61A Lecture 18

Announcements

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

Recursive Computation of the Fibonacci Sequence

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Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

fib(5)

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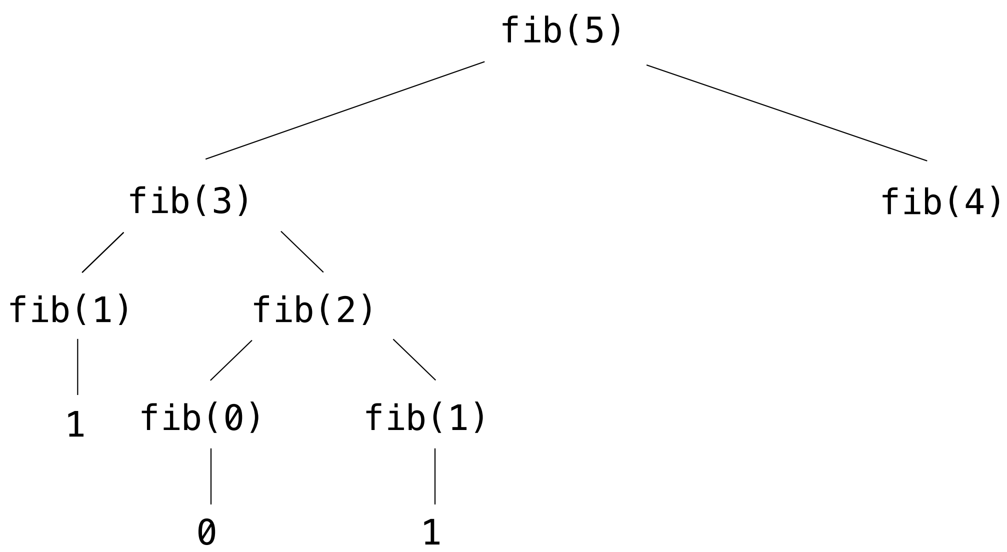


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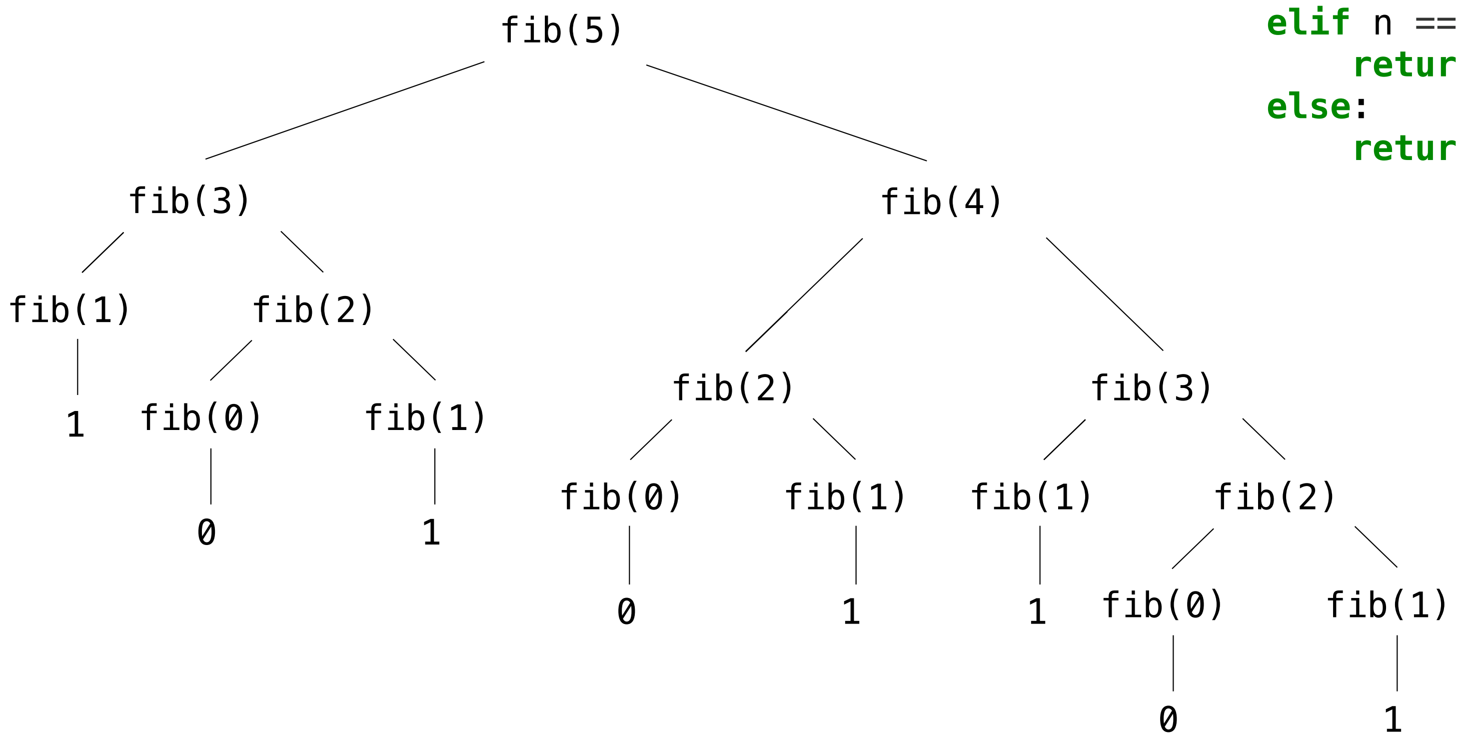

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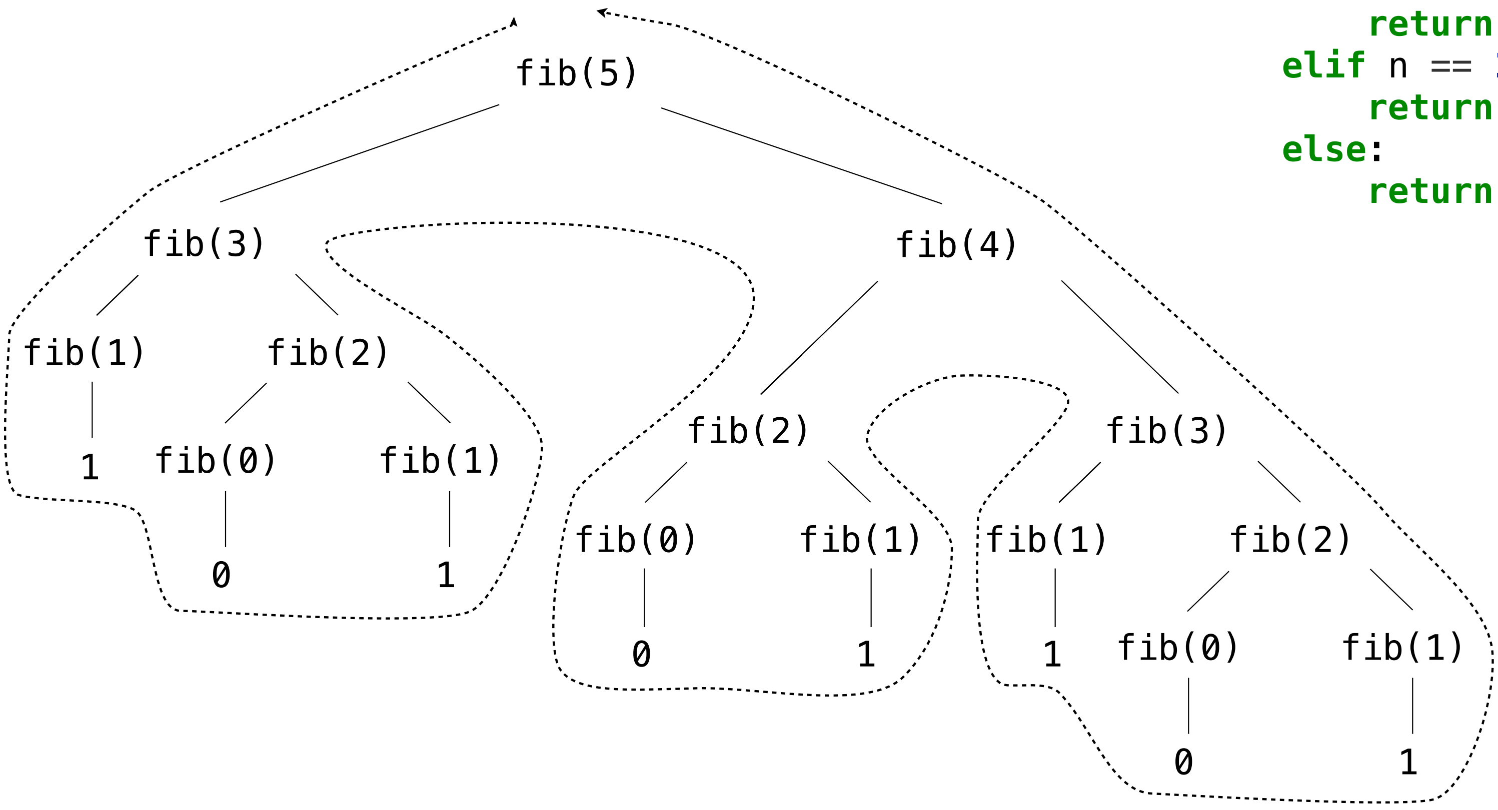

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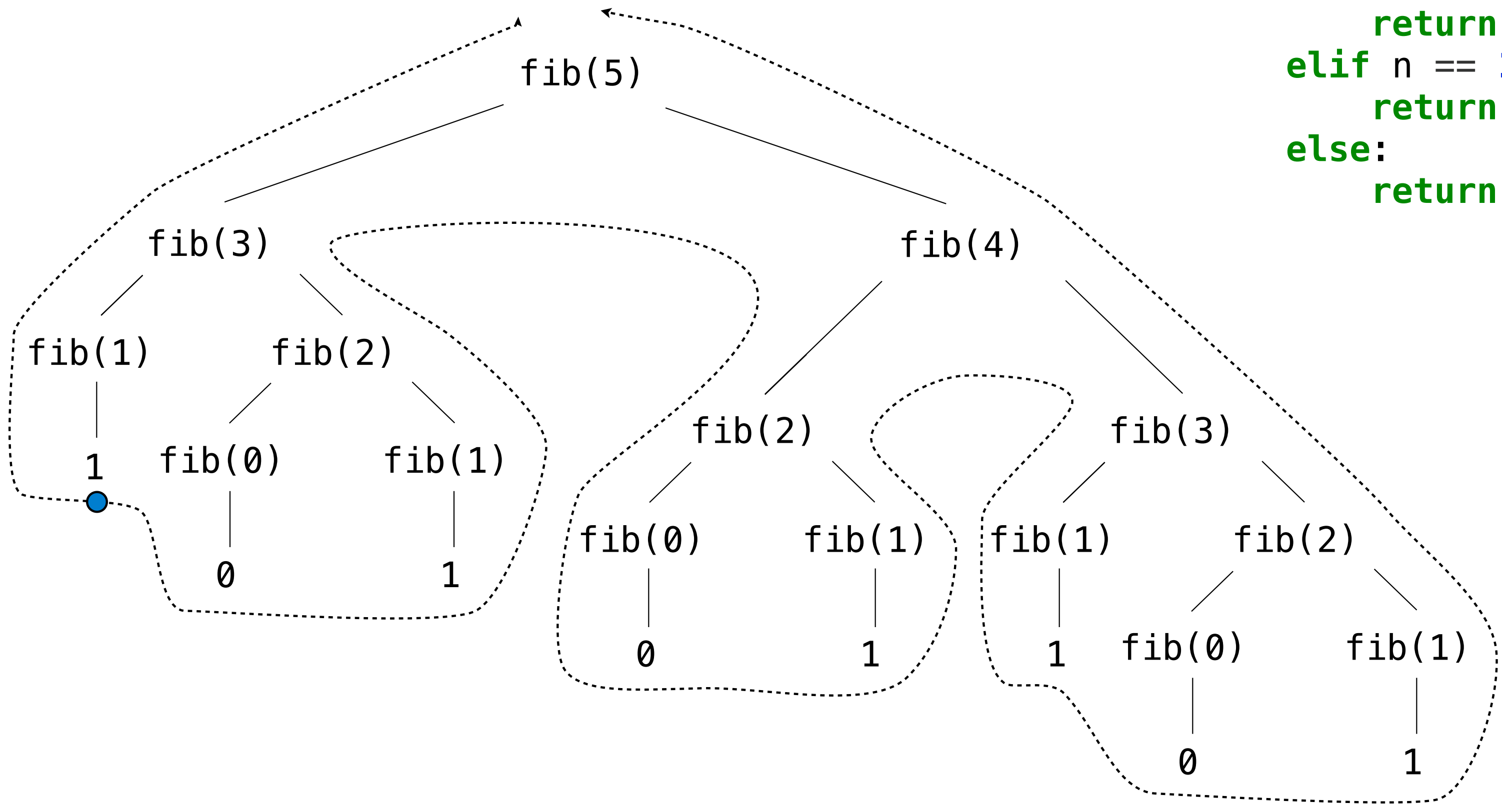

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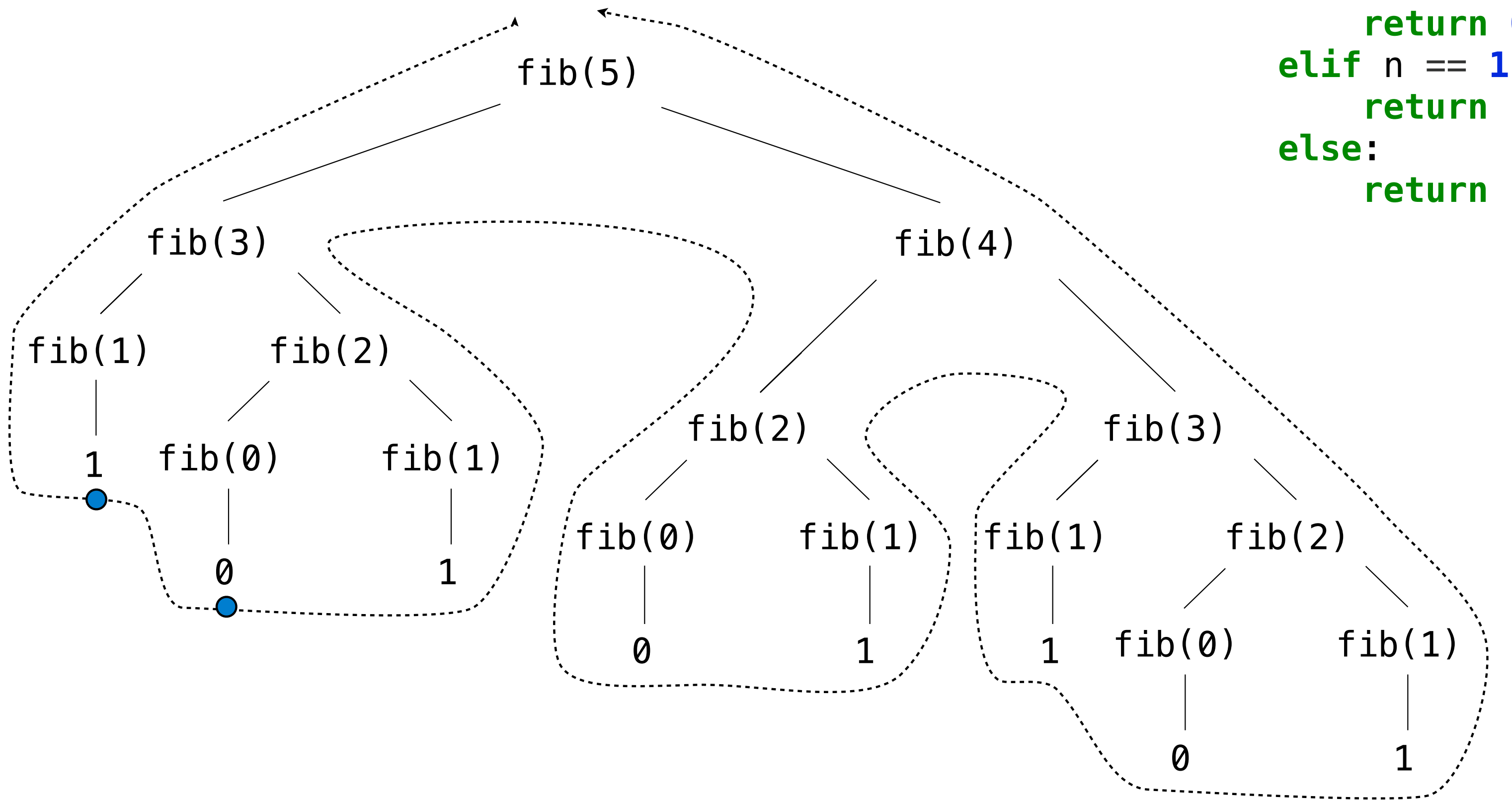

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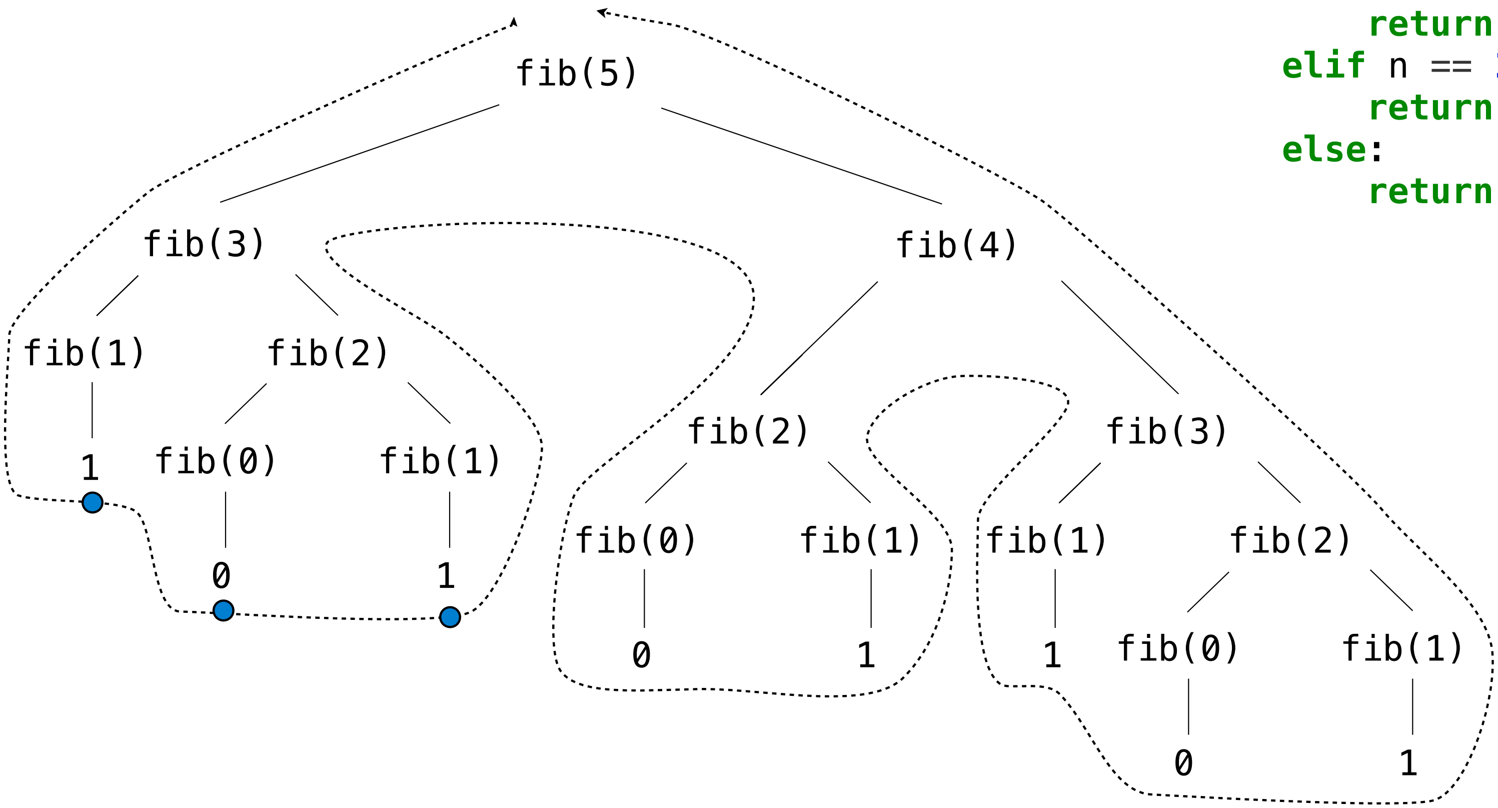

