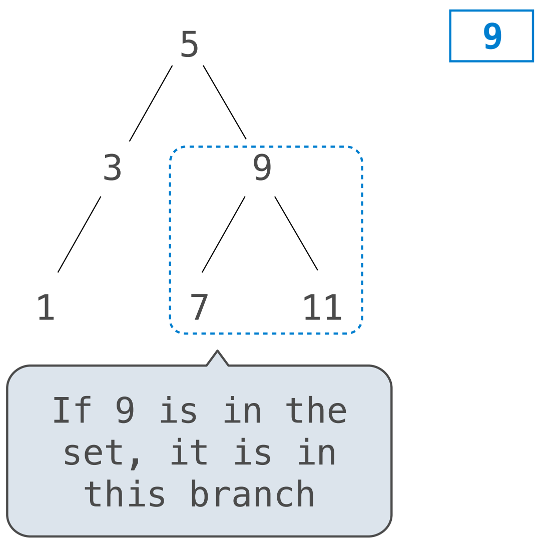
        return set_contains(s.left, v)
```



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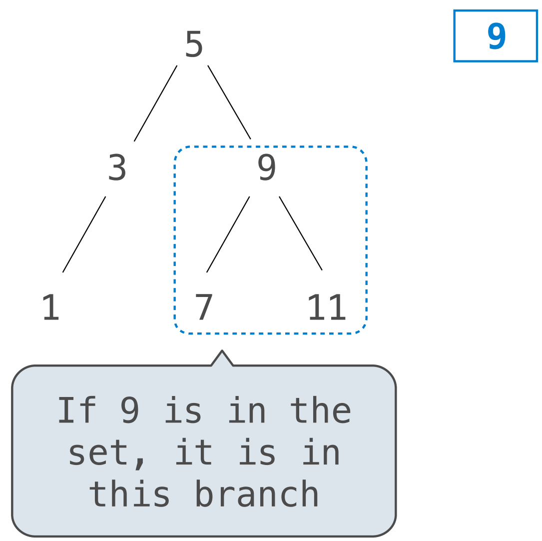

Order of growth?

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