61A Lecture 13

Announcements

Mutable Functions

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```
>>> withdraw(25)
```

A Function with Behavior That Varies Over Time

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>>> withdraw(25)
75
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A Function with Behavior That Varies Over Time

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>>> withdraw(25)
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Argument:
amount to withdraw

A Function with Behavior That Varies Over Time

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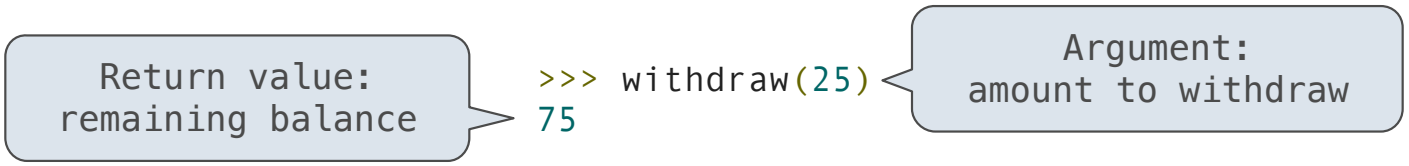

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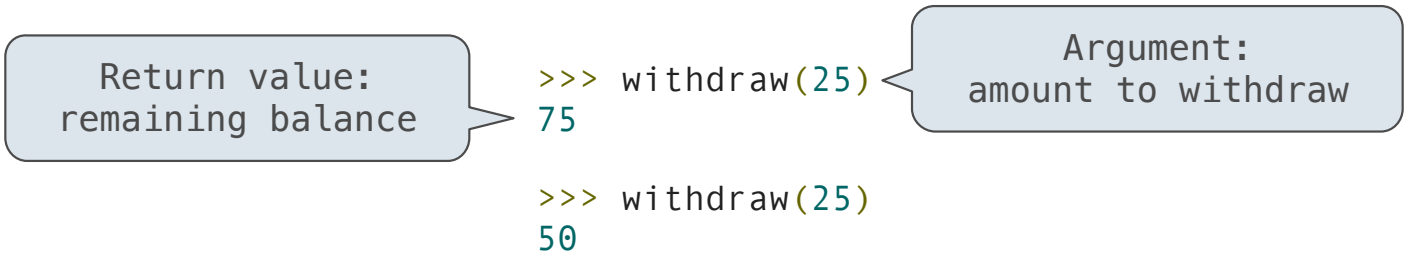

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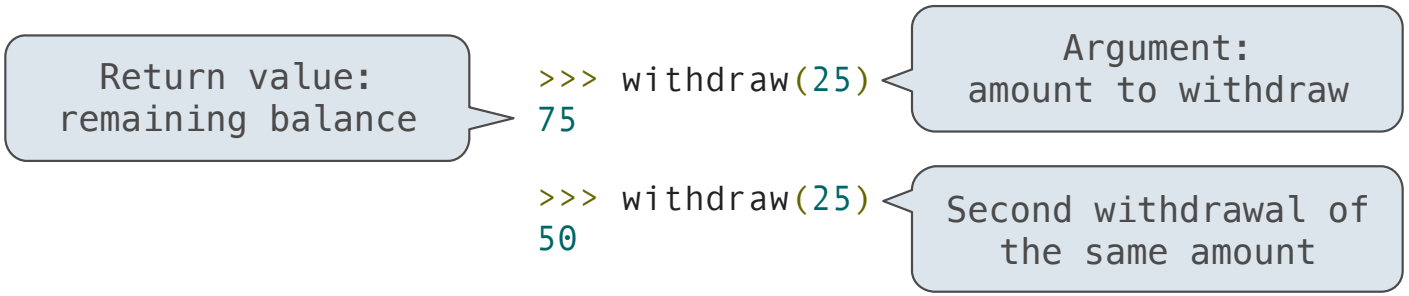

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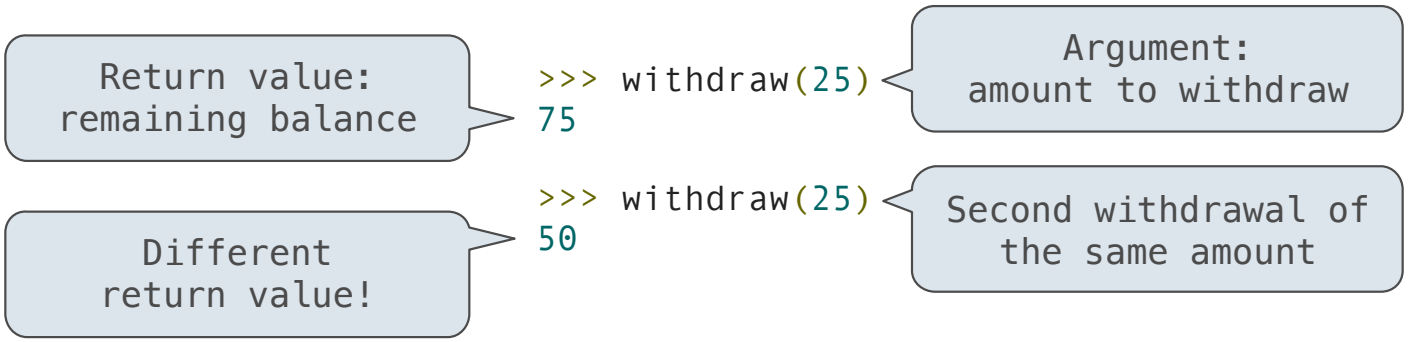

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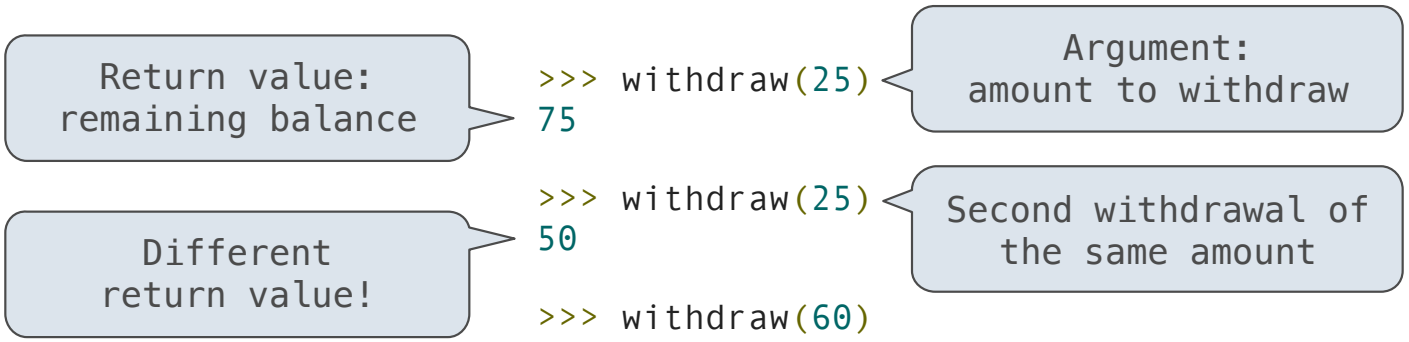