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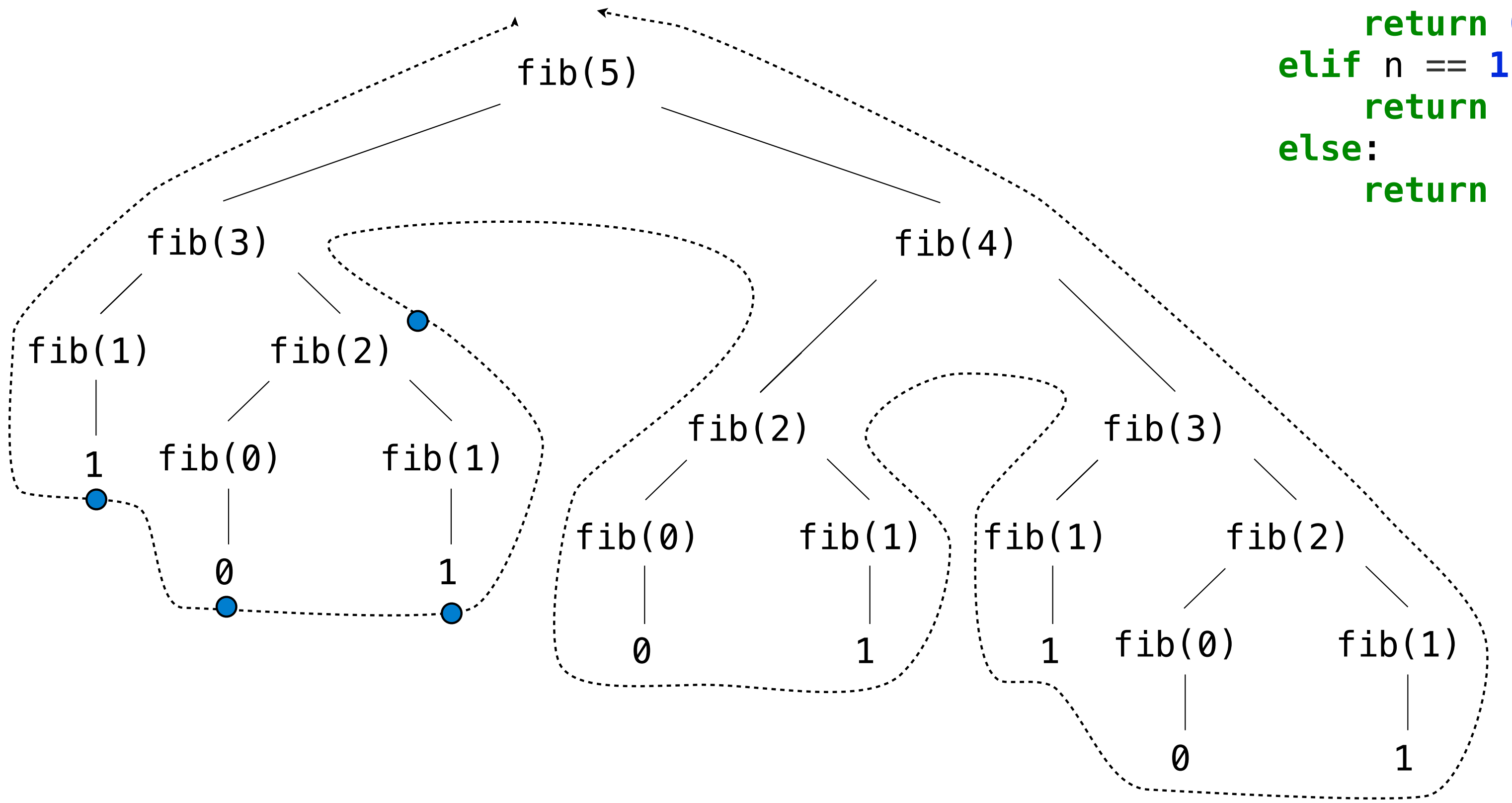

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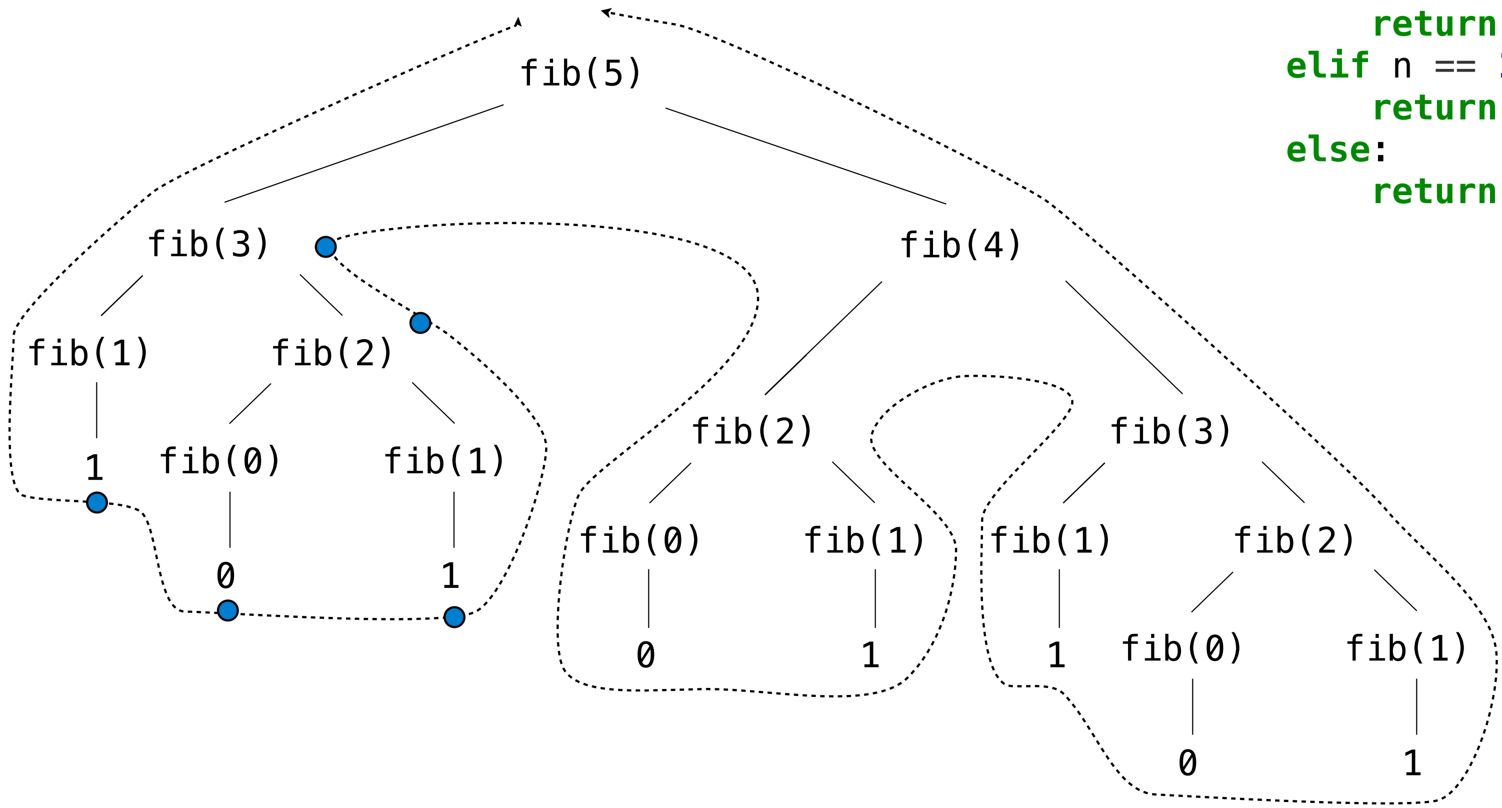


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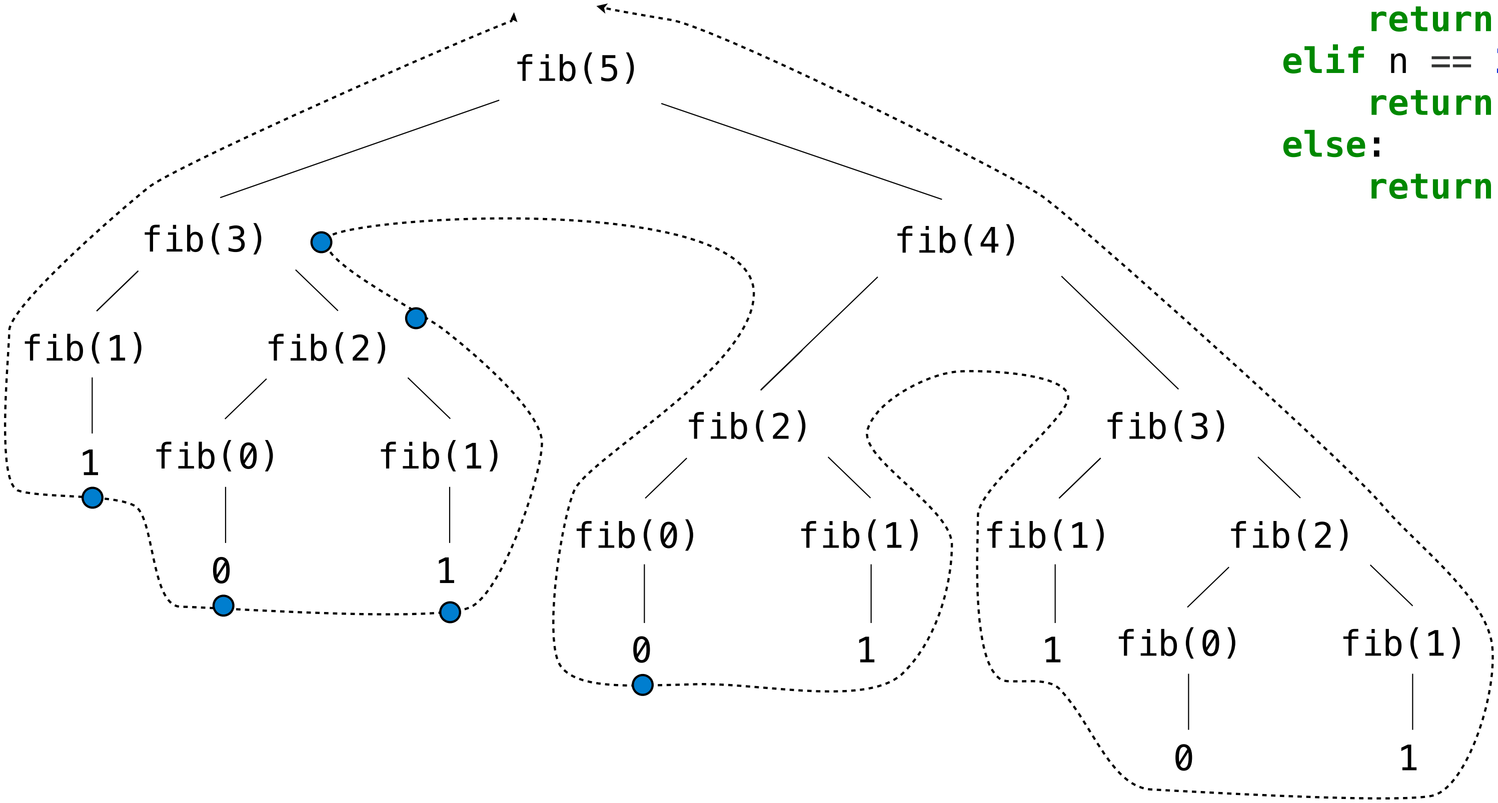

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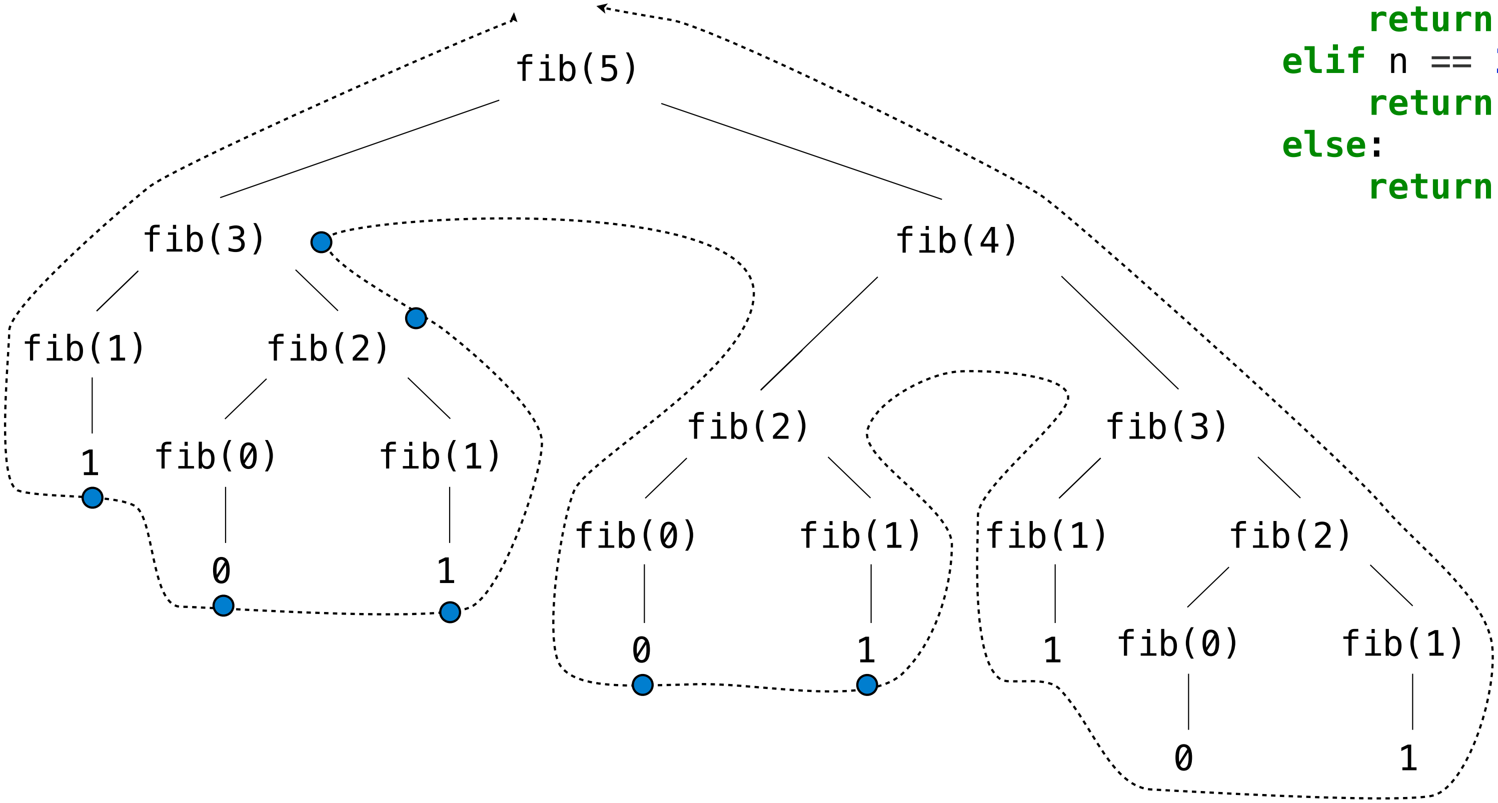


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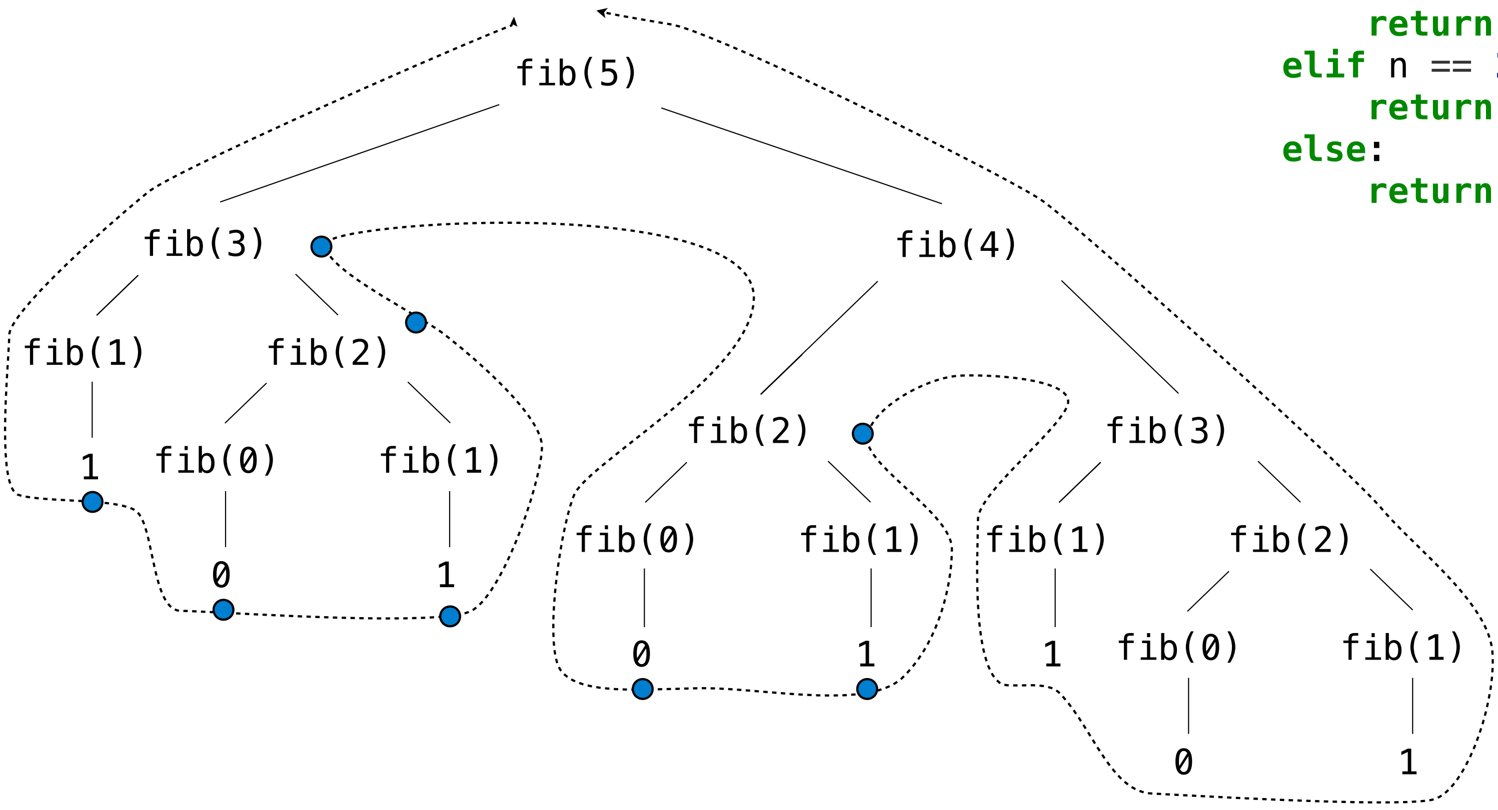