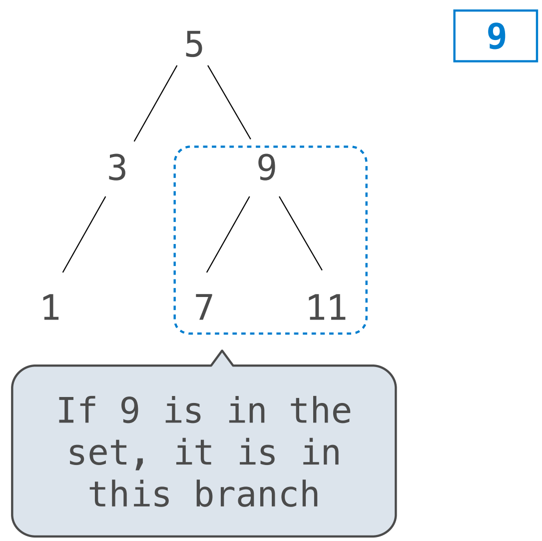

Order of growth? $\Theta(h)$ on average

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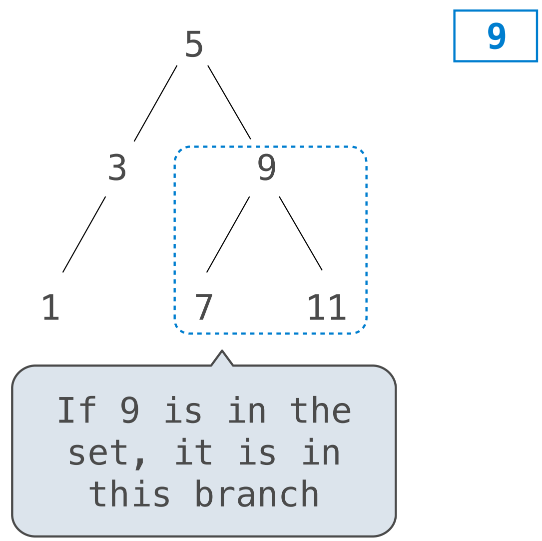

Order of growth? $\Theta(h)$ on average $\Theta(\log n)$ on average for a balanced tree