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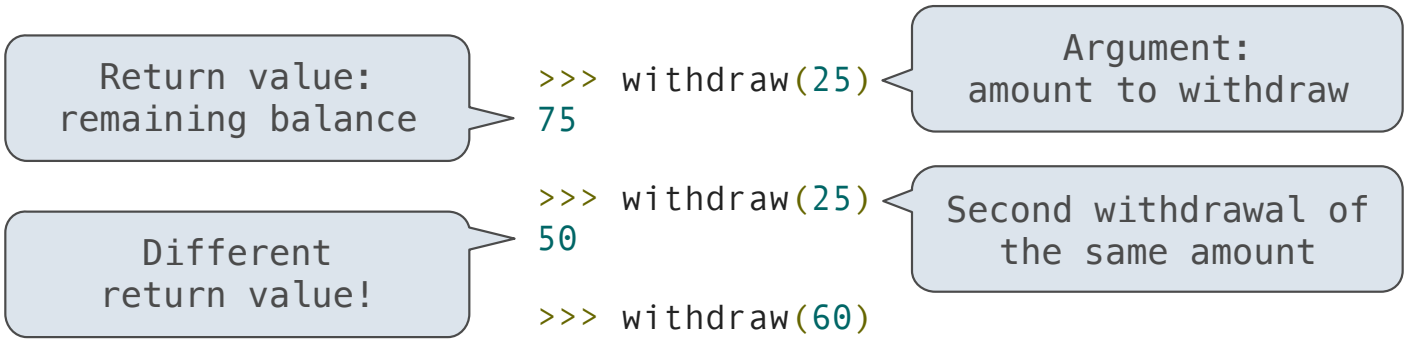

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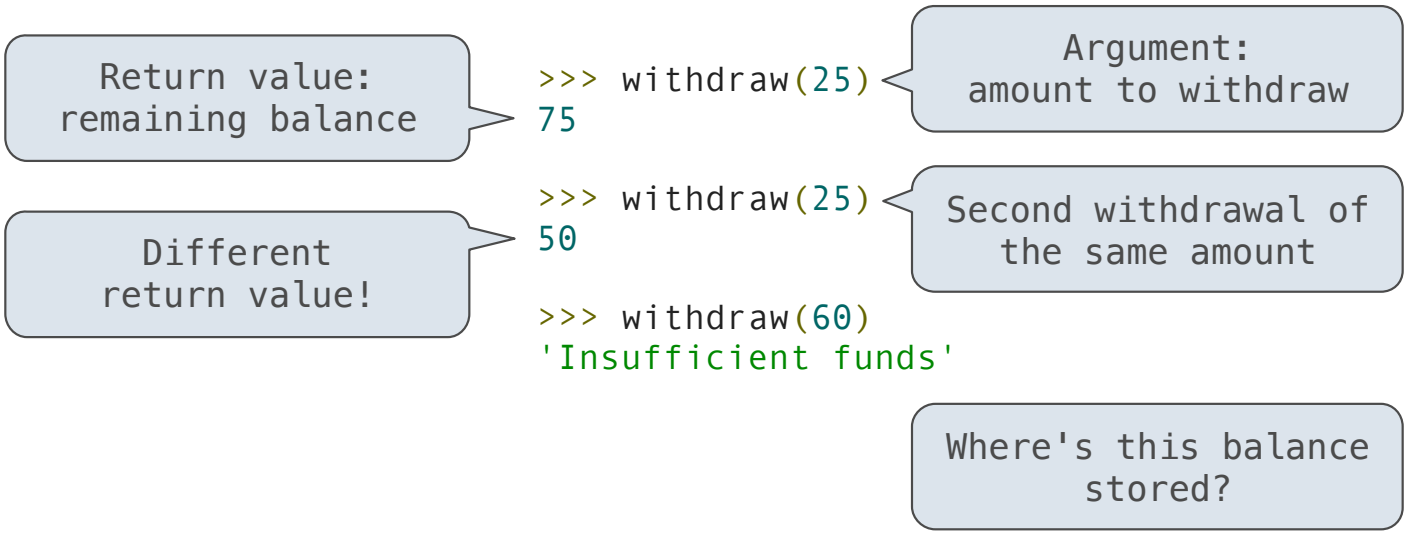

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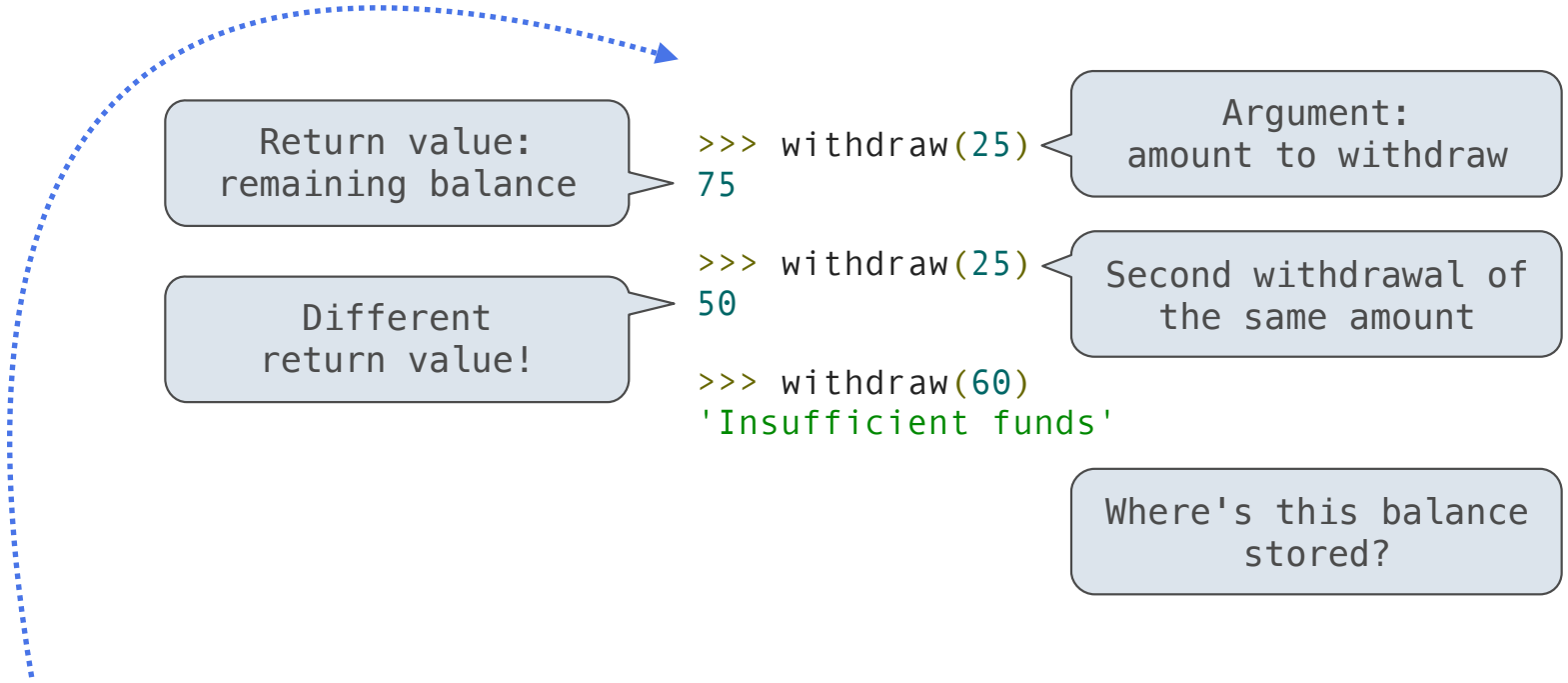

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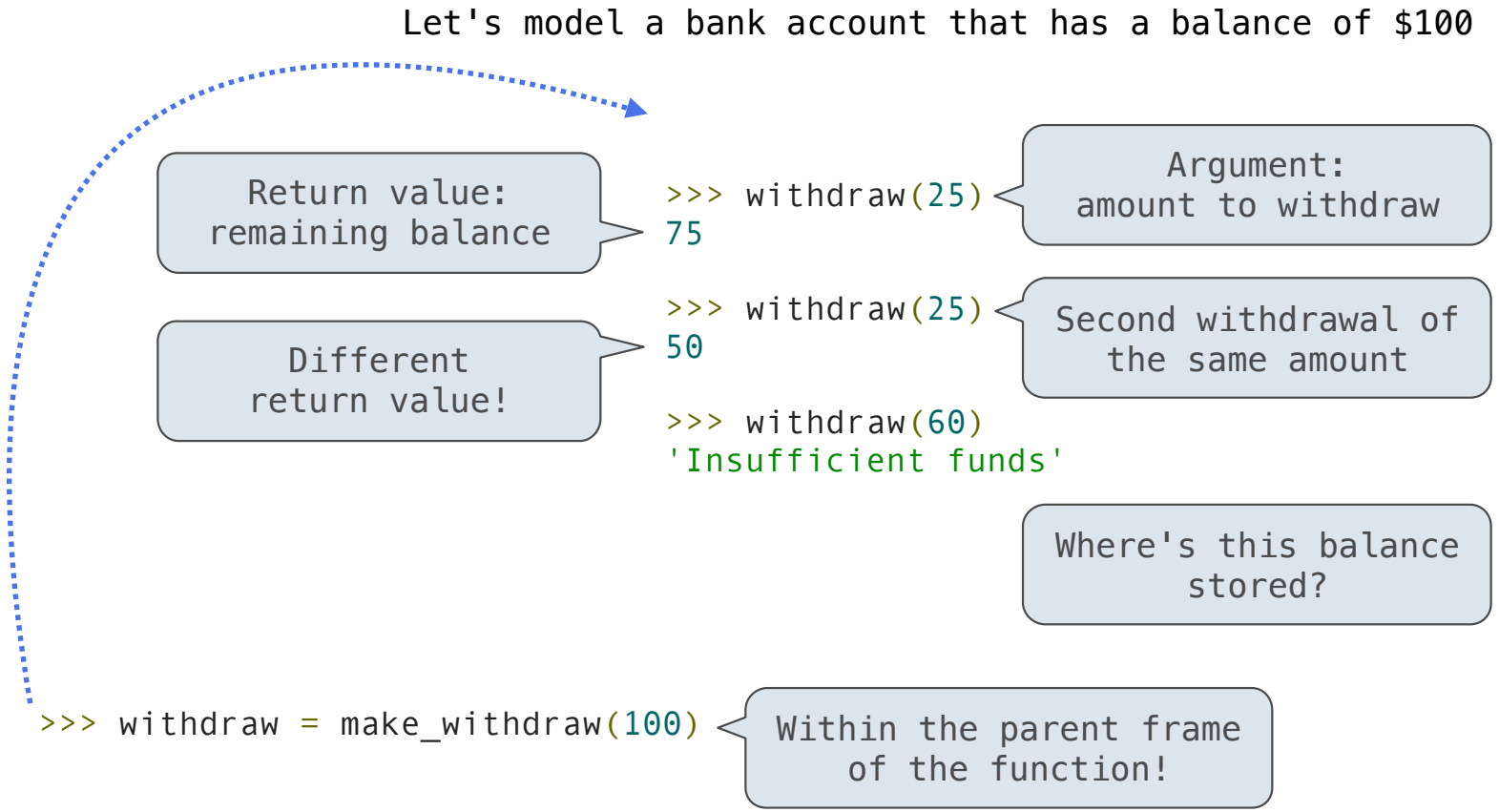


A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```
>>> withdraw(25)
75
>>> withdraw(25)
50
>>> withdraw(60)
'Insufficient funds'
```