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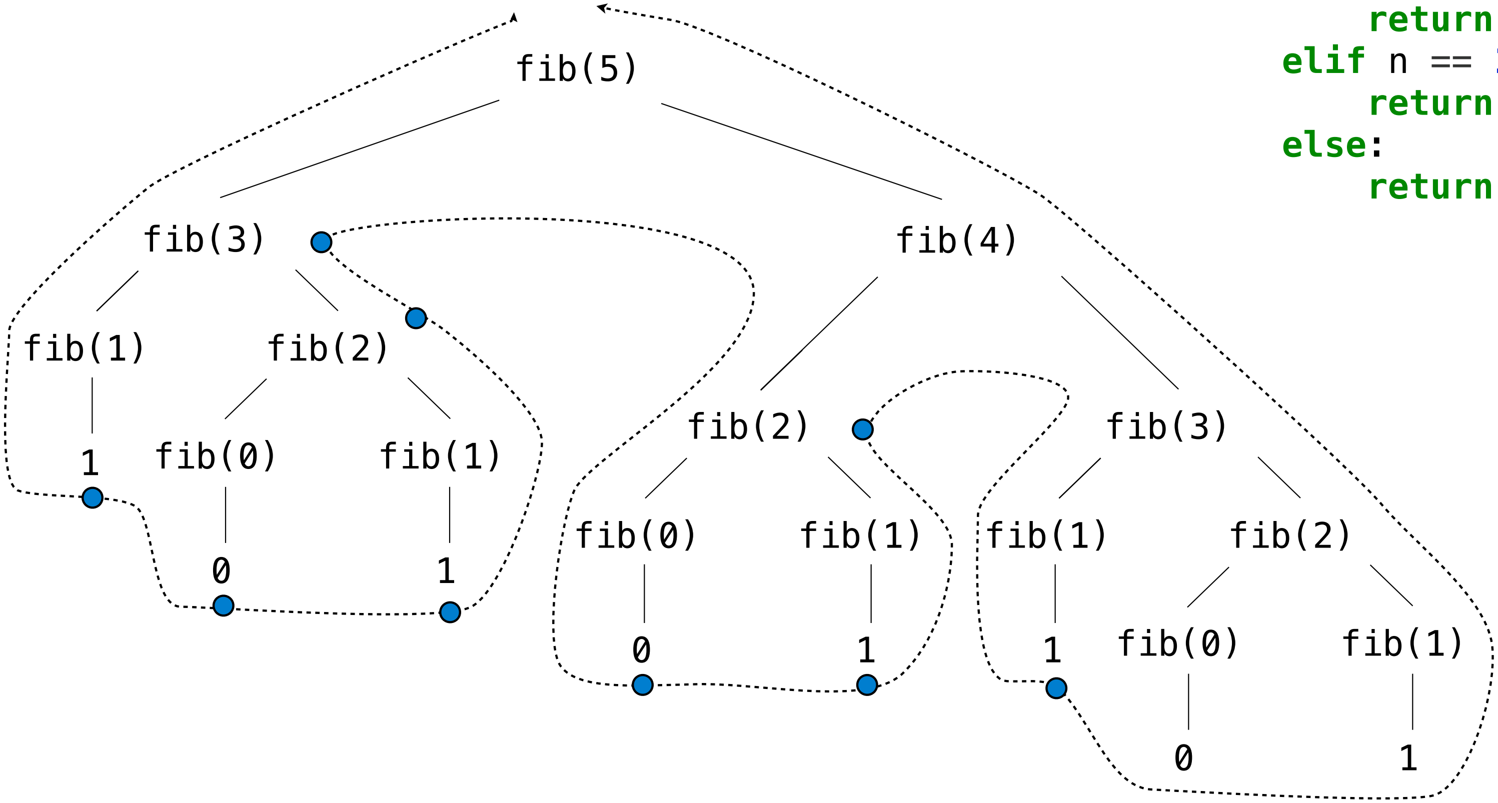

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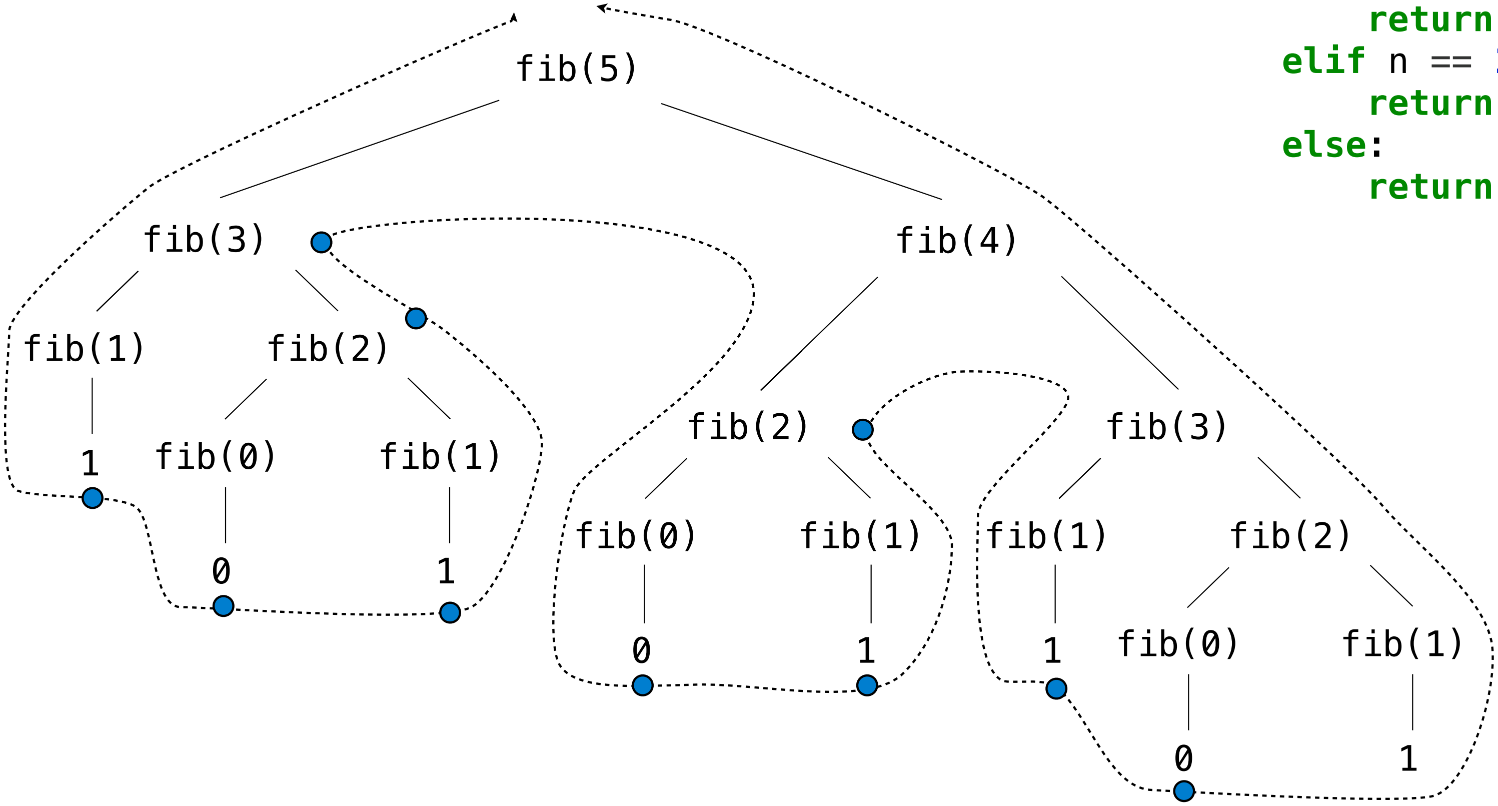

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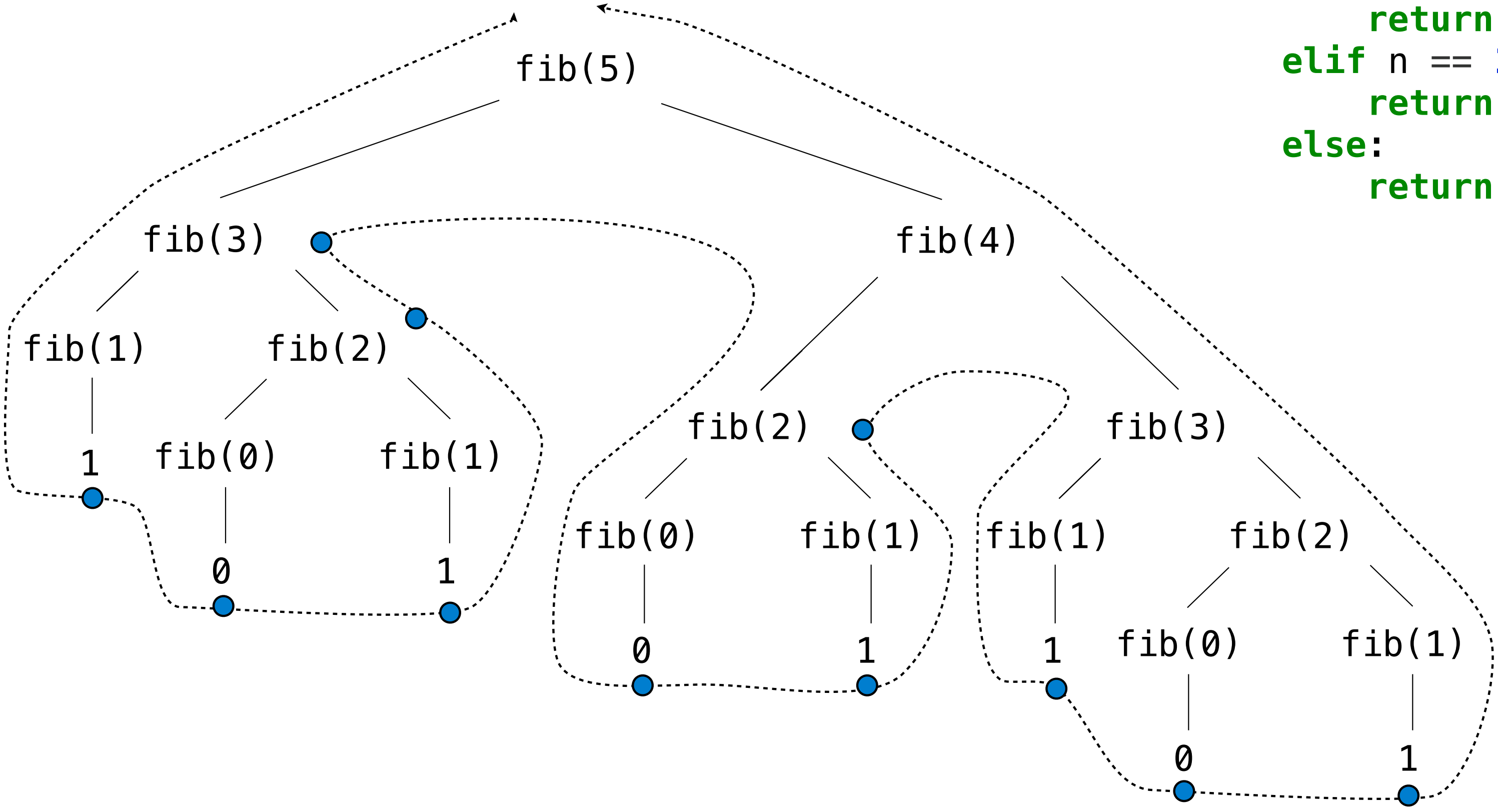

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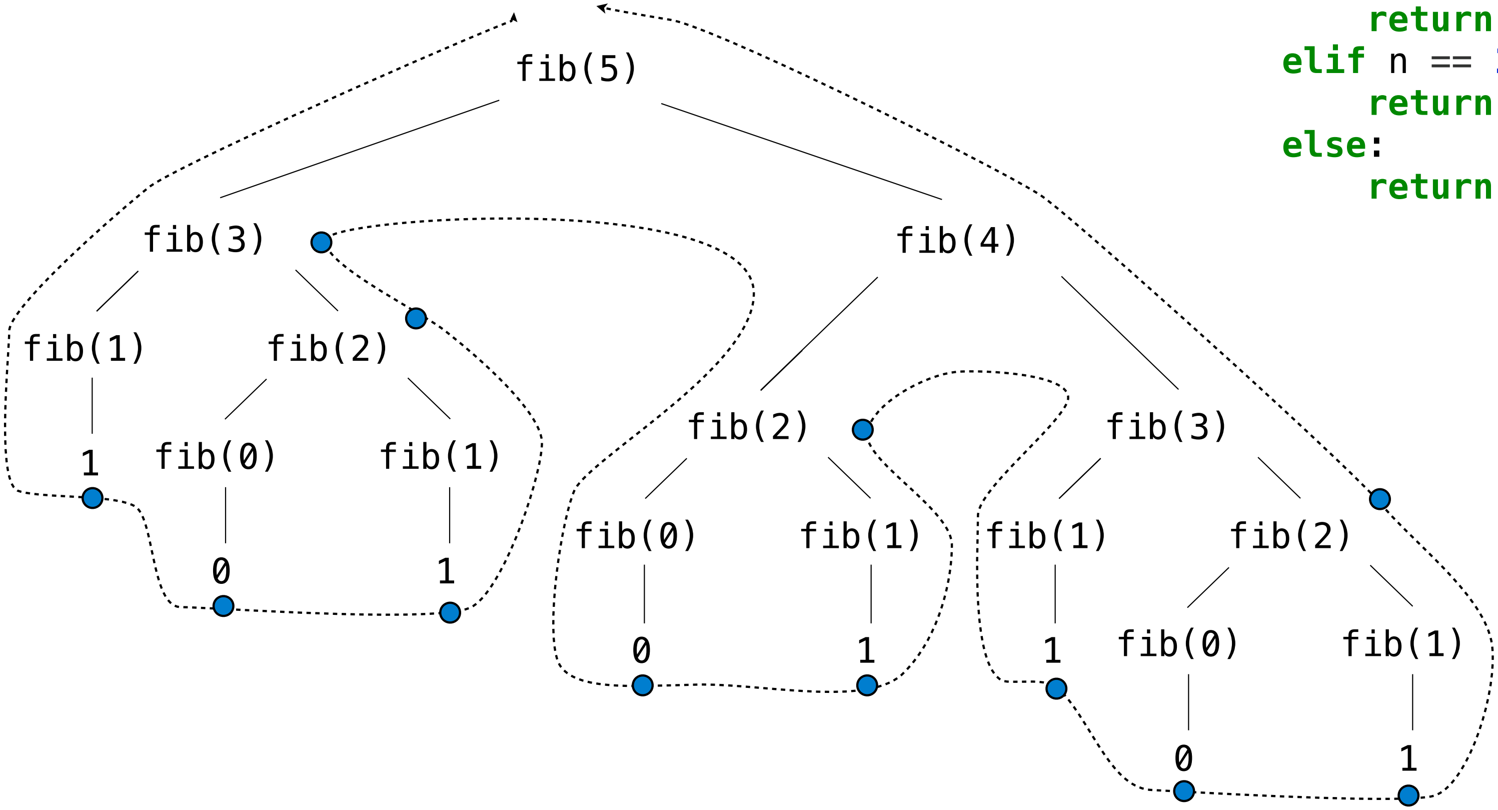

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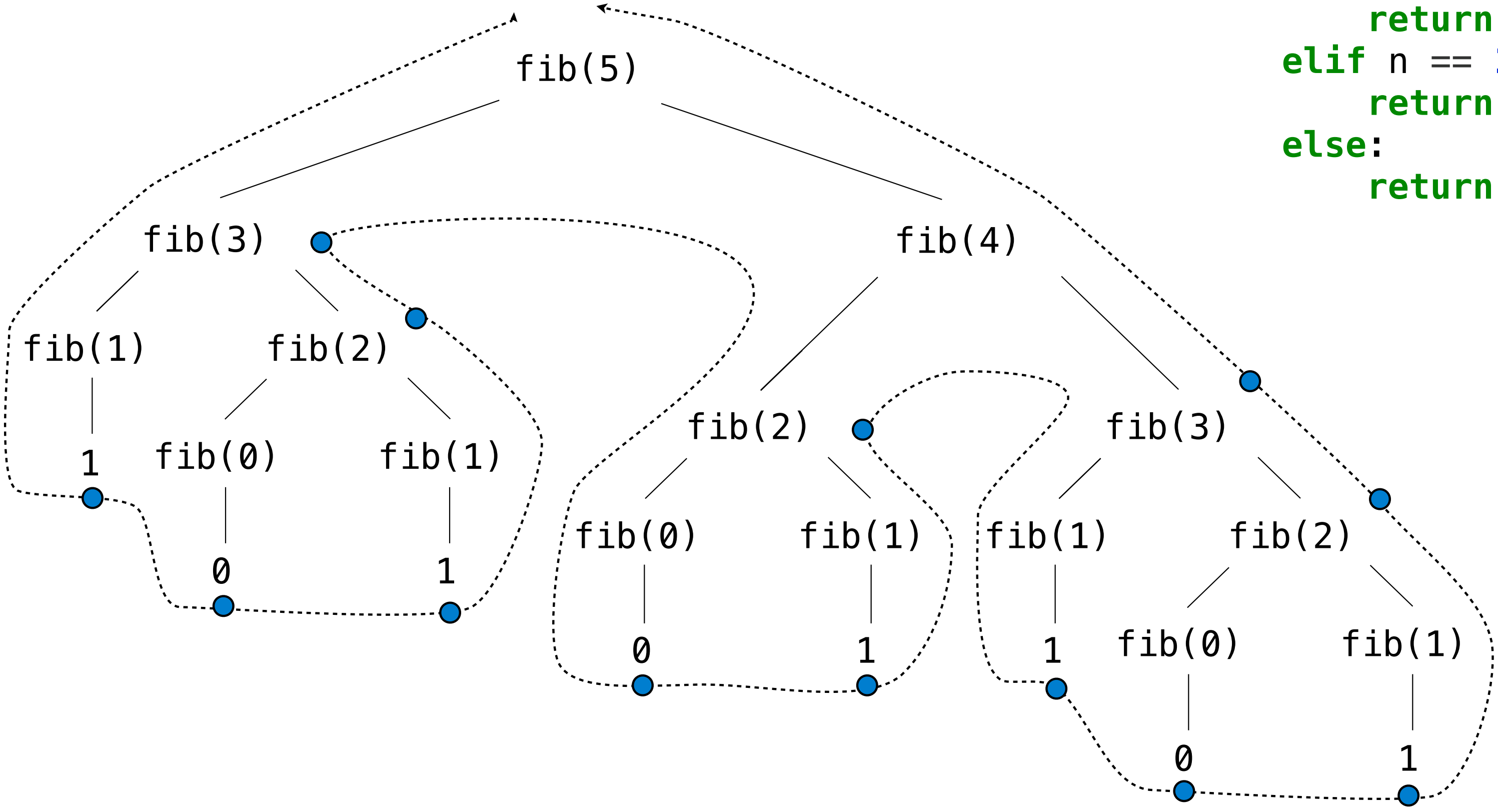