Adjoining to a Tree Set

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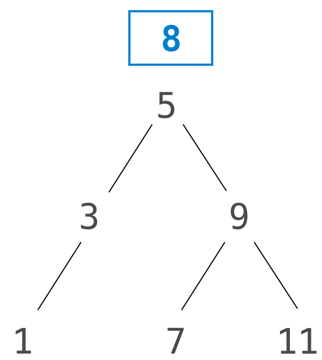

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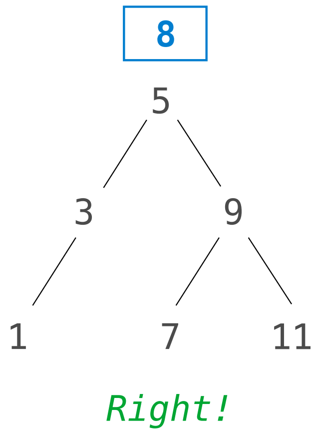

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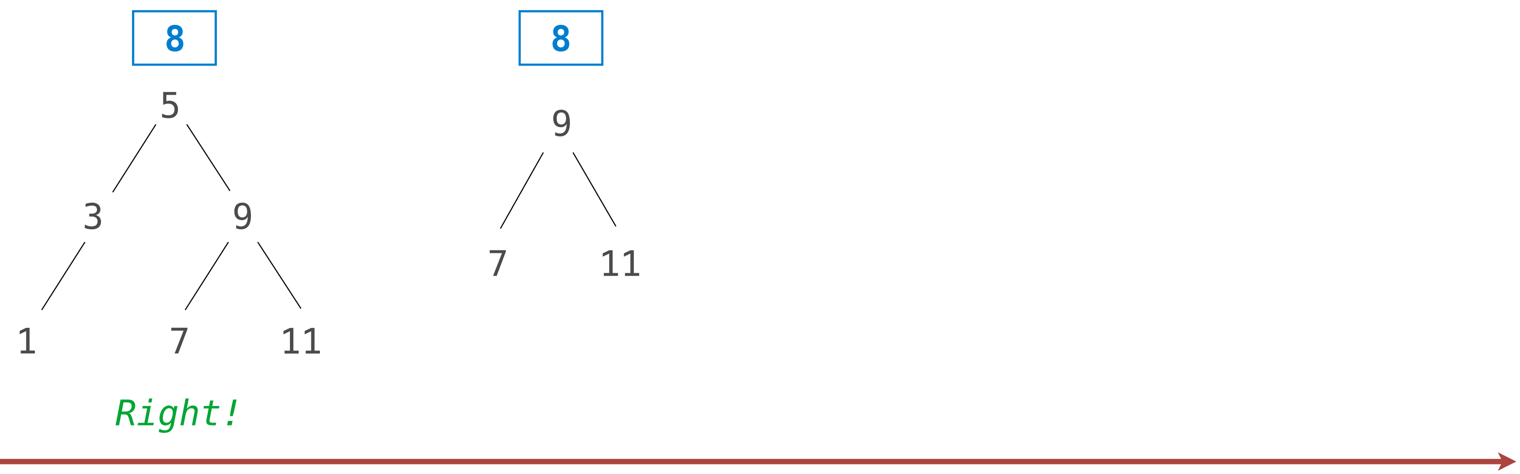

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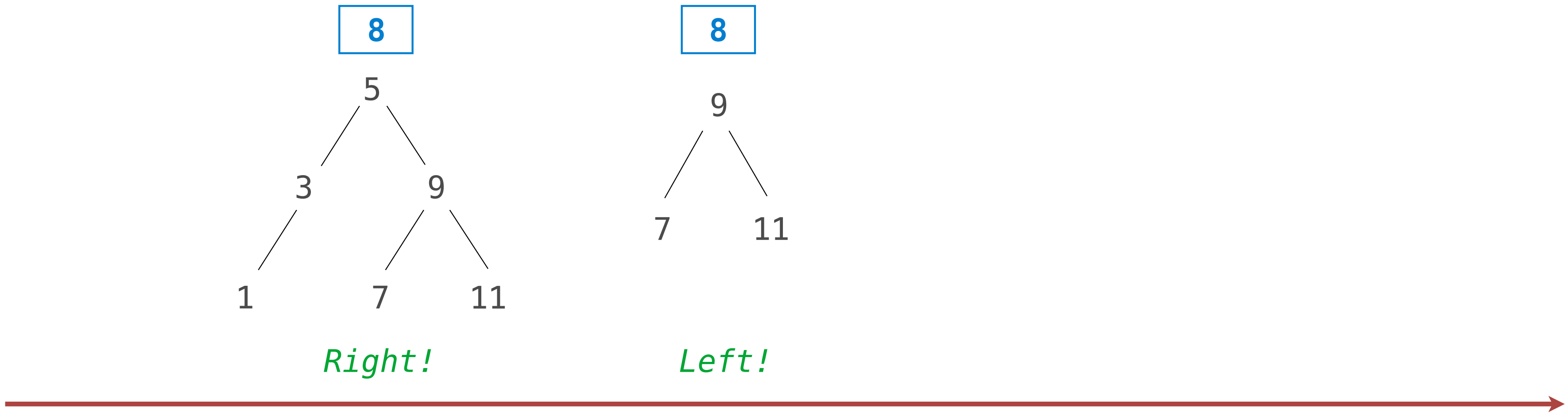

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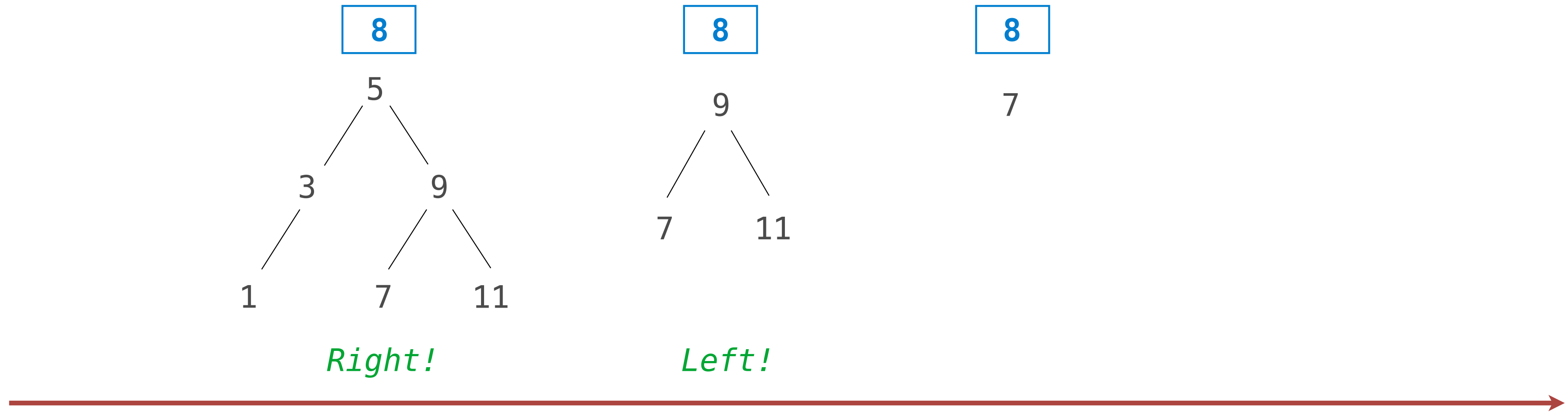