Return value: remaining balance

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

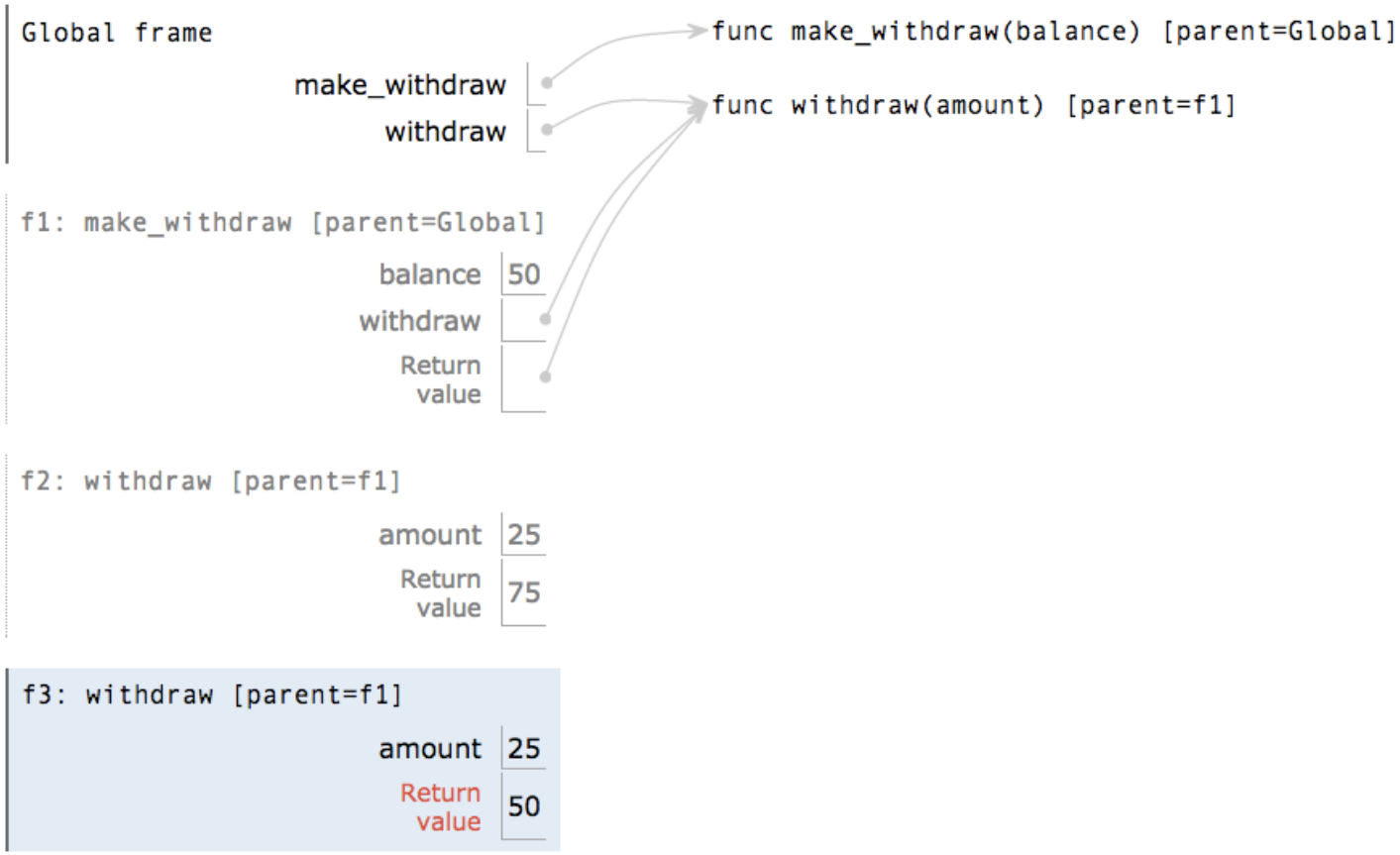
Where's this balance stored?

```
>>> withdraw = make_withdraw(100)
```

Within the parent frame of the function!