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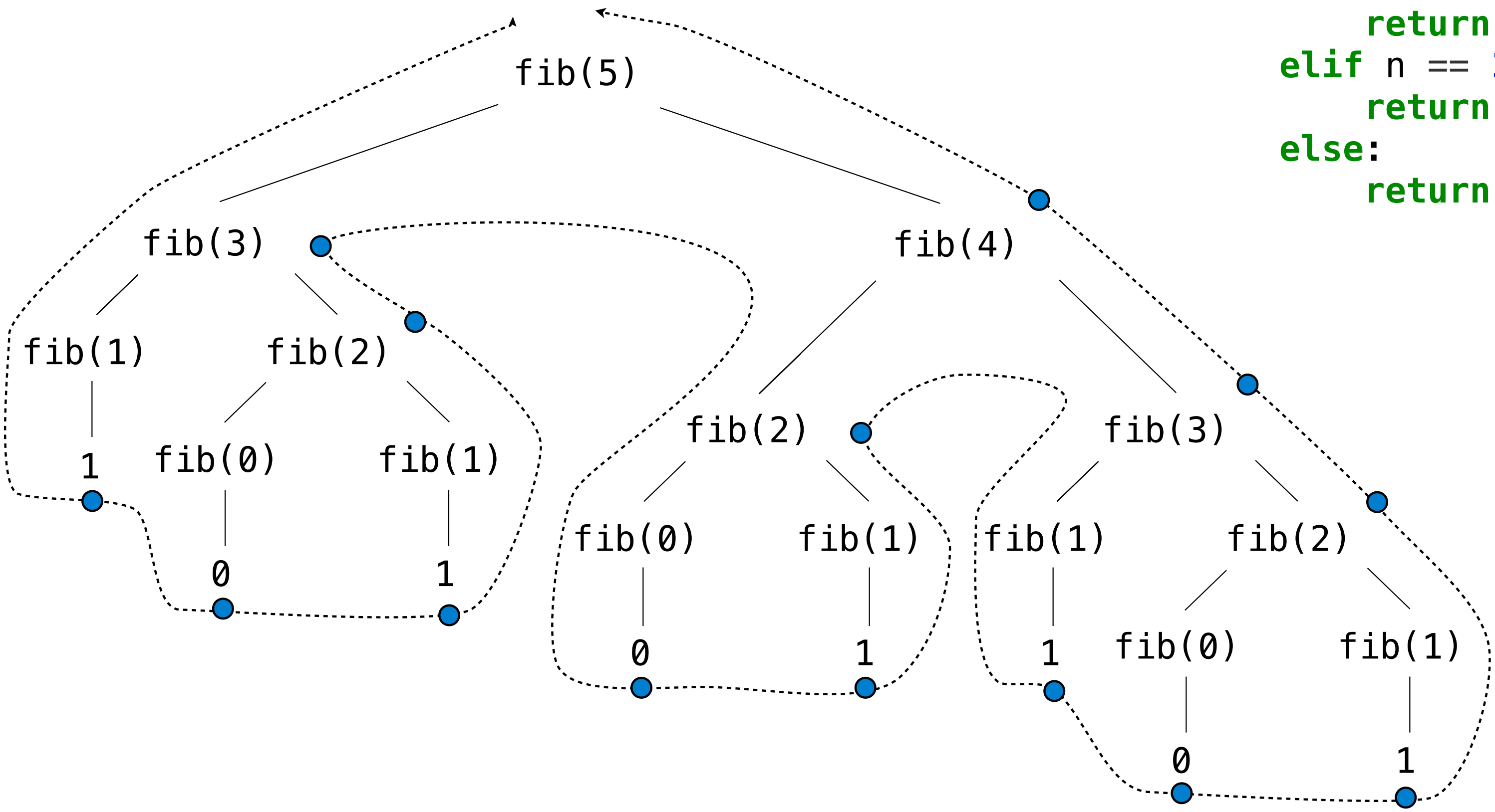

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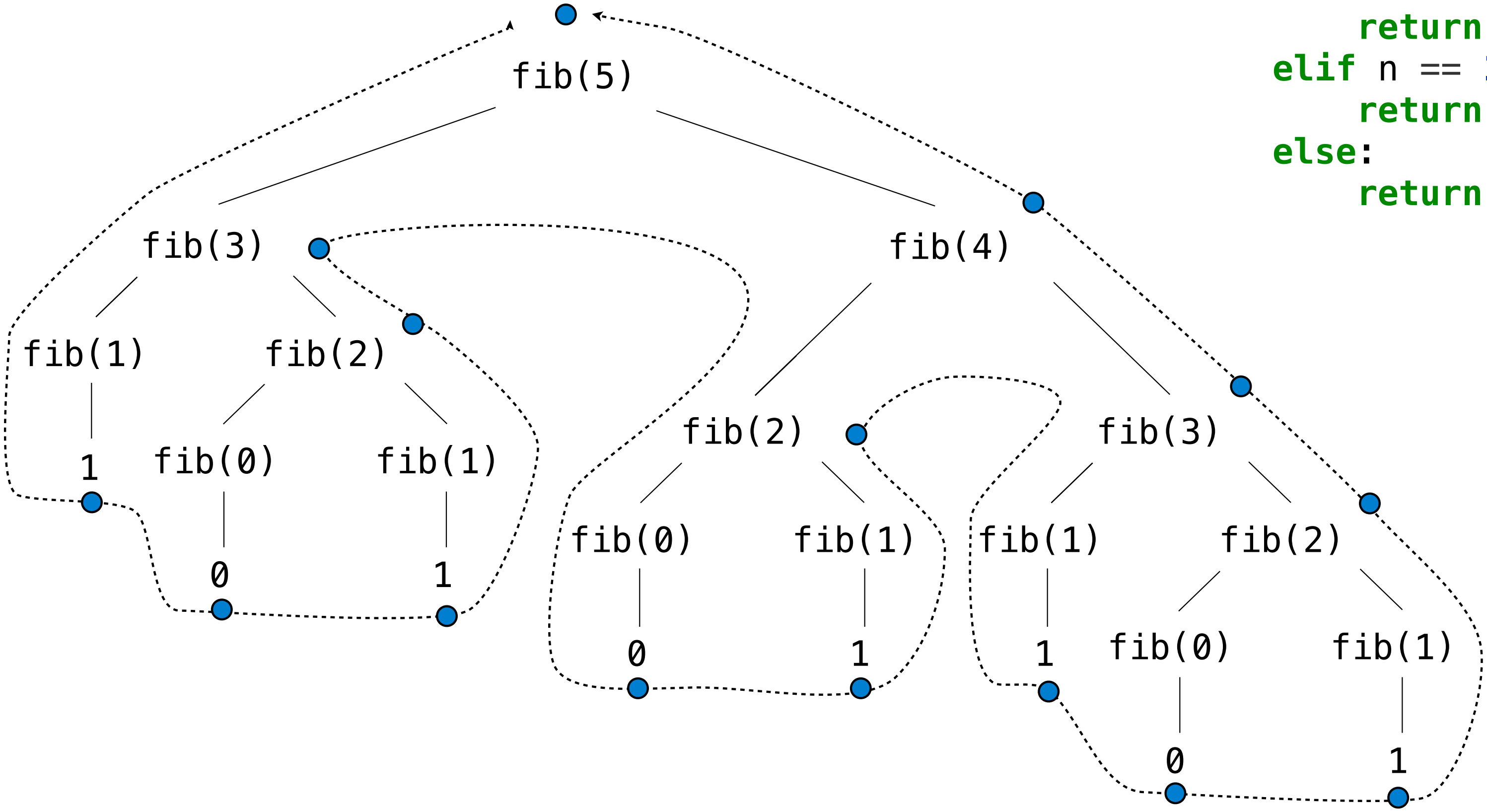

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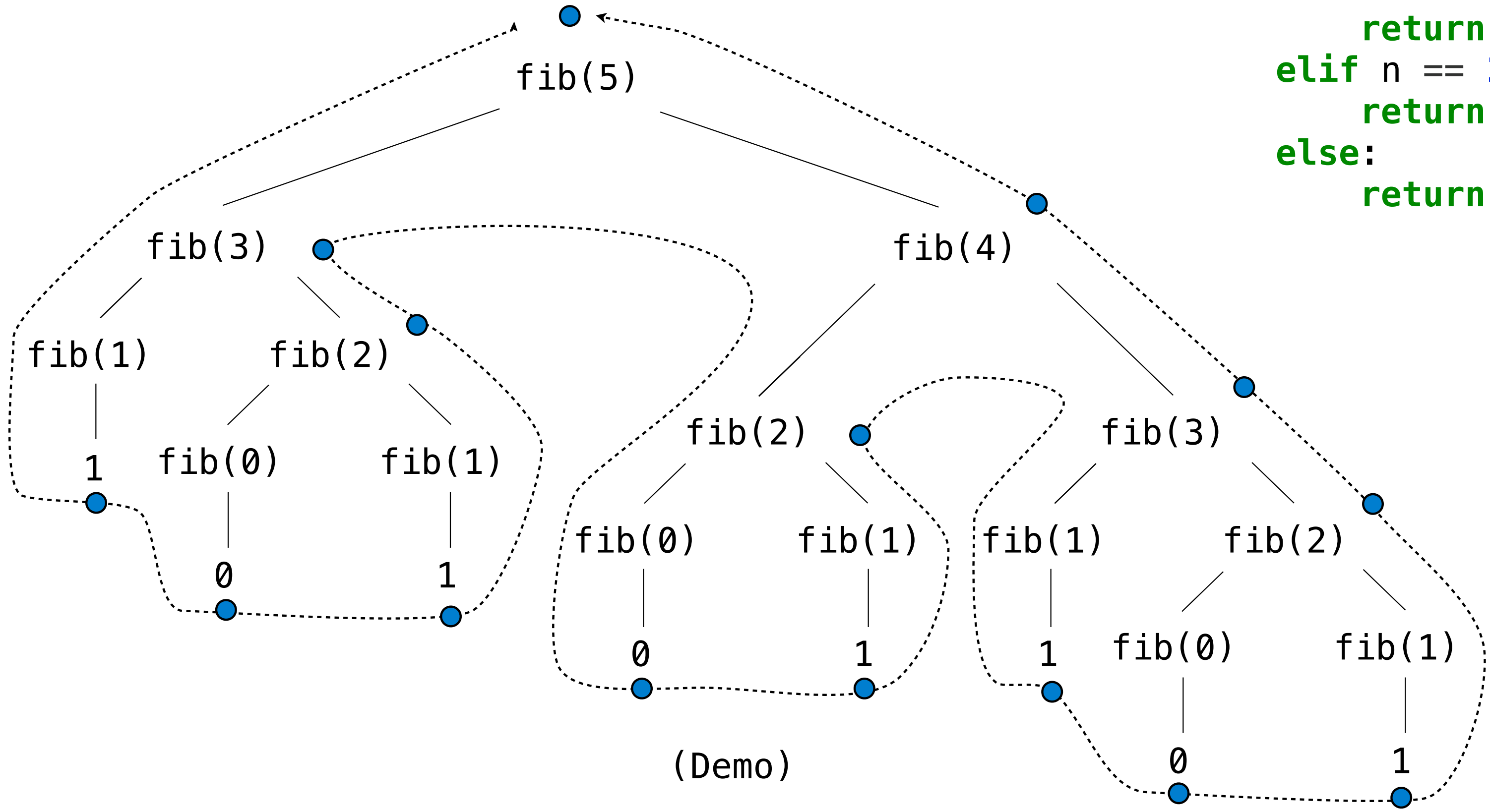

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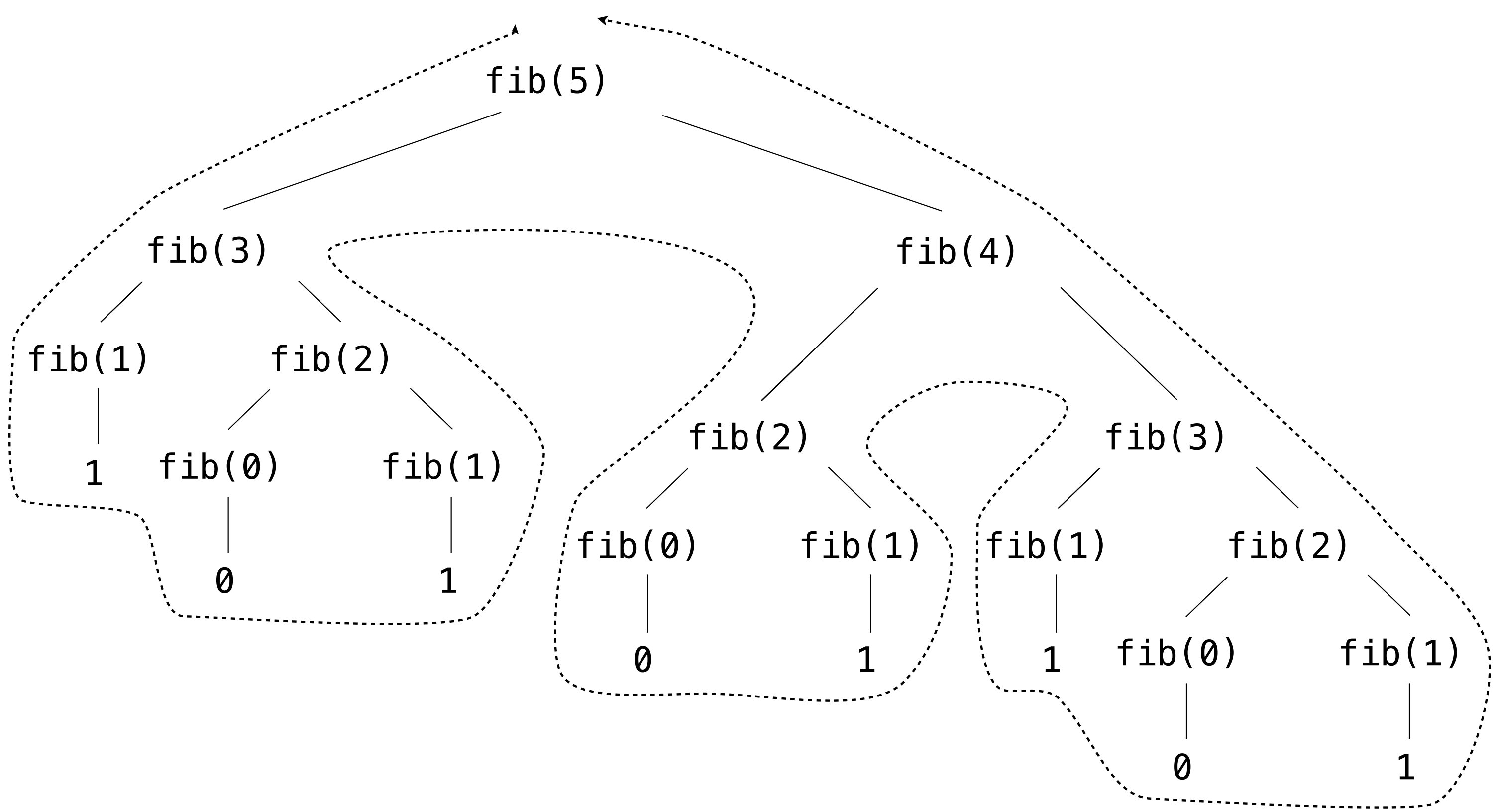