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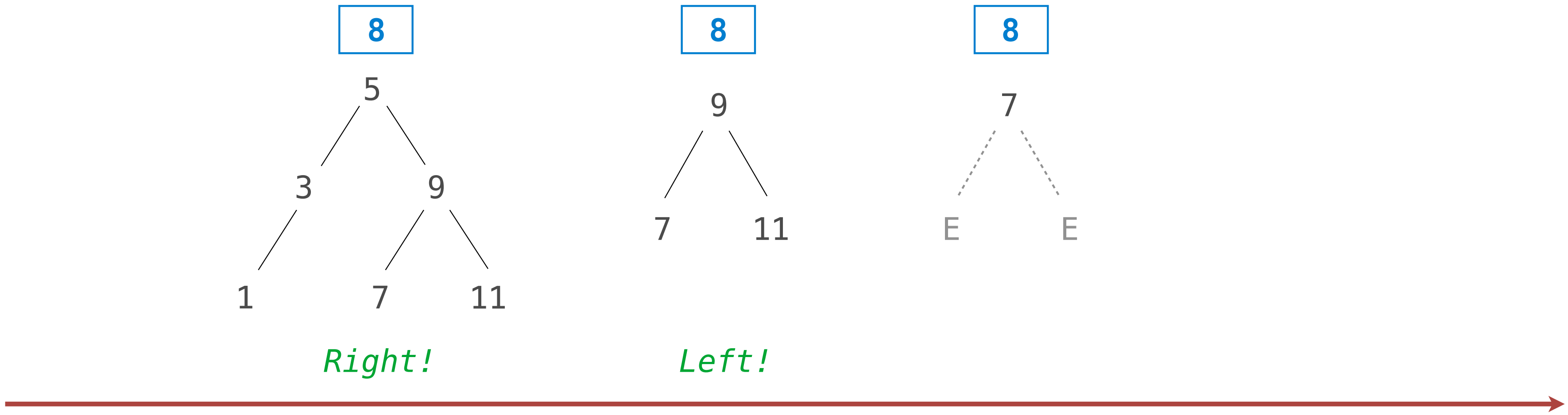

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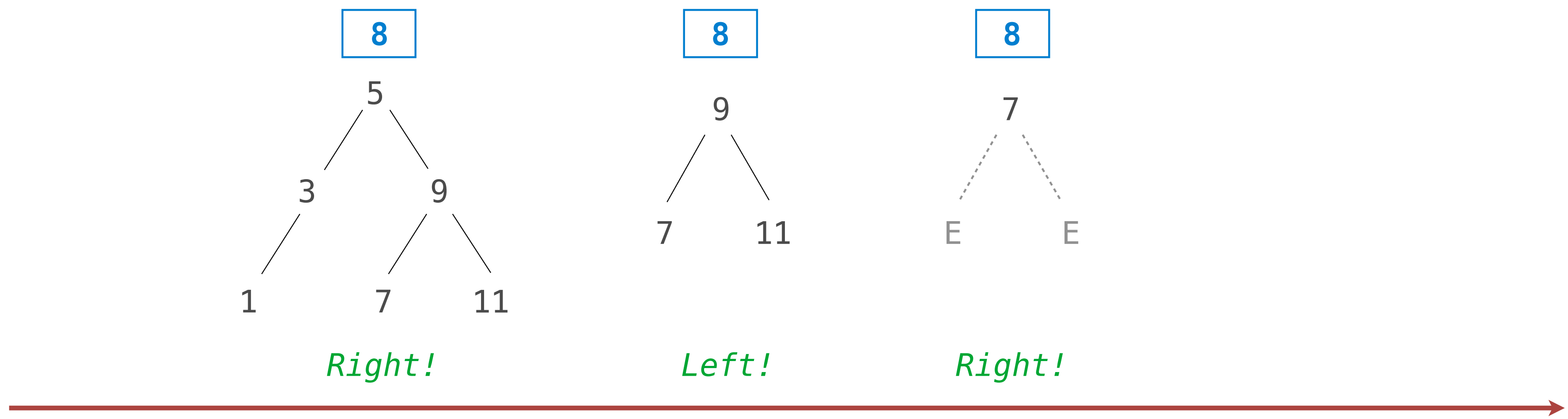