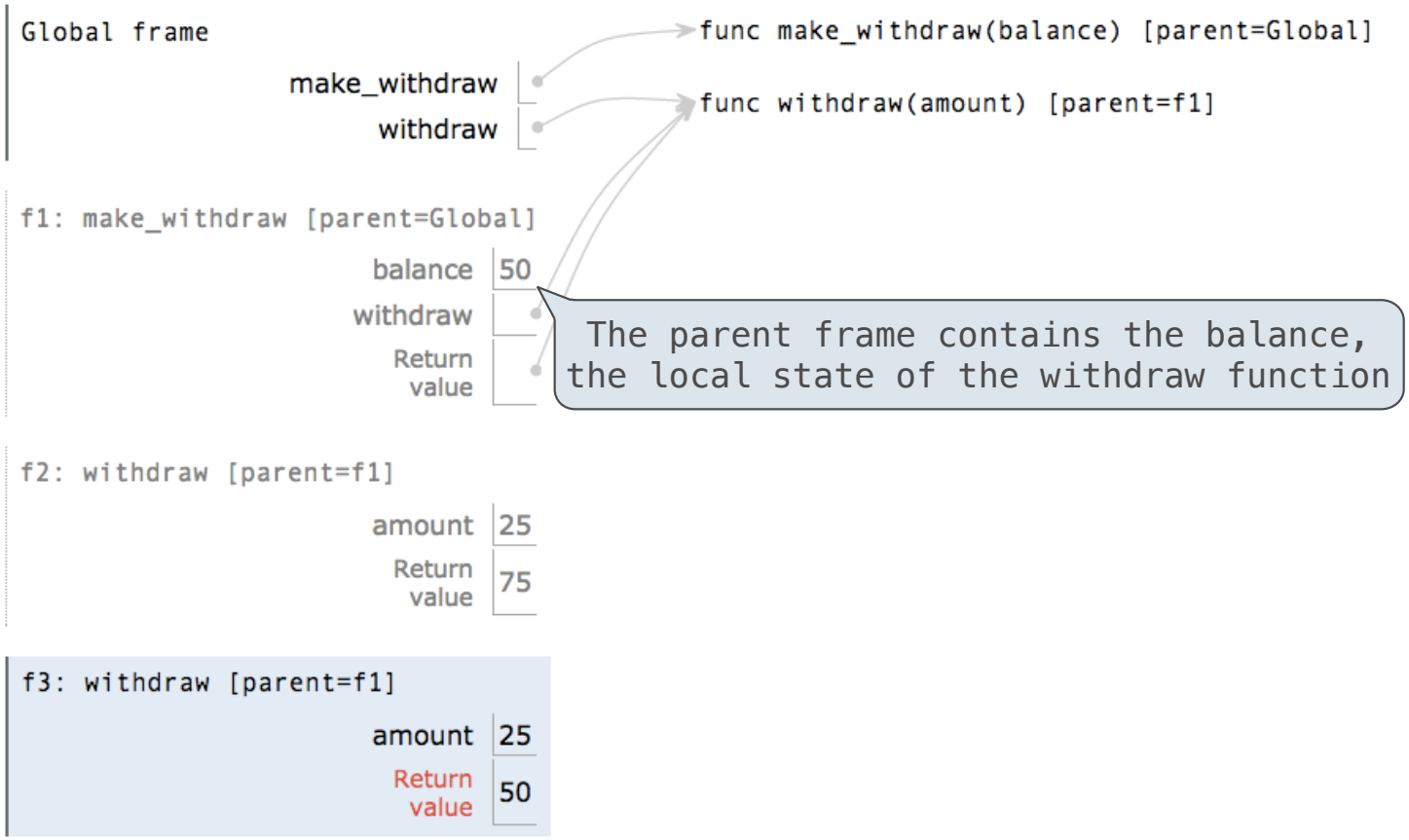
A function has a body and a parent environment