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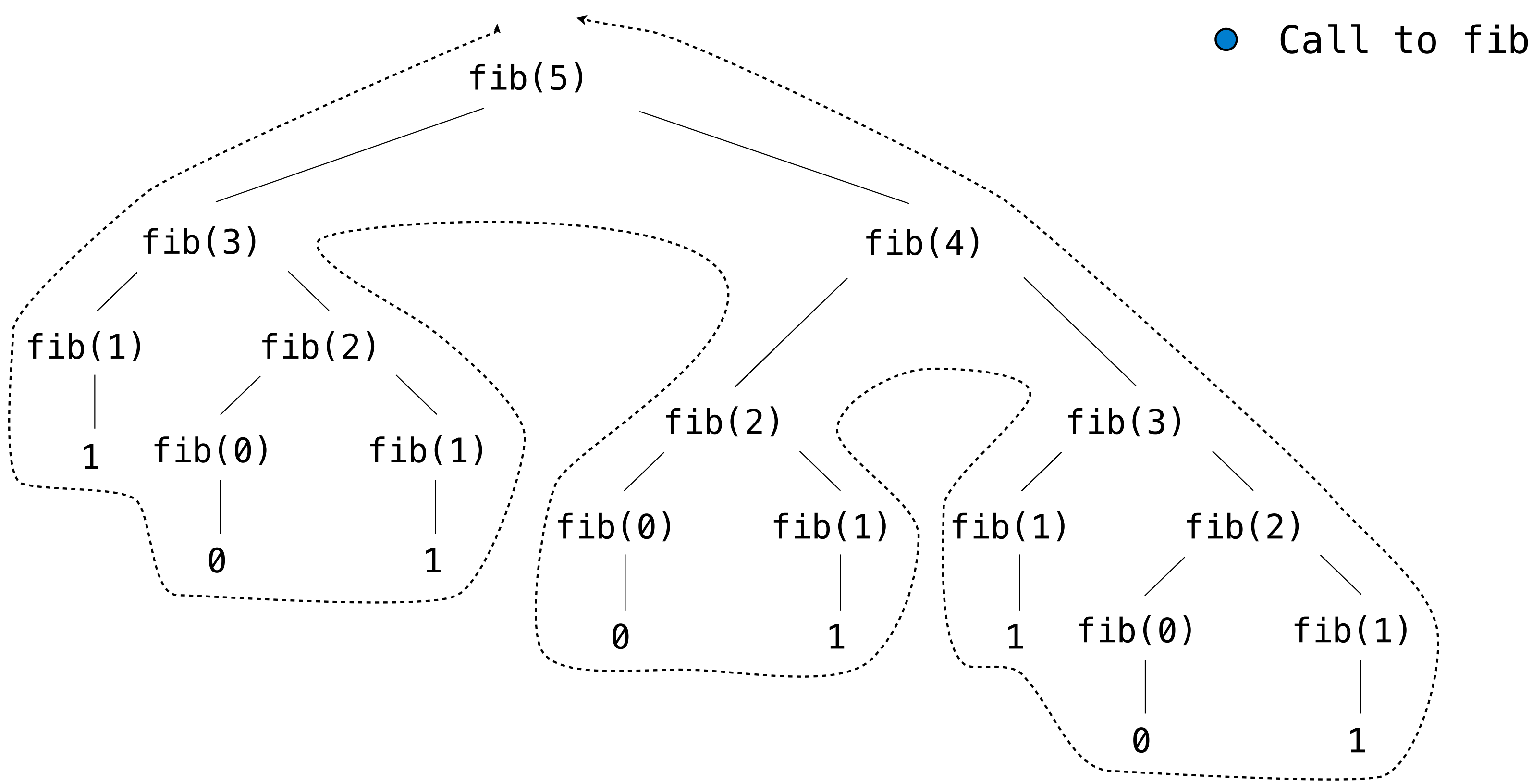

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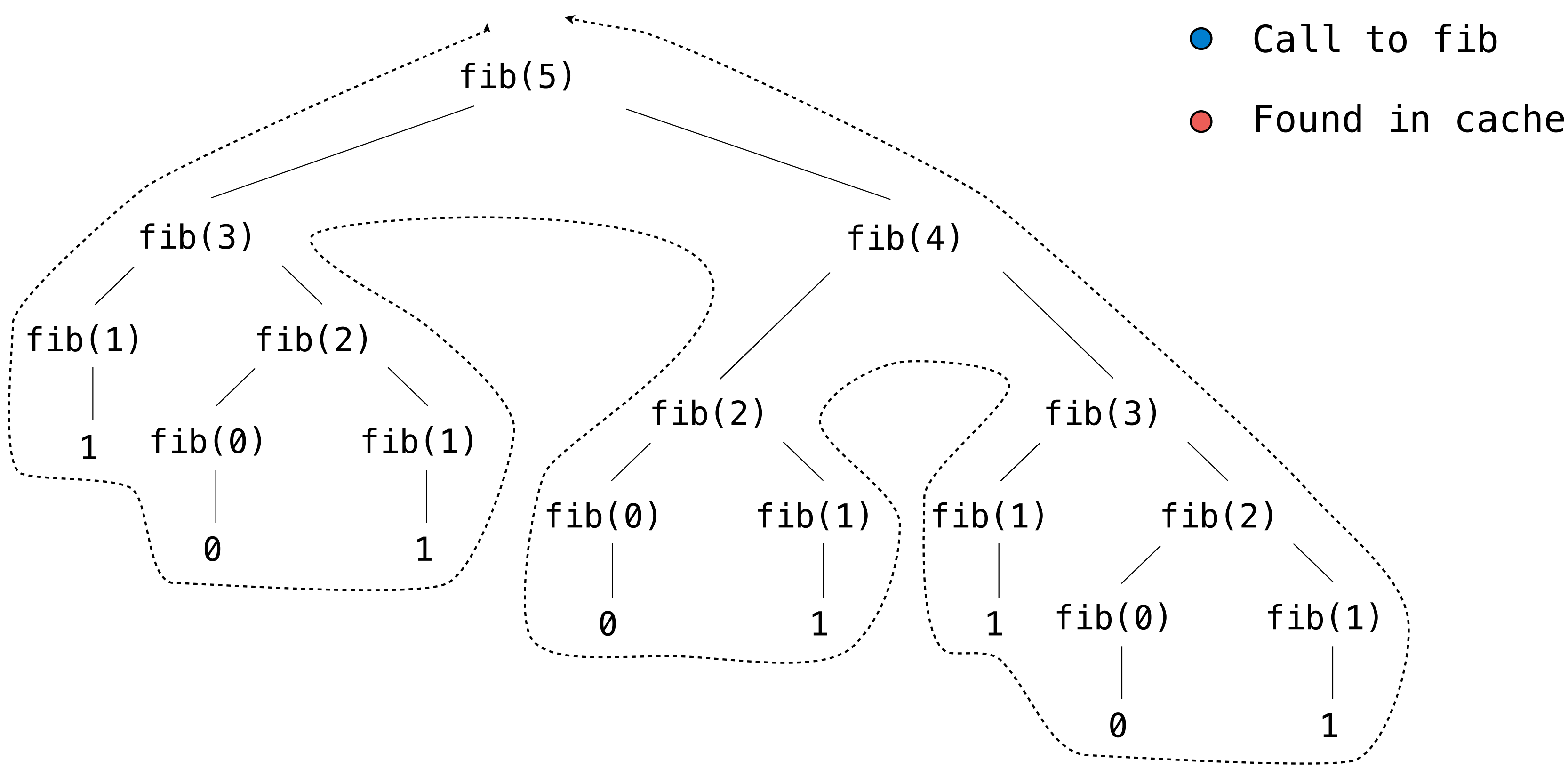

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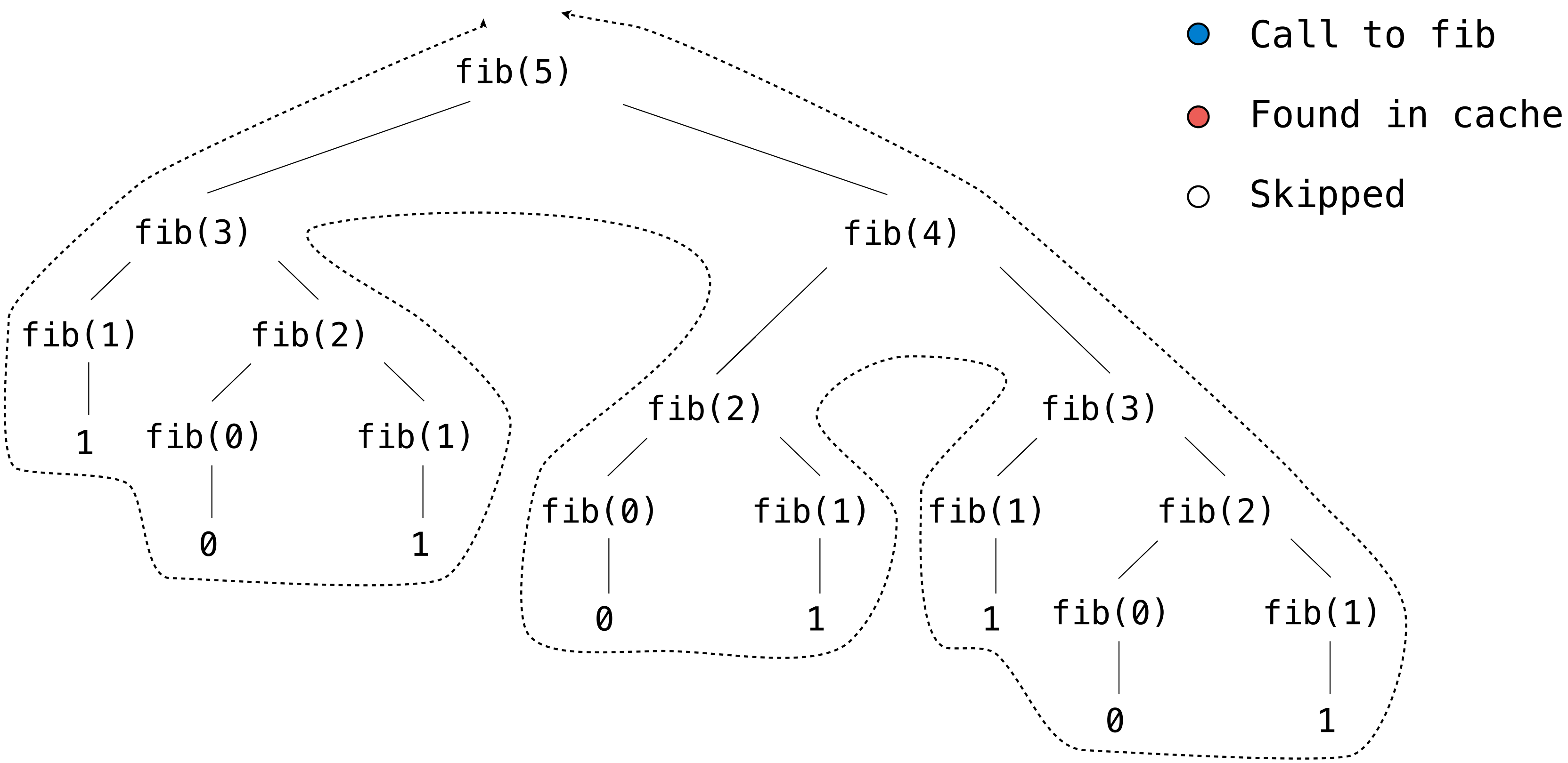

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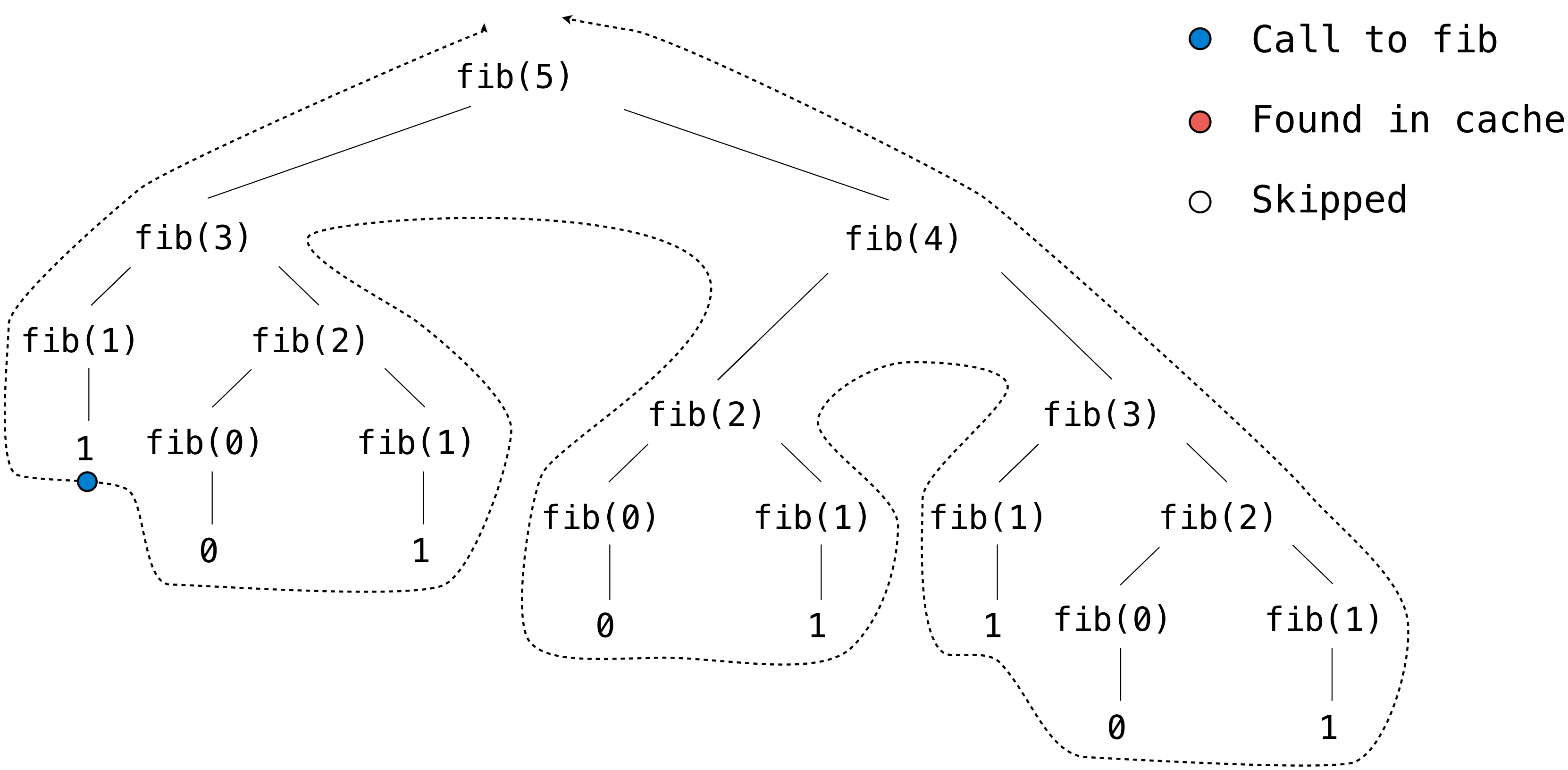

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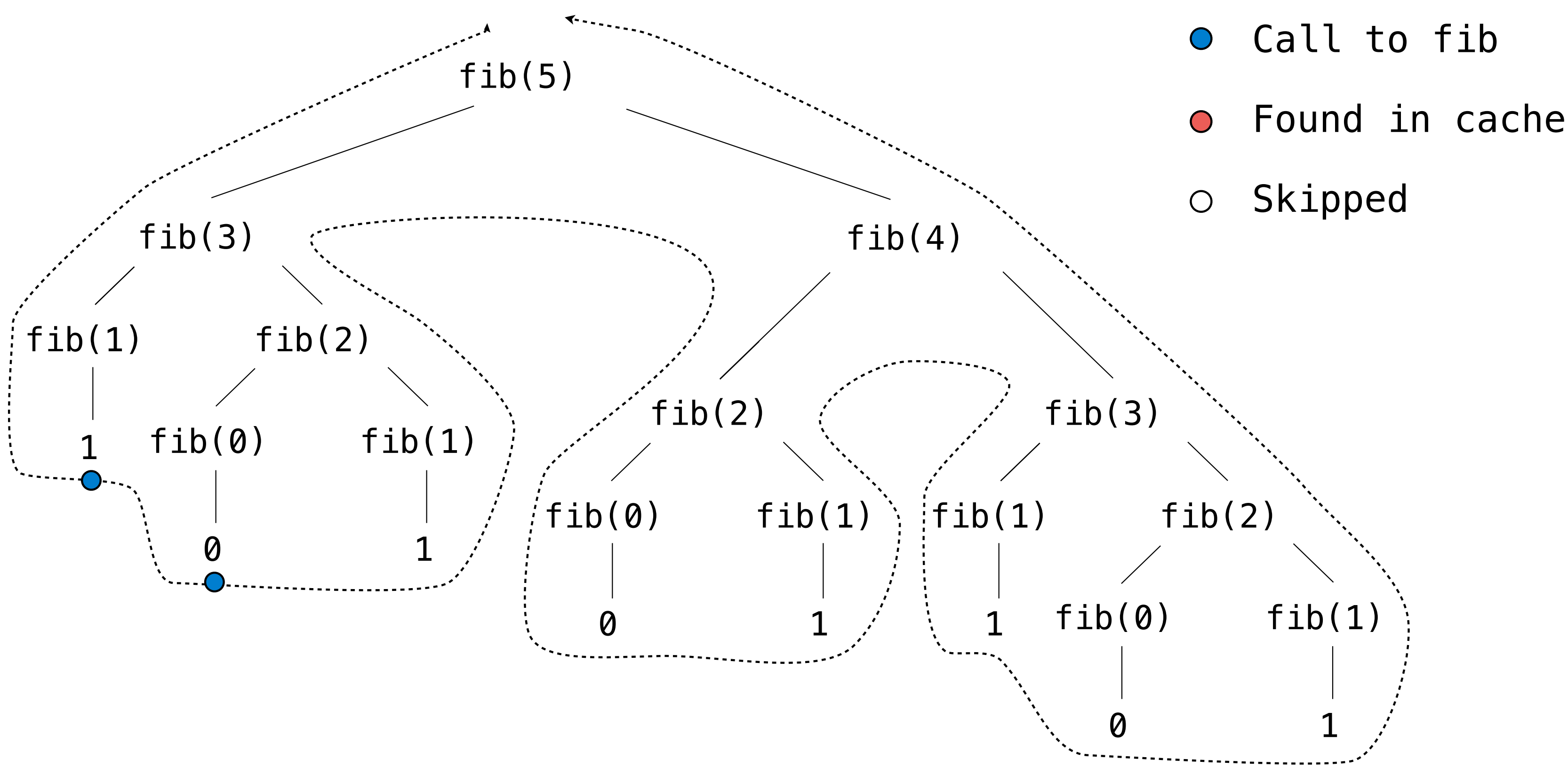