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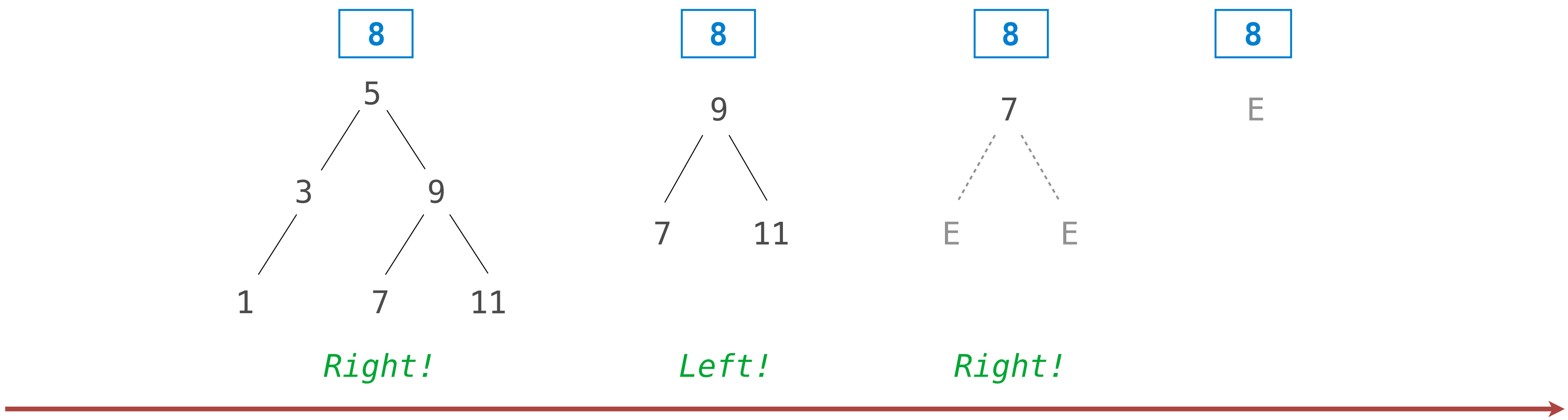

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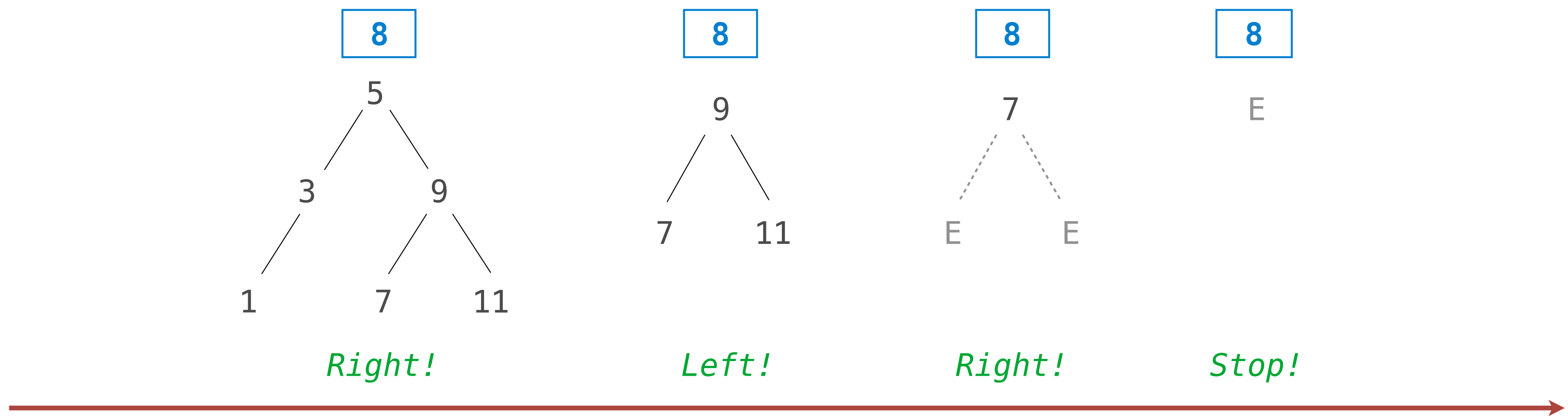

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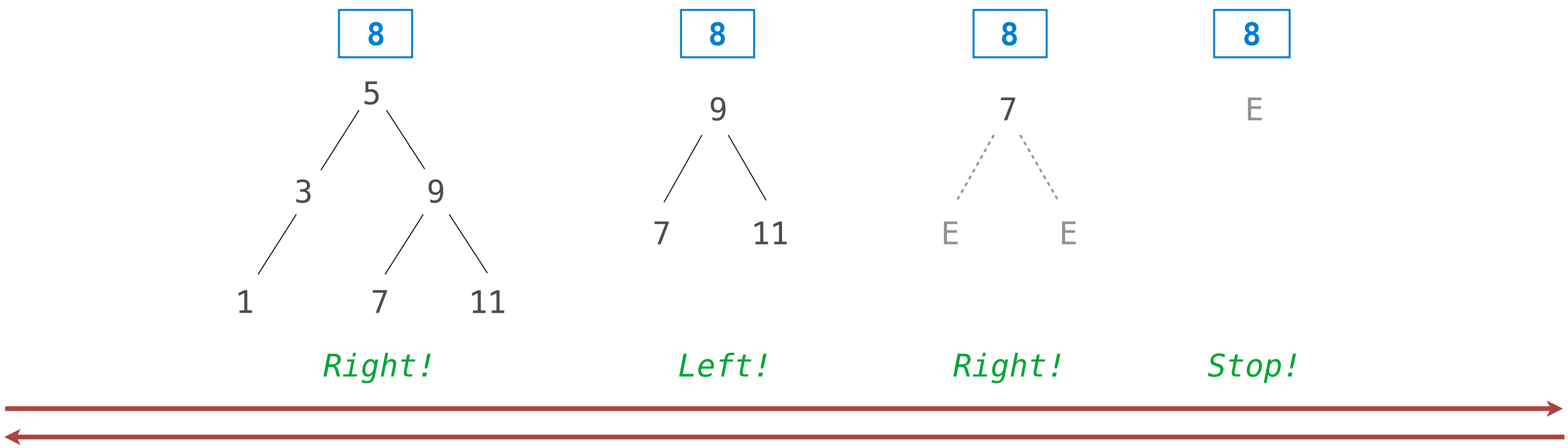