Persistent Local State Using Environments



Interactive Diagram

Persistent Local State Using Environments



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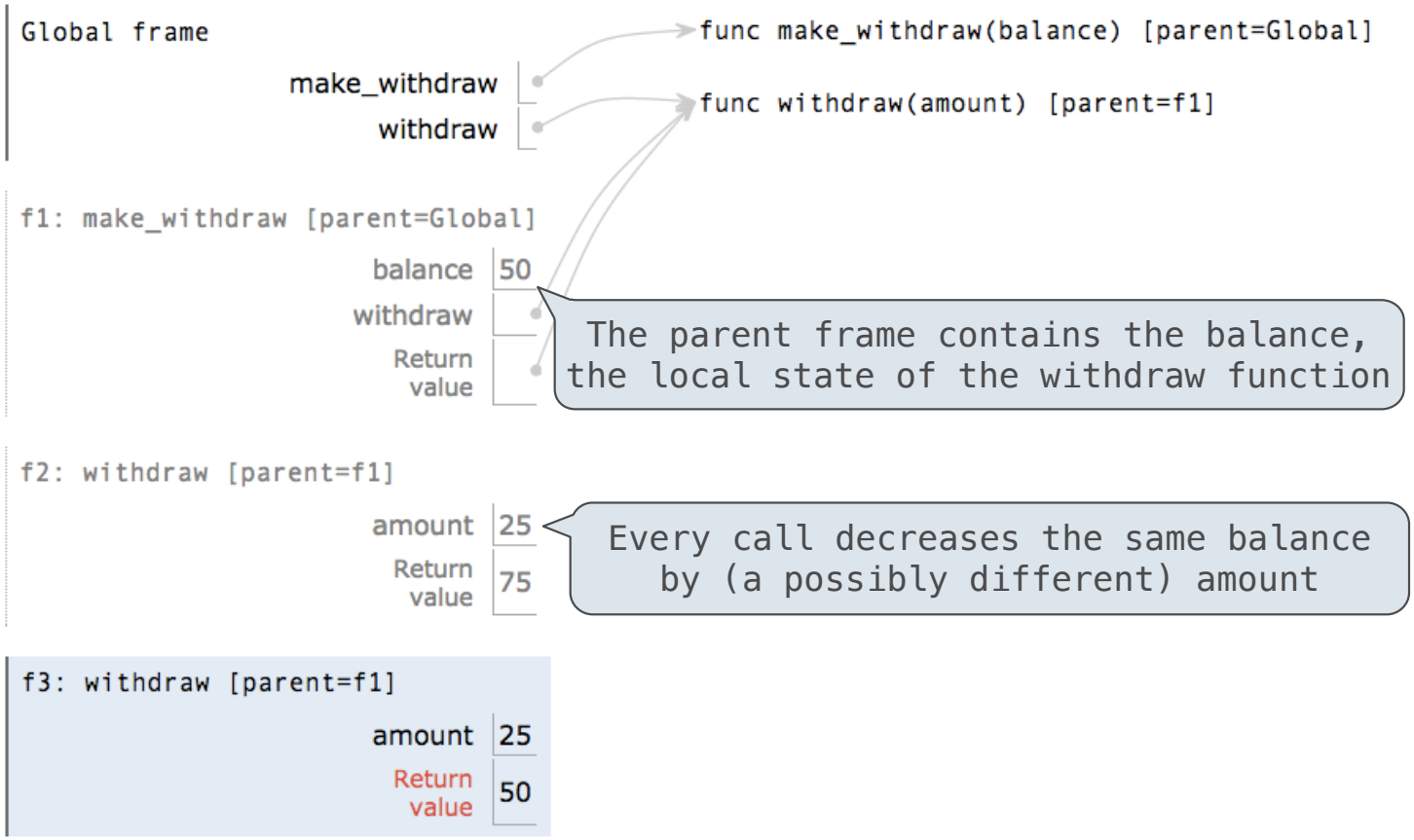

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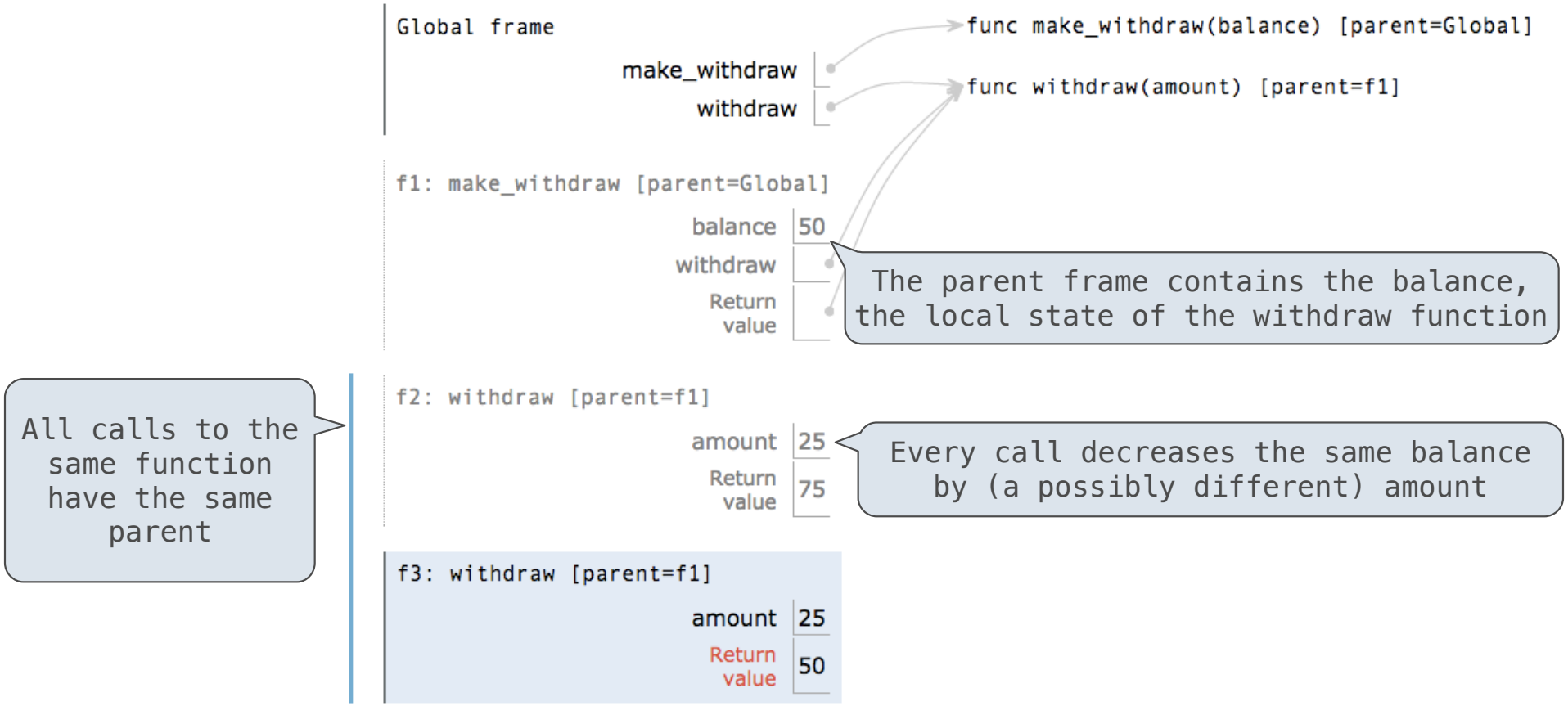
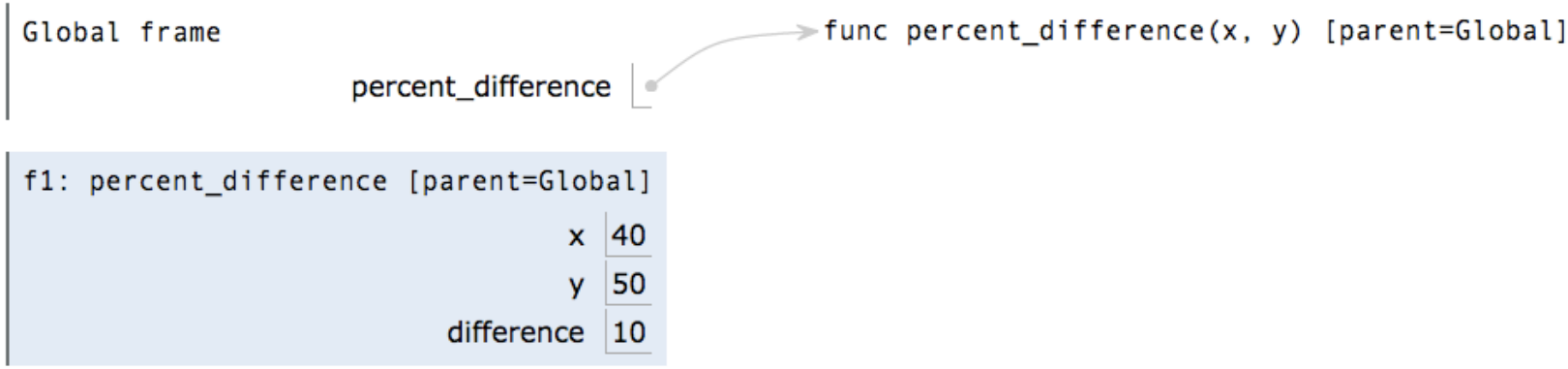