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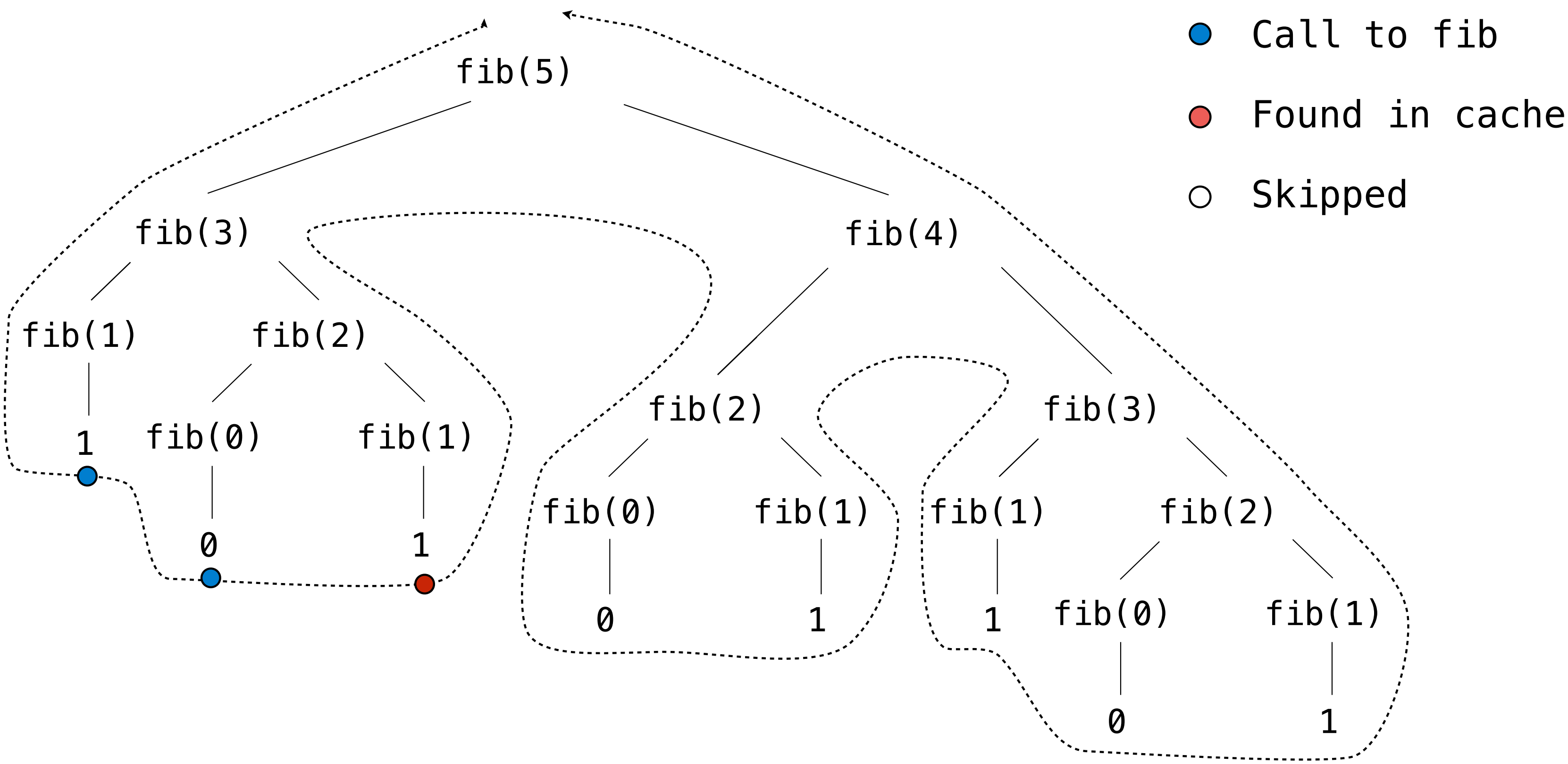

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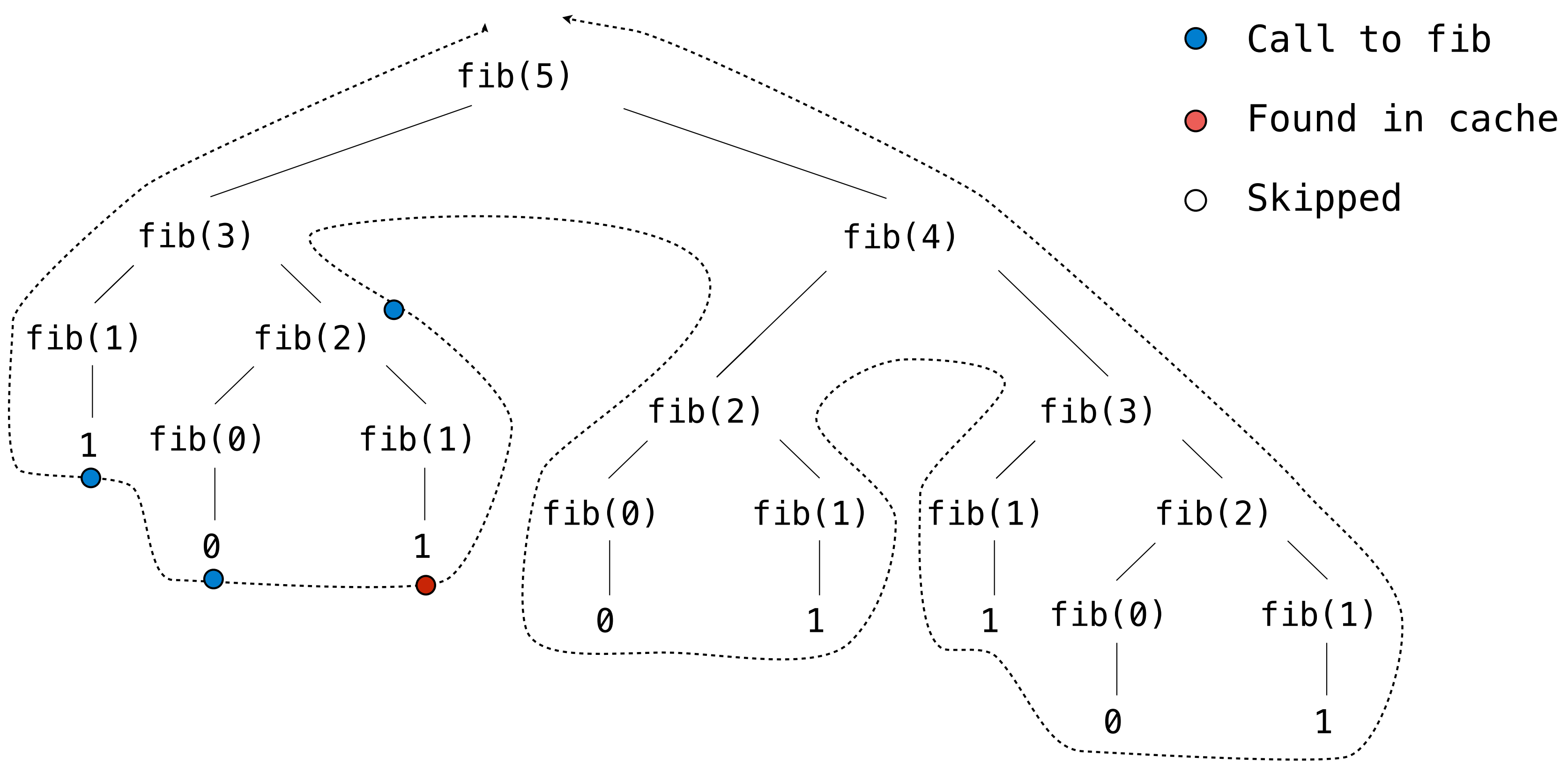

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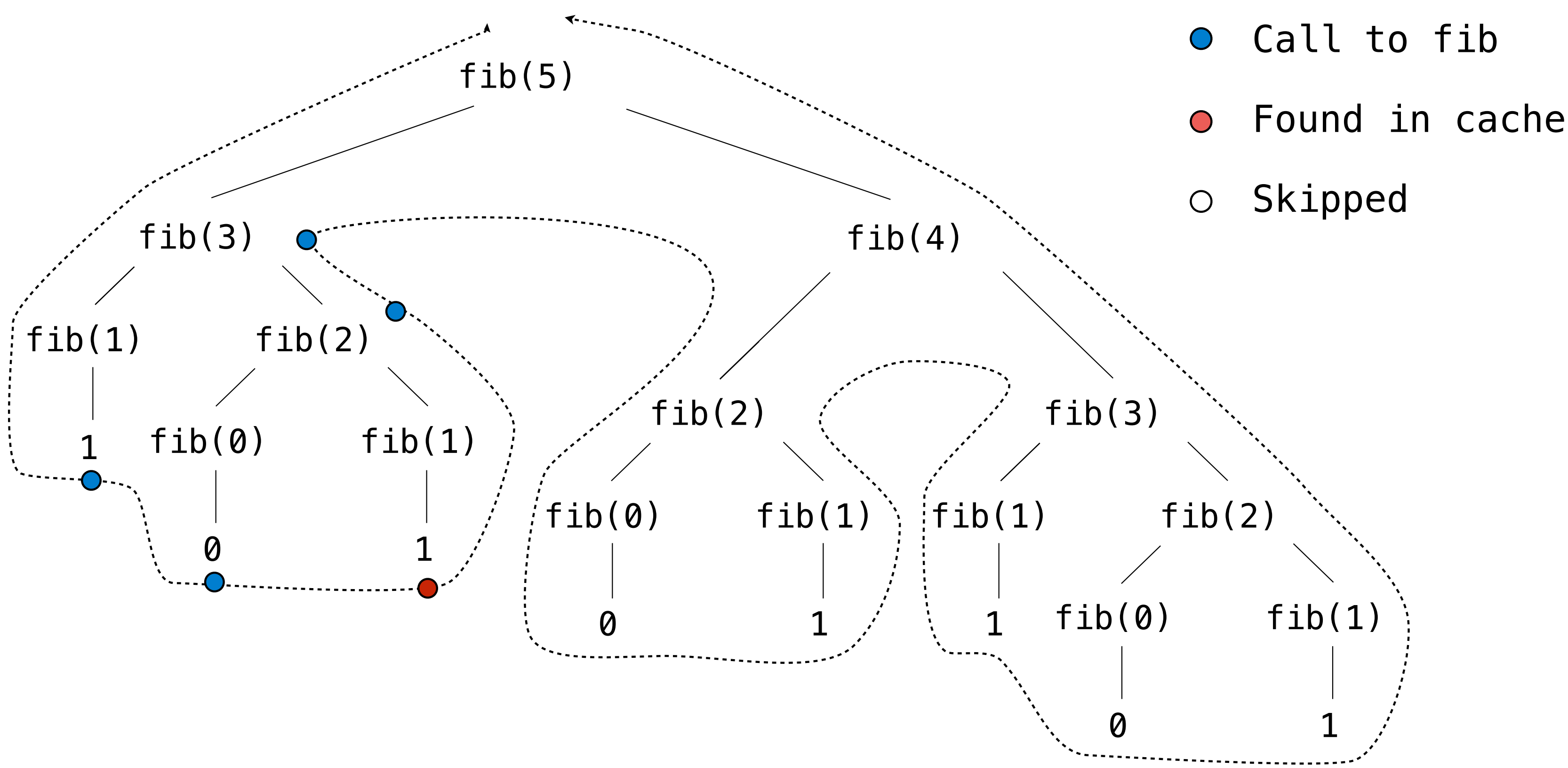

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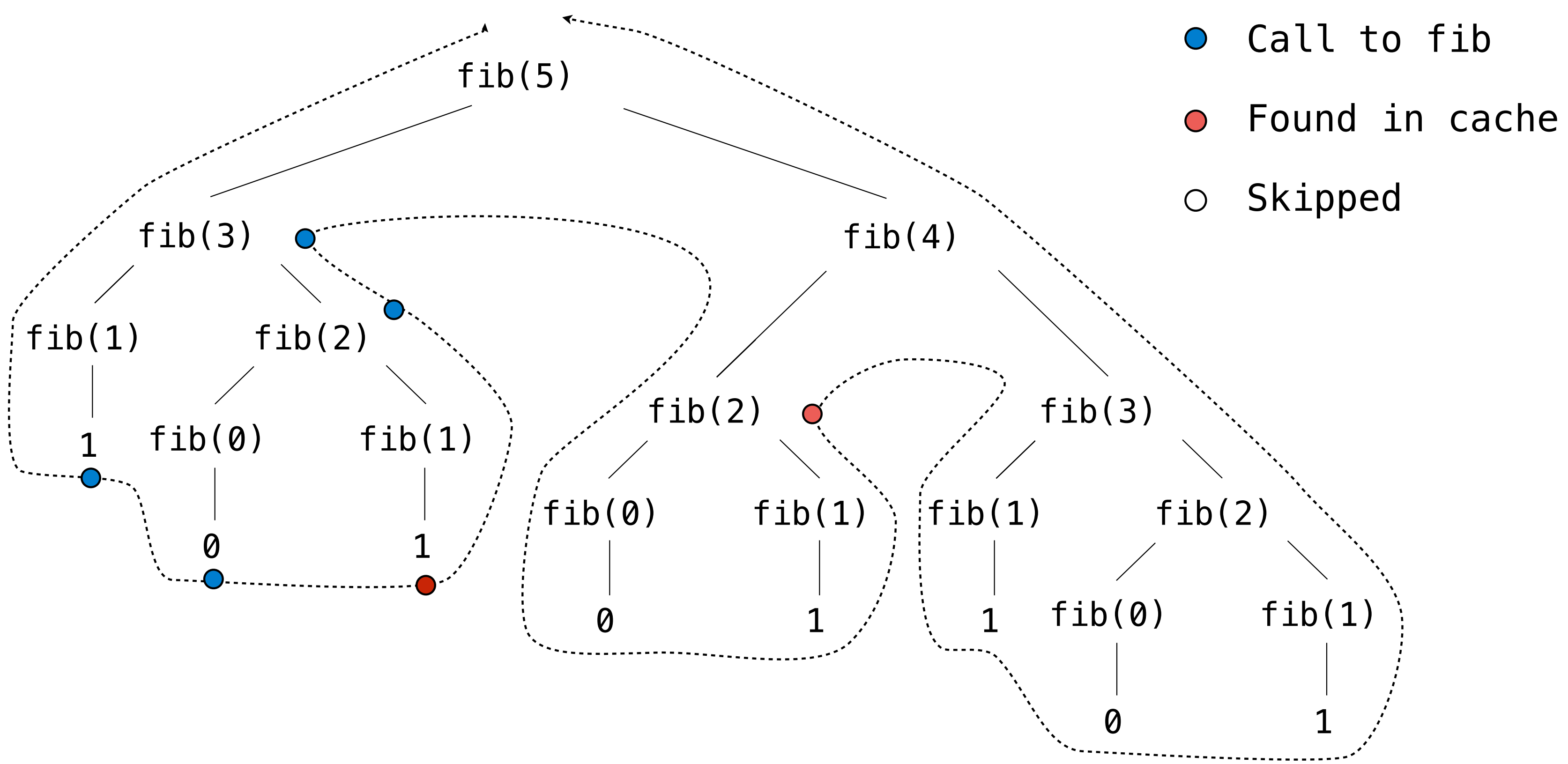

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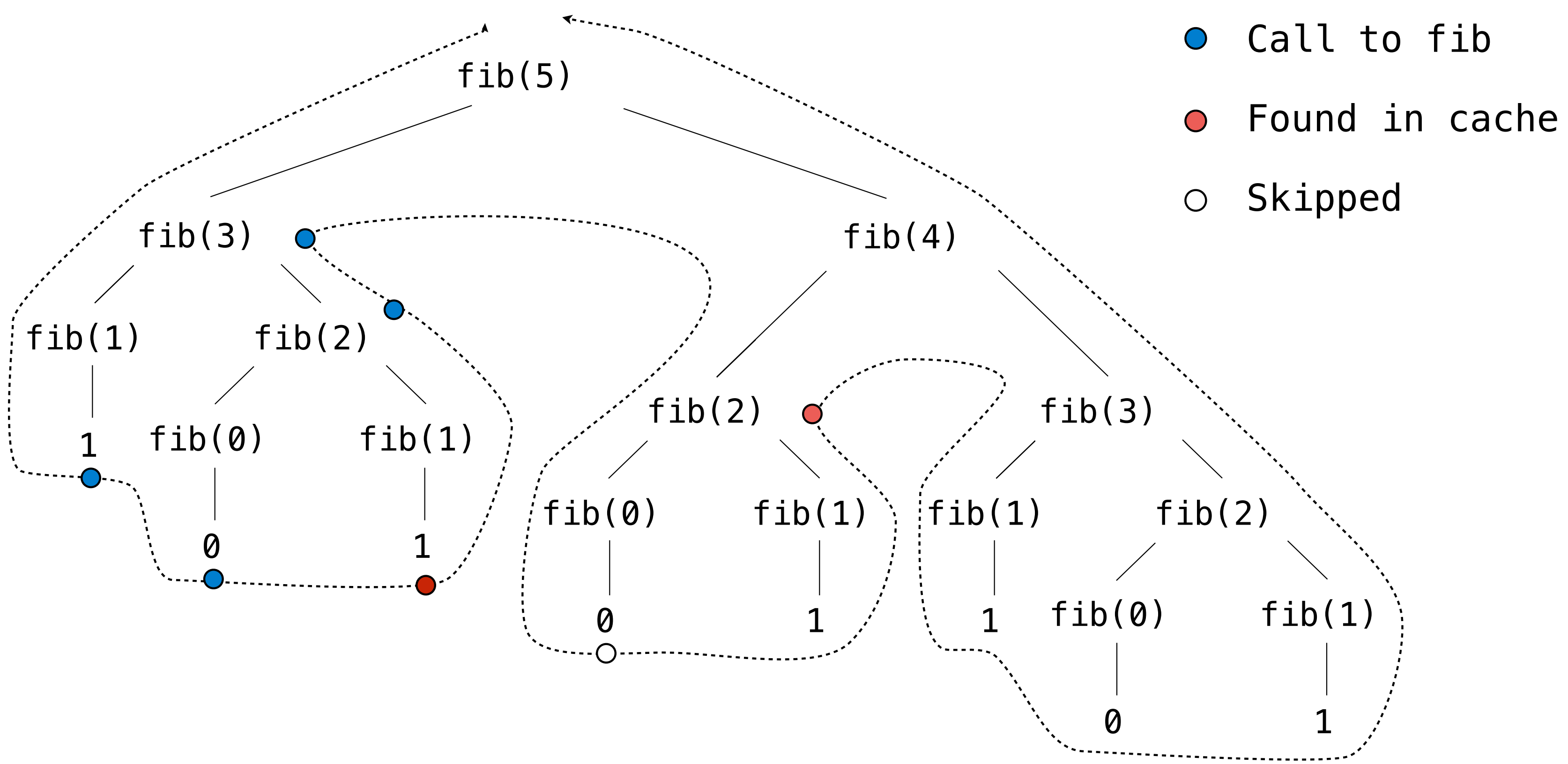