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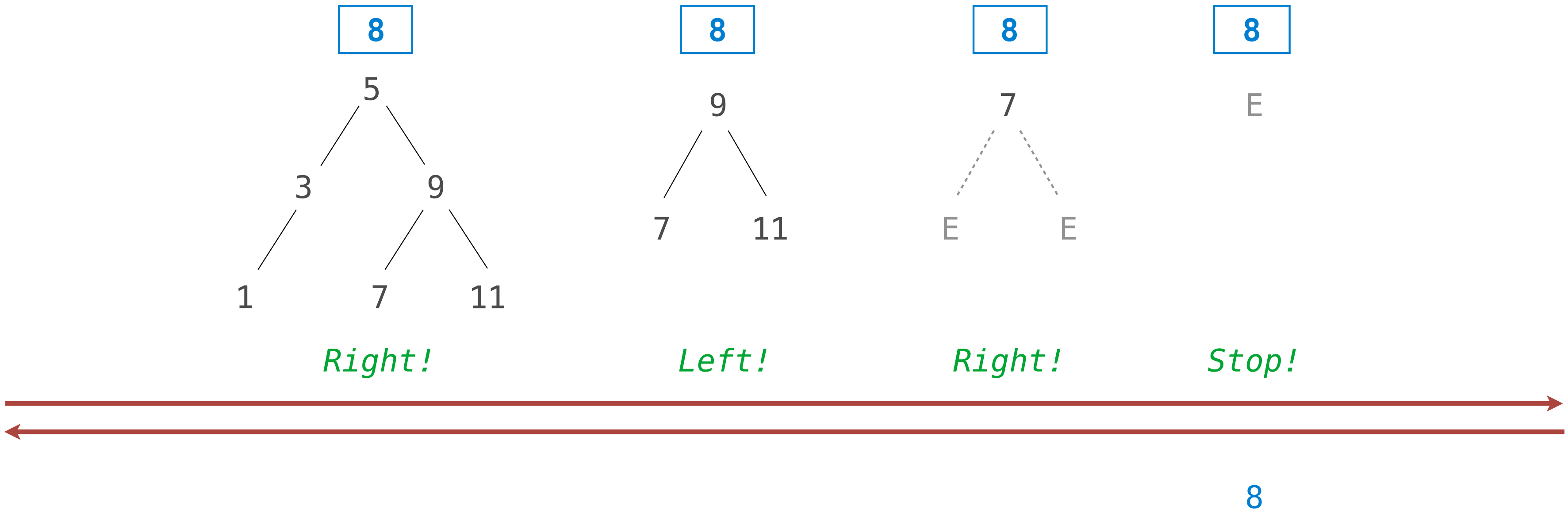

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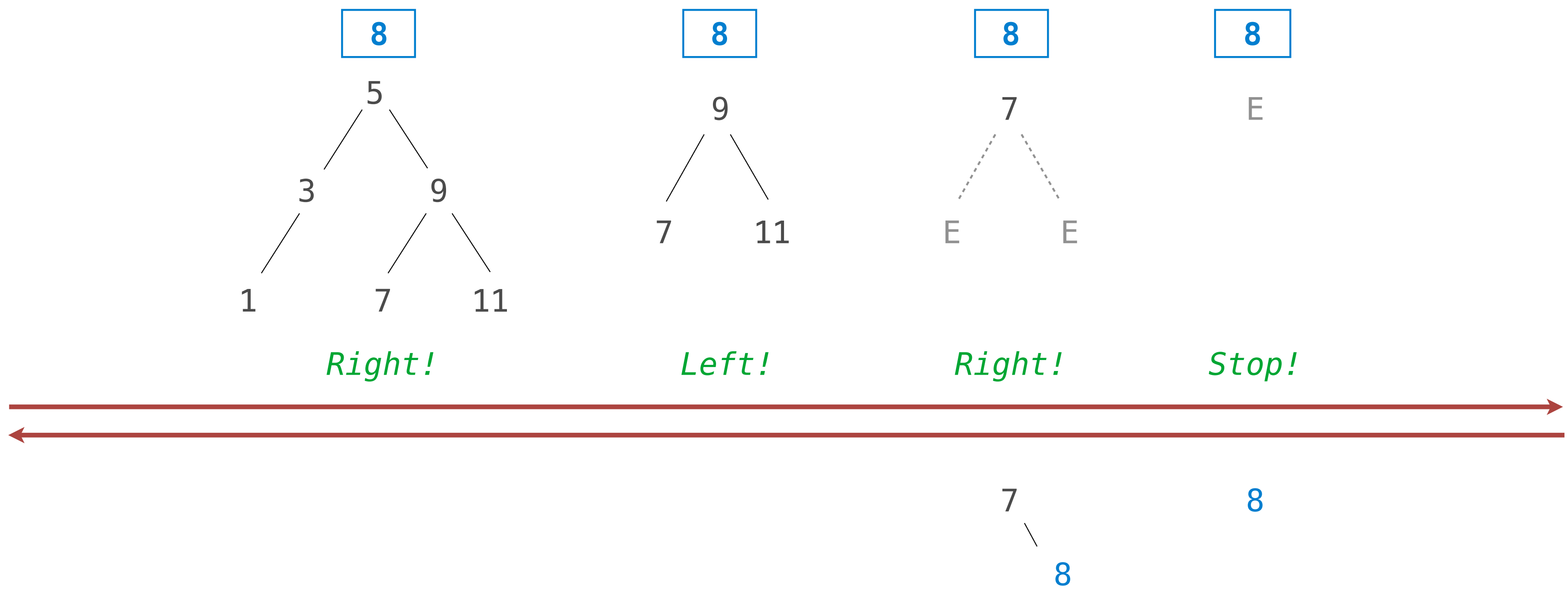

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