Interactive Diagram

Reminder: Local Assignment

```
def percent_difference(x, y):
        difference = abs(x-y)
        return 100 * difference / x
diff = percent_difference(40, 50)
```



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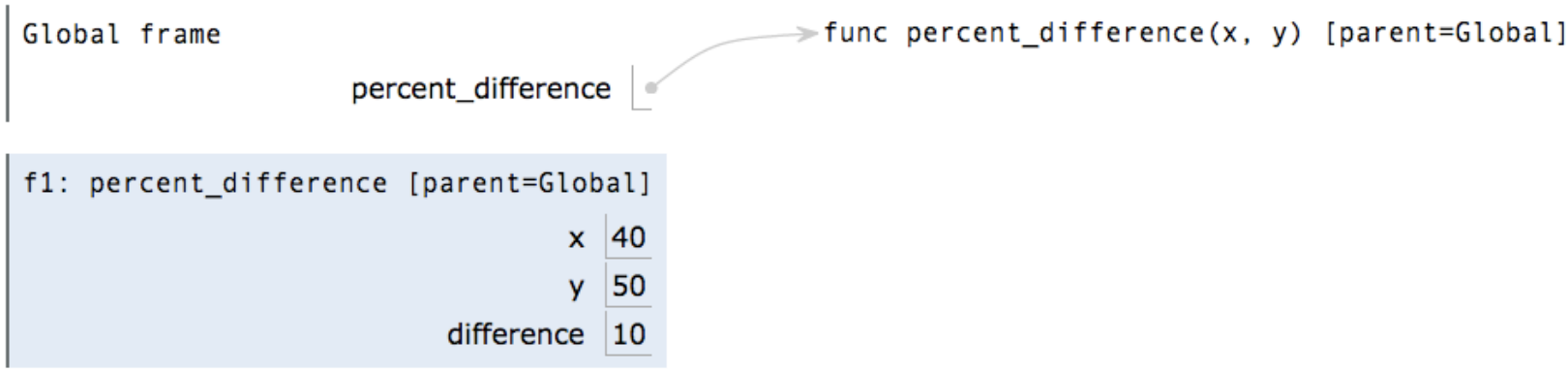

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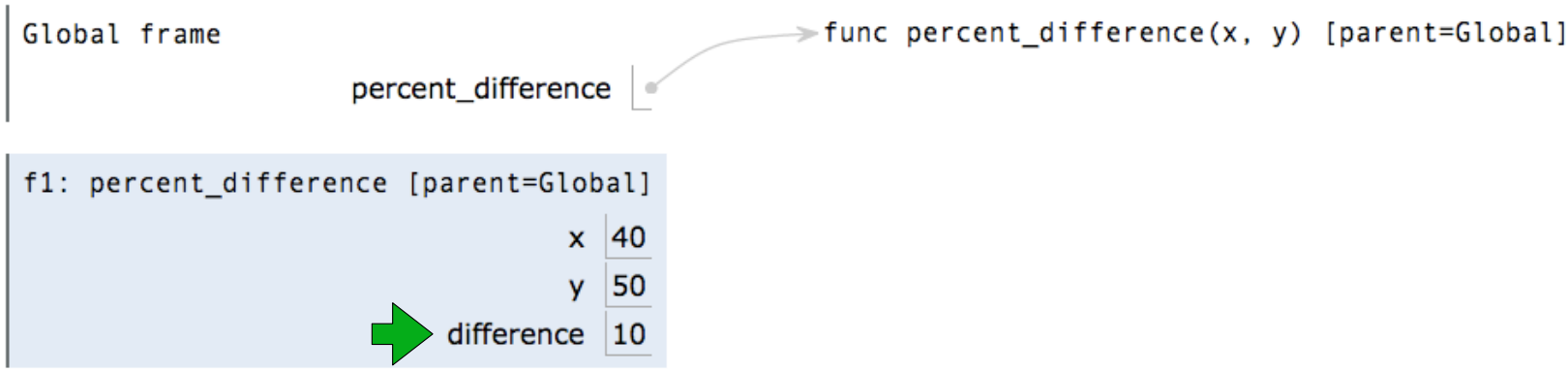