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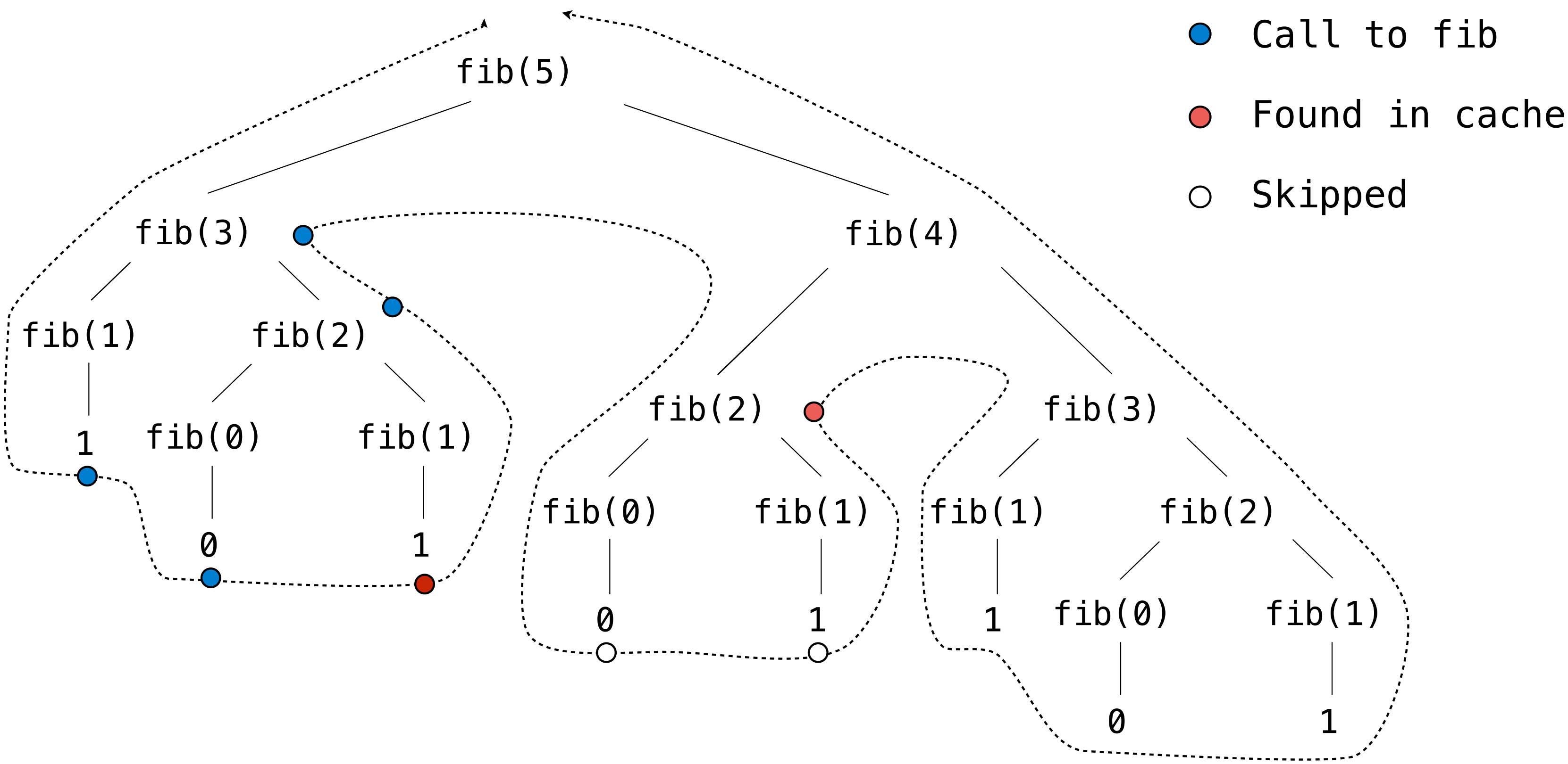

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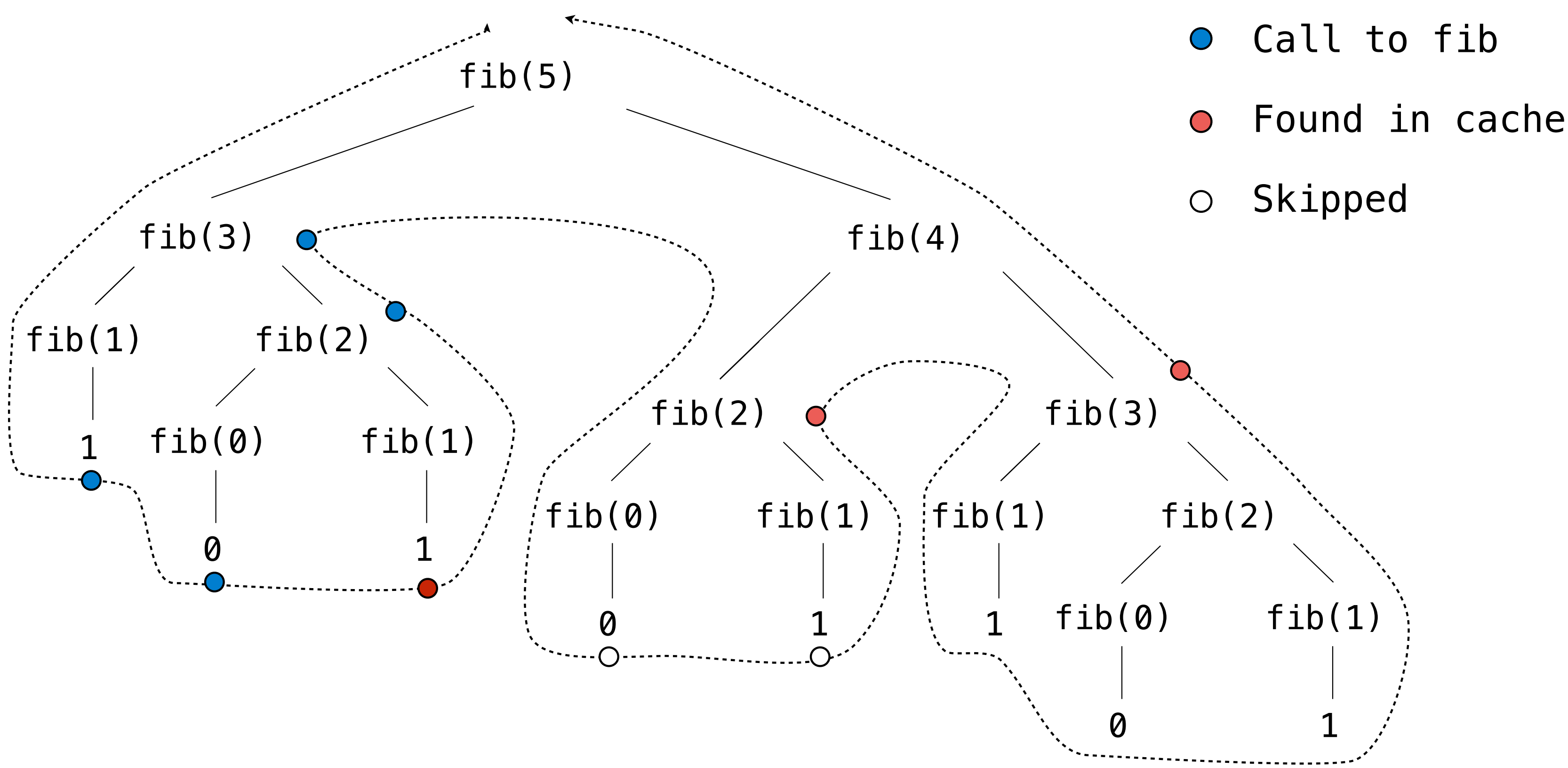

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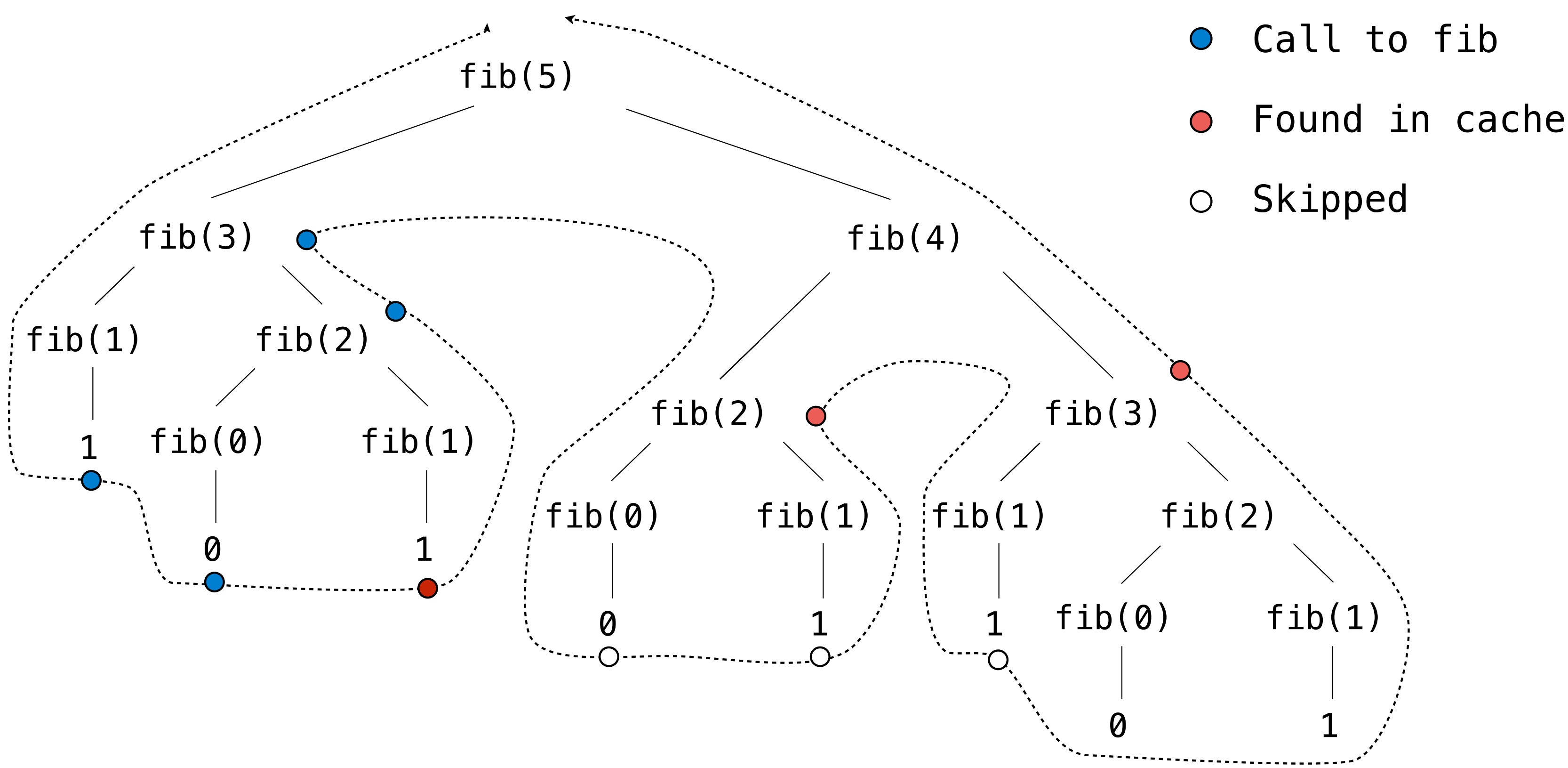

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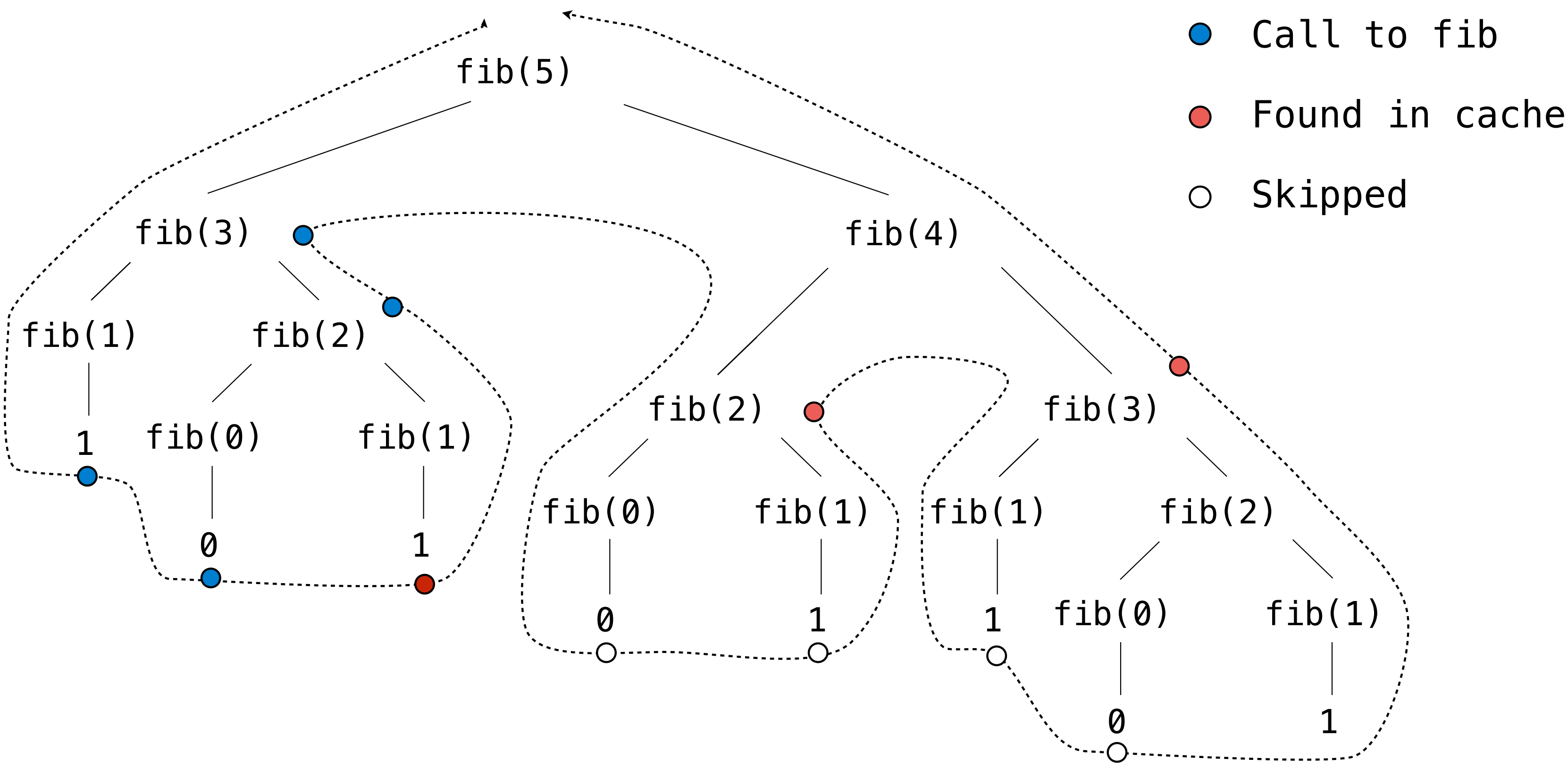

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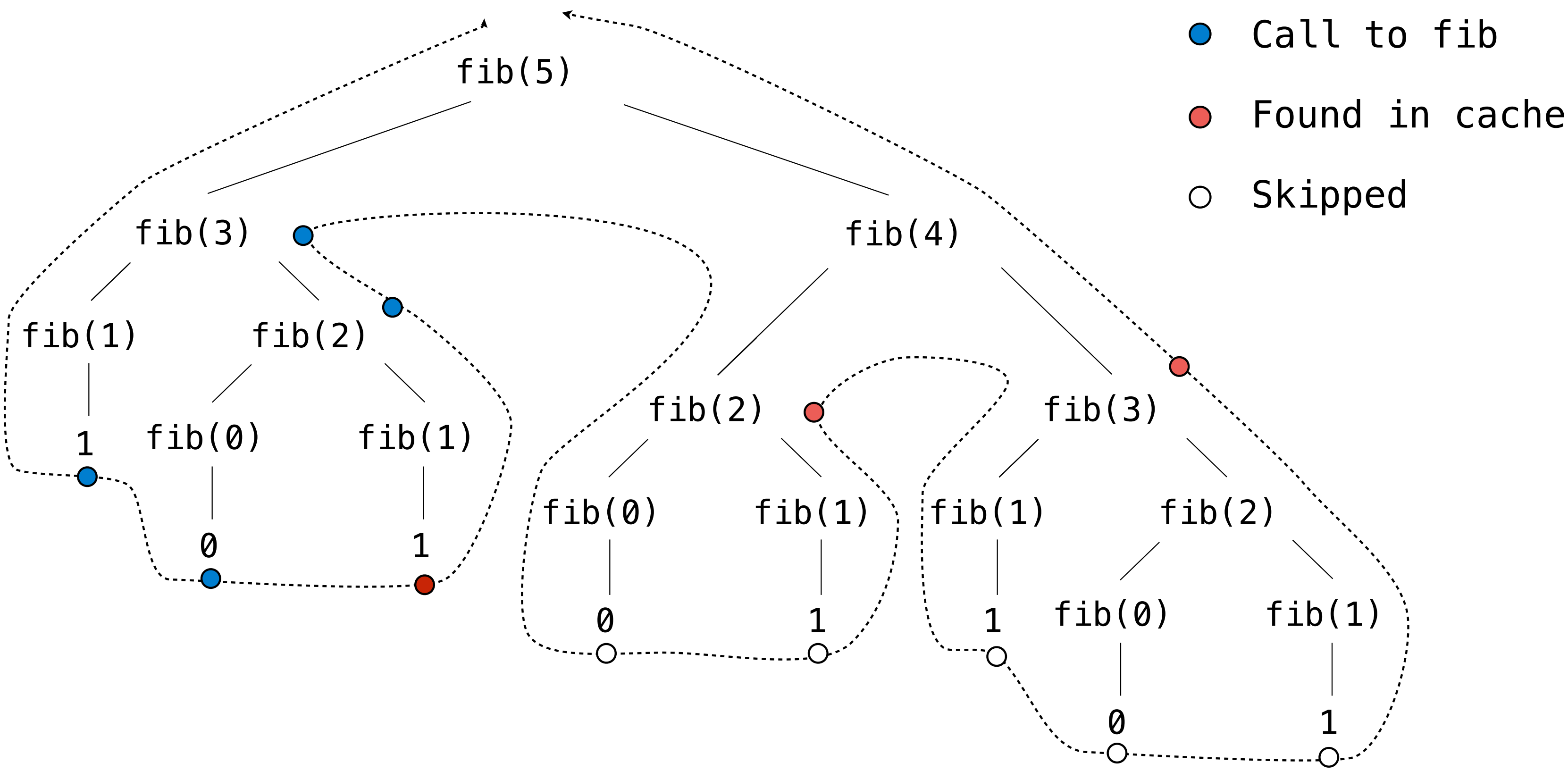