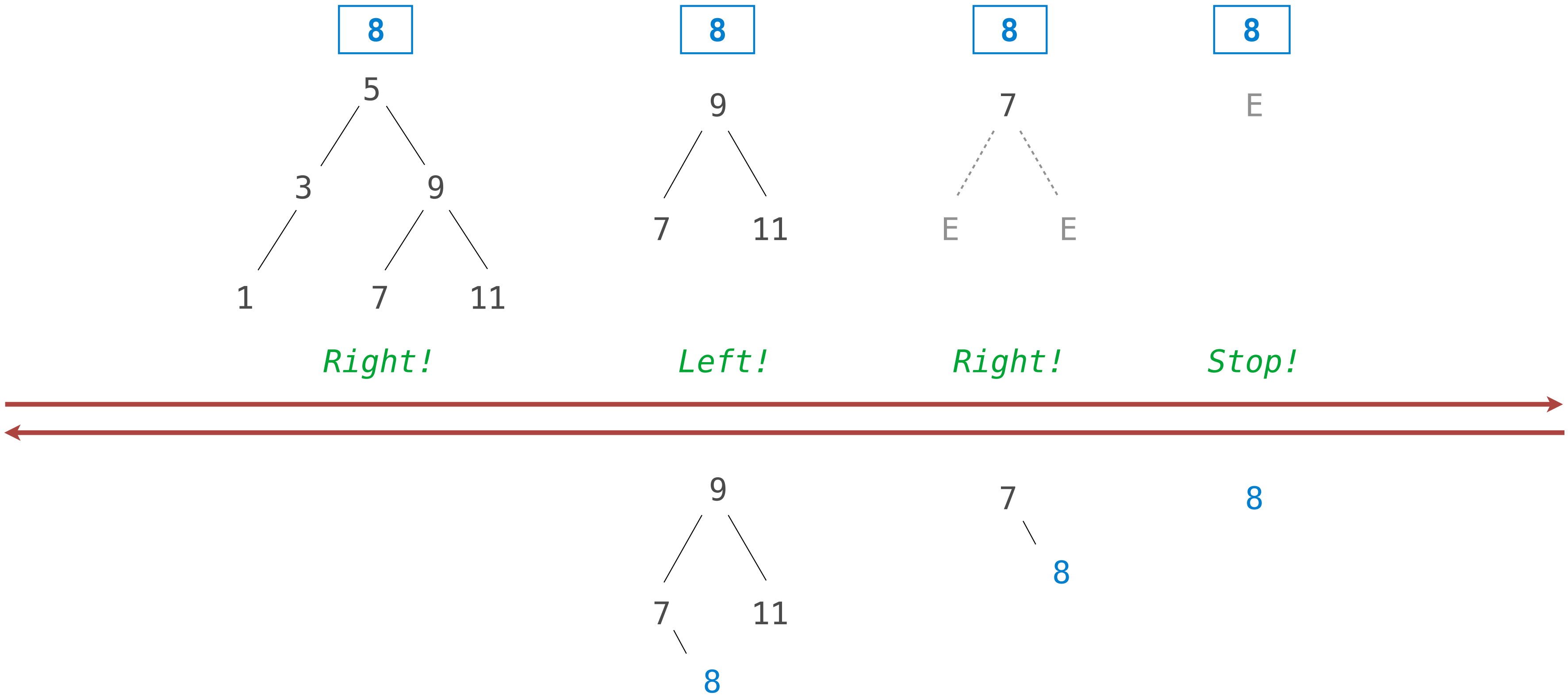

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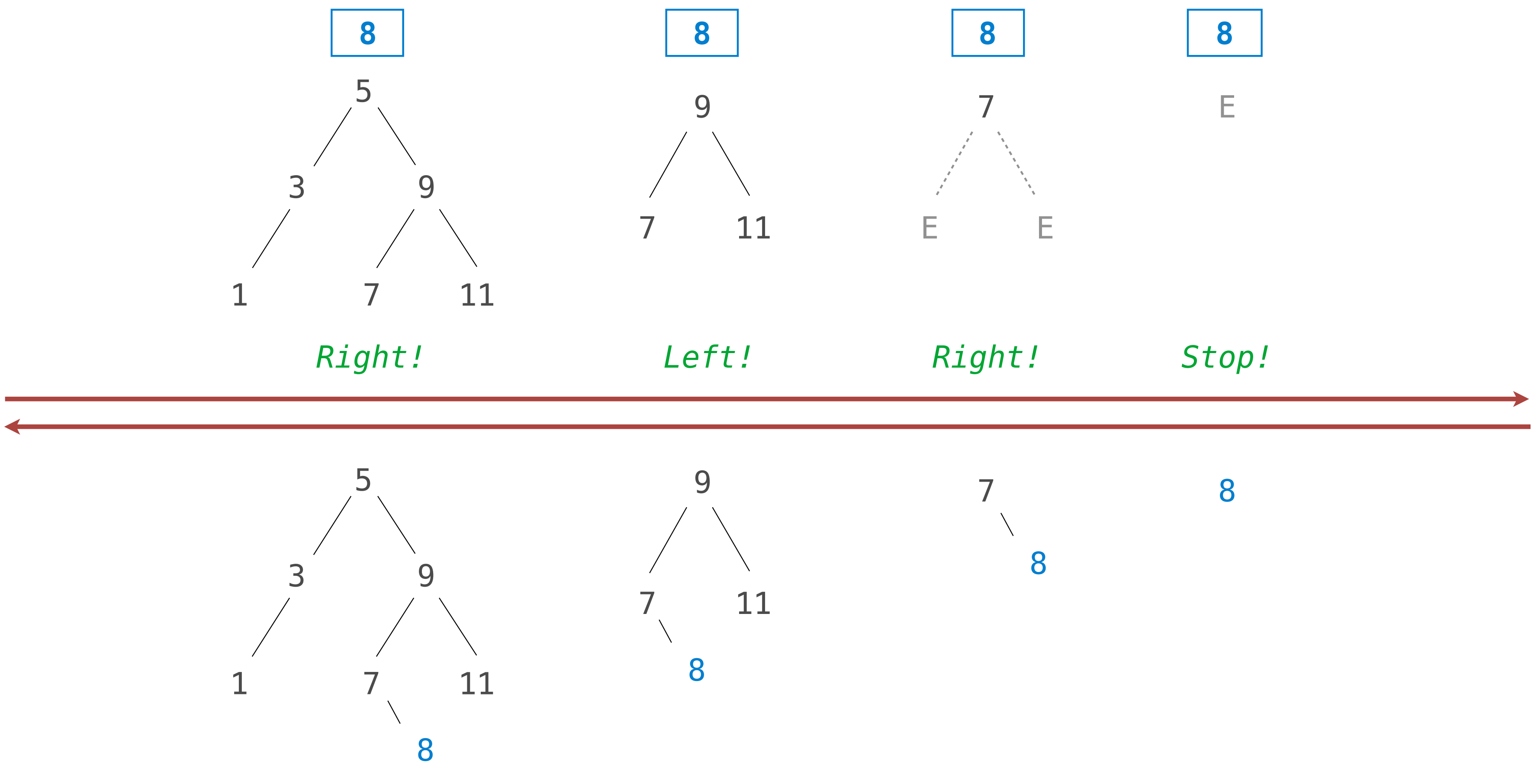

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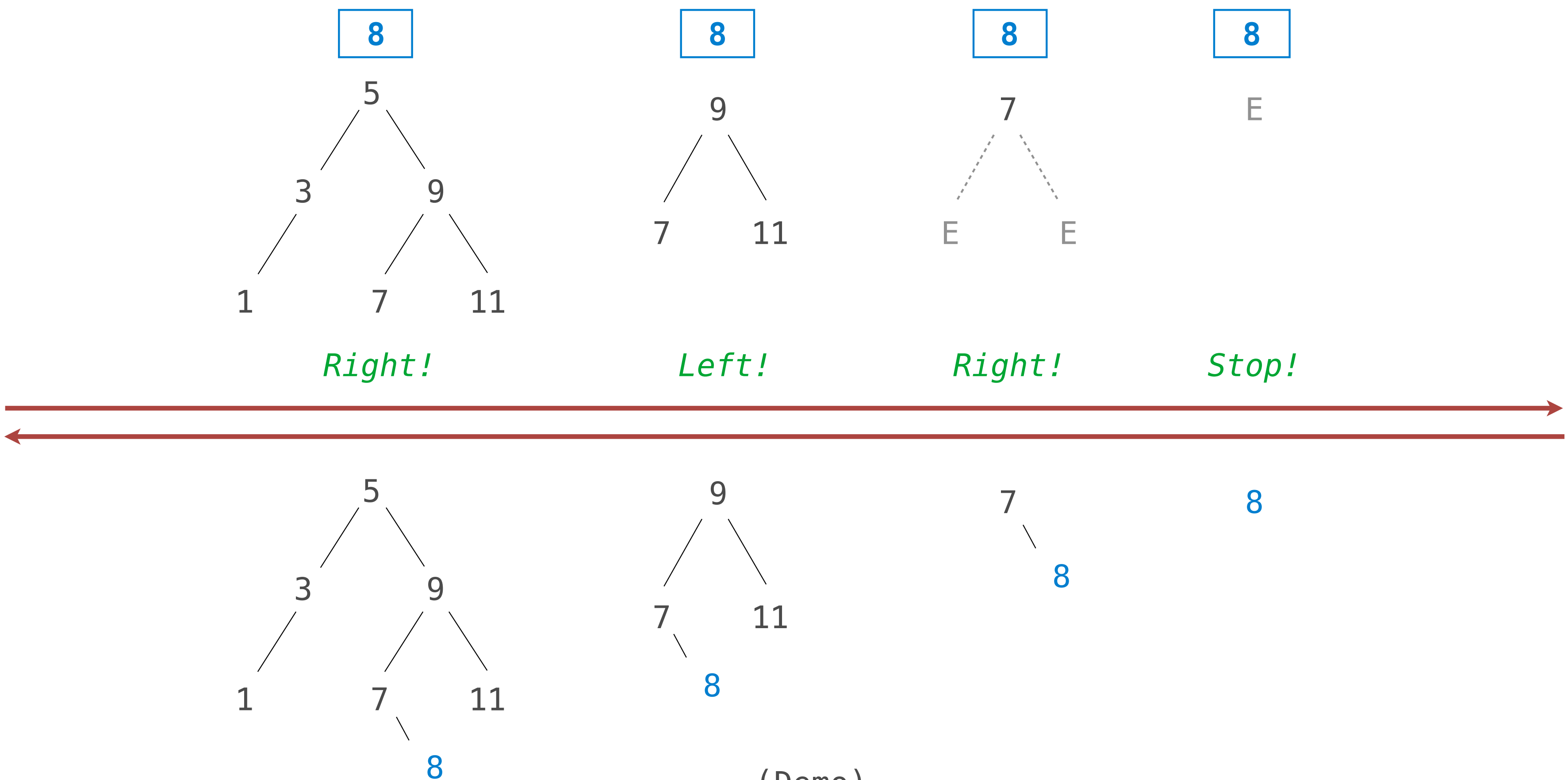

(Demo)