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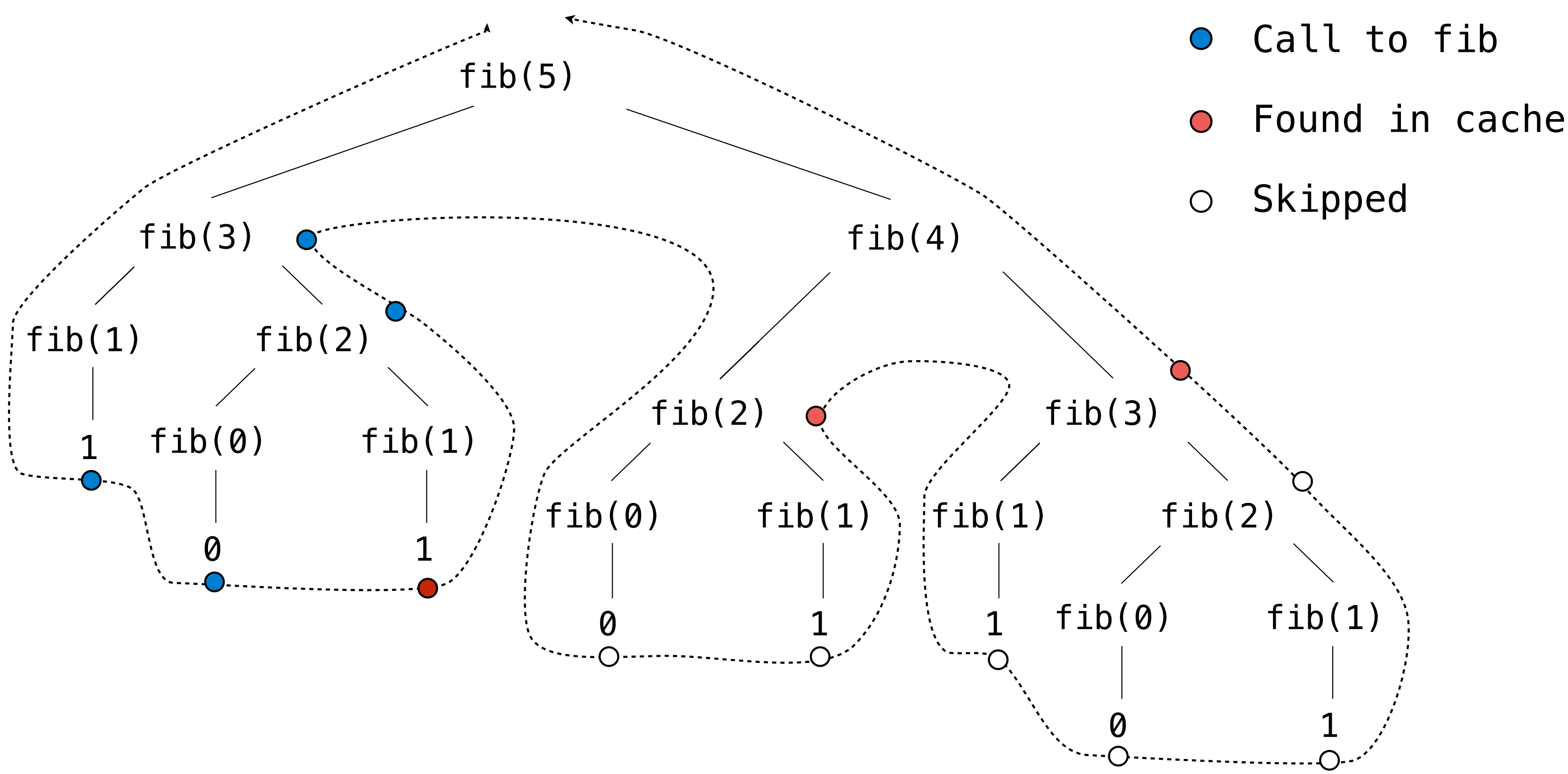

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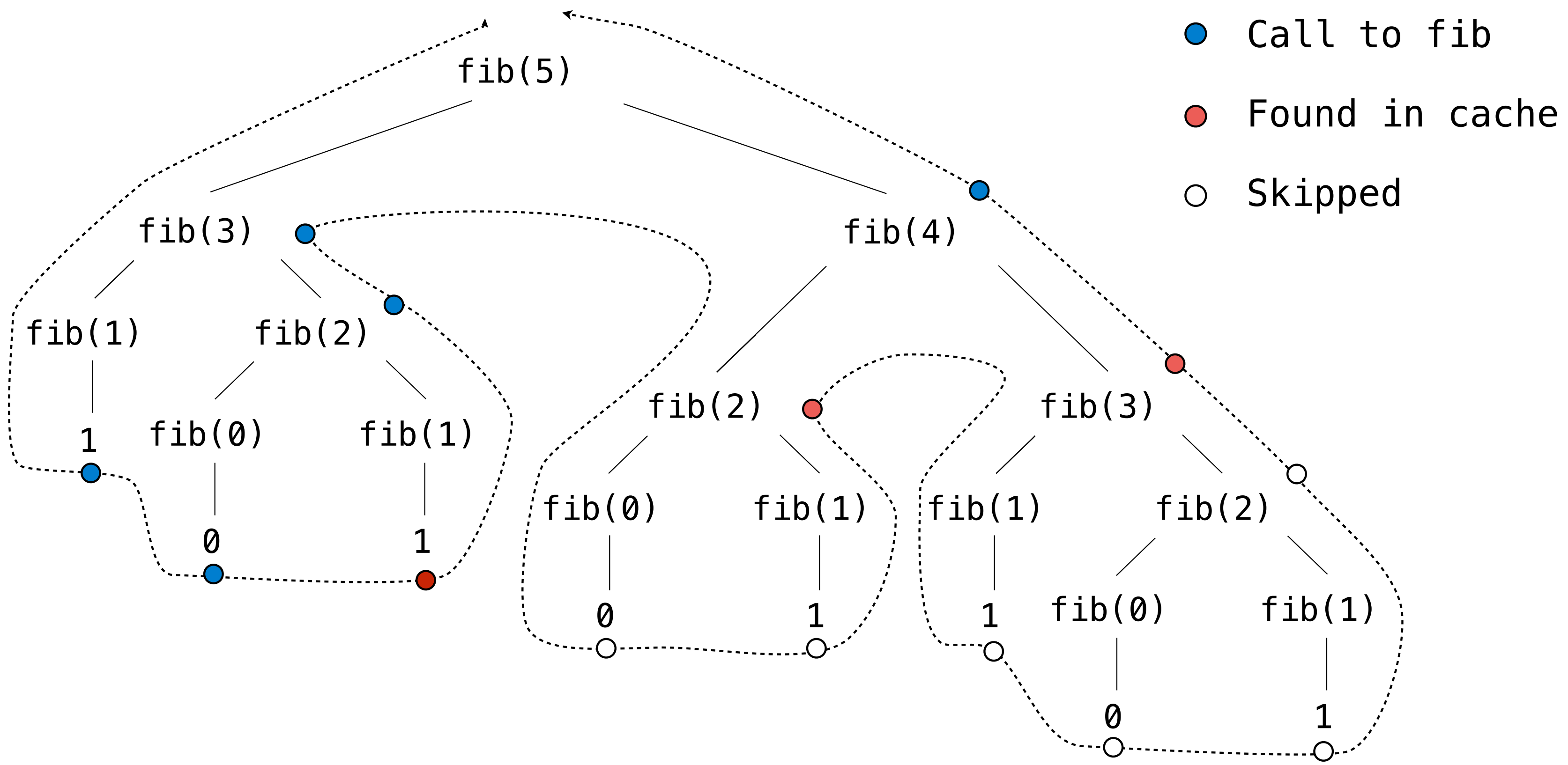

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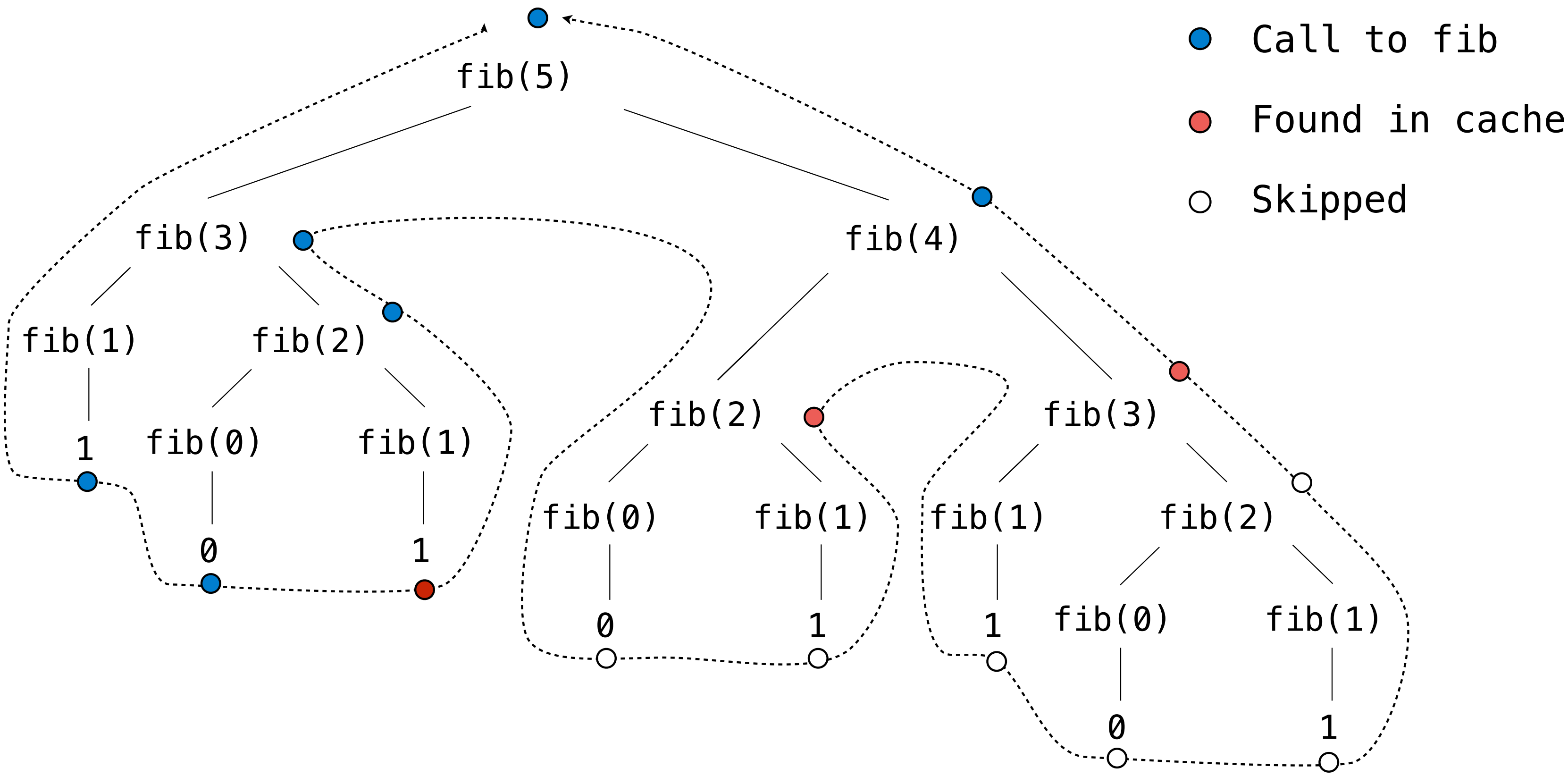

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```

Built-in isinstance function: returns True if branch has a class that *is* **or** *inherits from* Tree

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
```

Tree Class

A Tree has an entry (any value) at its root and a list of branches

```
class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
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Built-in isinstance function: returns True if branch has a class that *is* **or** *inherits from* Tree

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def fib_tree(n):
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        return Tree(left.entry + right.entry, (left, right))
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```

(Demo)

Hailstone Trees

Hailstone Trees

Hailstone Trees

```
Pick a positive integer n as the start
```

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

1

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

1

2

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

1

2

4

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

1

2

4

8

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

1

2

4

8

16

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

1

2

4

8

16

32

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

1

2

4

8

16

32

64

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

1

2

4

8

16

32

64

128

Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```

```
 1
 |
 2
 |
 4
 |
 8
 |
 16
 |
 32
 |
 64
 |
128
```

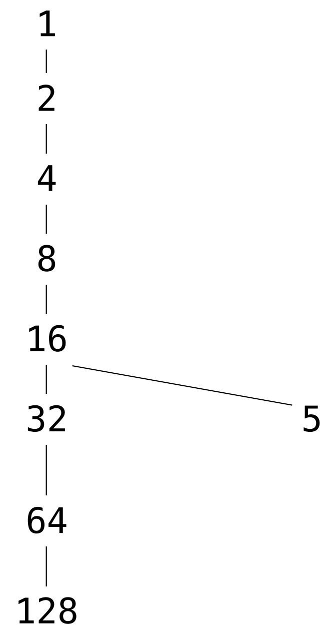
Hailstone Trees

```
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1
```



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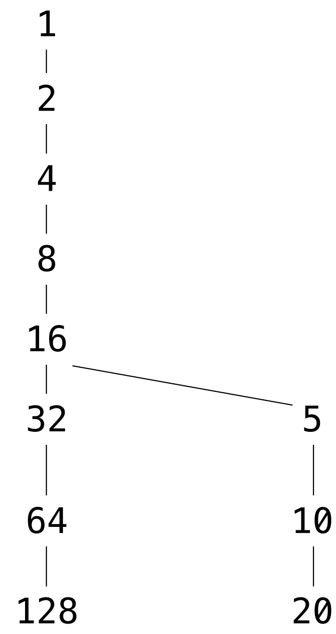
If n is odd, multiply it by 3 and add 1

Continue this process until n is 1



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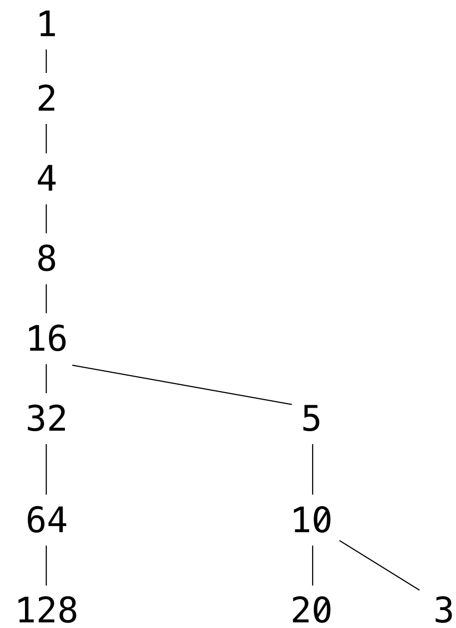

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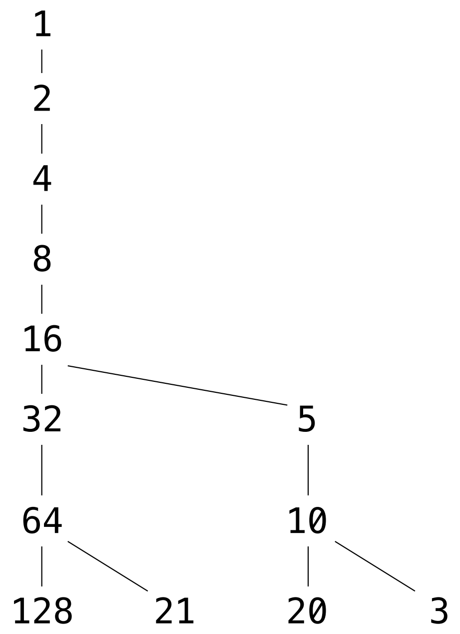

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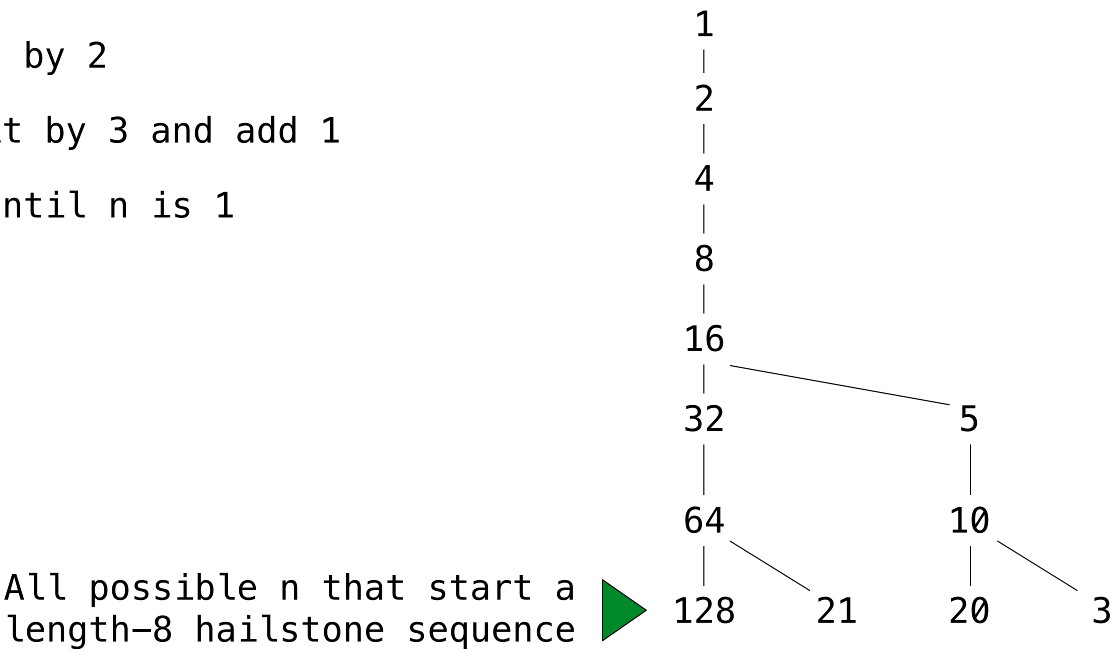


Hailstone Trees

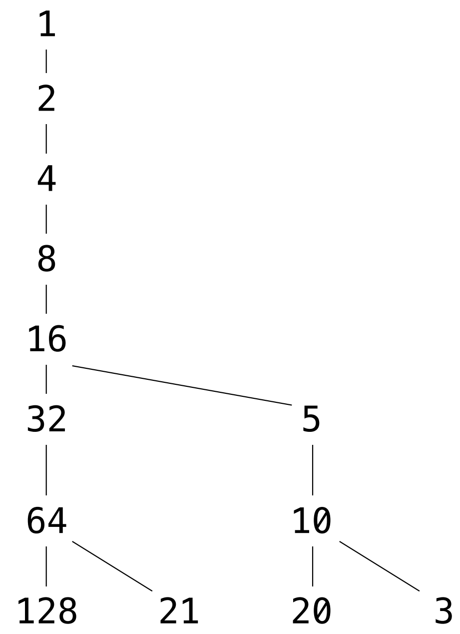
Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

```
def hailstone_tree(k, n=1):
    """Return a Tree in which the paths from the
    leaves to the root are all possible hailstone
    sequences of length k ending in n."""
```



All possible n that start a
length-8 hailstone sequence

Hailstone Trees

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If n is odd, multiply it by 3 and add 1

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All possible n that start a
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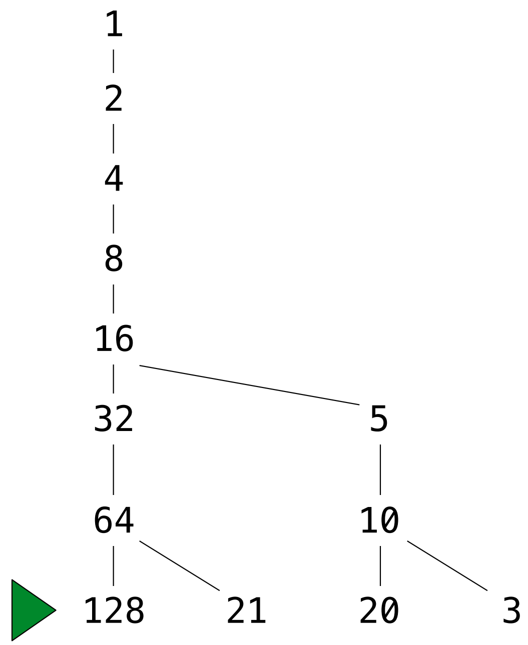

(Demo)