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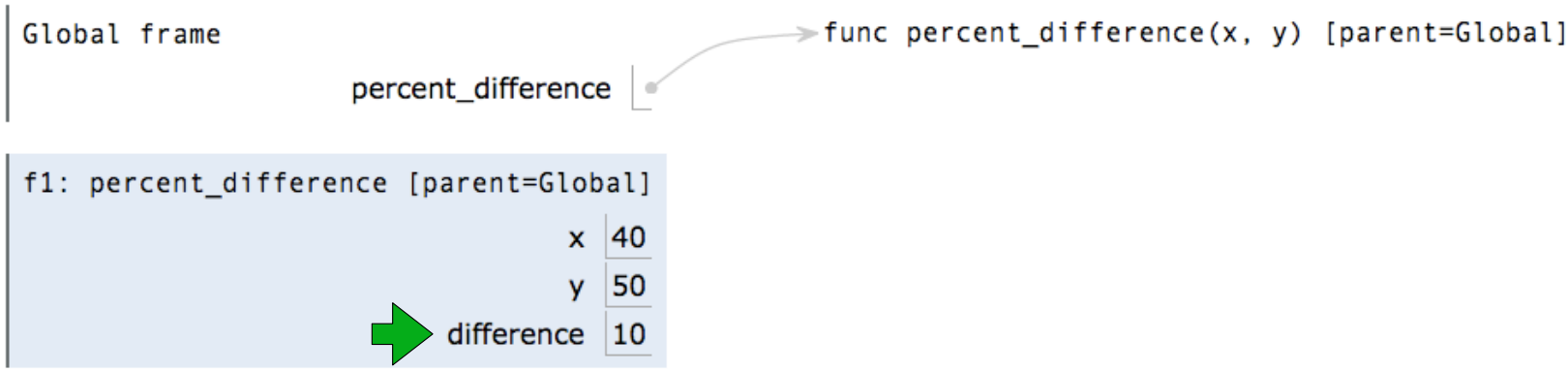

Execution rule for assignment statements:

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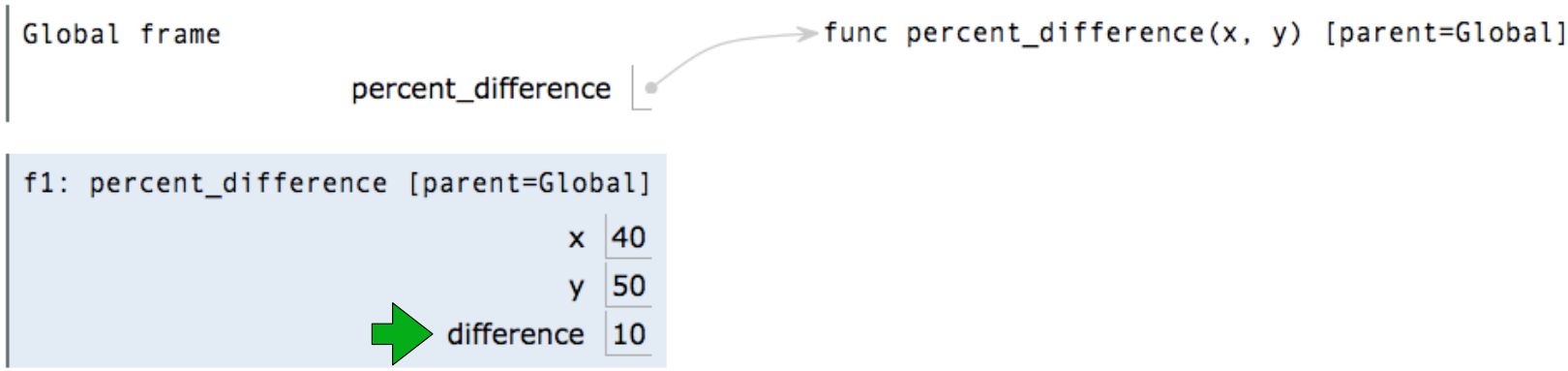

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right

2. Bind the names on the left to the resulting values in the **current frame**

Interactive Diagram

Non-Local Assignment & Persistent Local State

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```
def make_withdraw(balance):
```

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        nonlocal balance
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def make_withdraw(balance):
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```

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def make_withdraw(balance):
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Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned

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(Demo)

Non-Local Assignment

The Effect of Nonlocal Statements

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The Many Meanings of Assignment Statements

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Python Particulars

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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
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        balance = balance - amount
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wd = make_withdraw(20)
wd(5)
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Interactive Diagram

Python Particulars

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Local assignment

Interactive Diagram

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Local assignment

UnboundLocalError: local variable 'balance' referenced before assignment

Interactive Diagram

Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

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```
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
withdraw(25)
```

Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

Name bound outside of withdraw def

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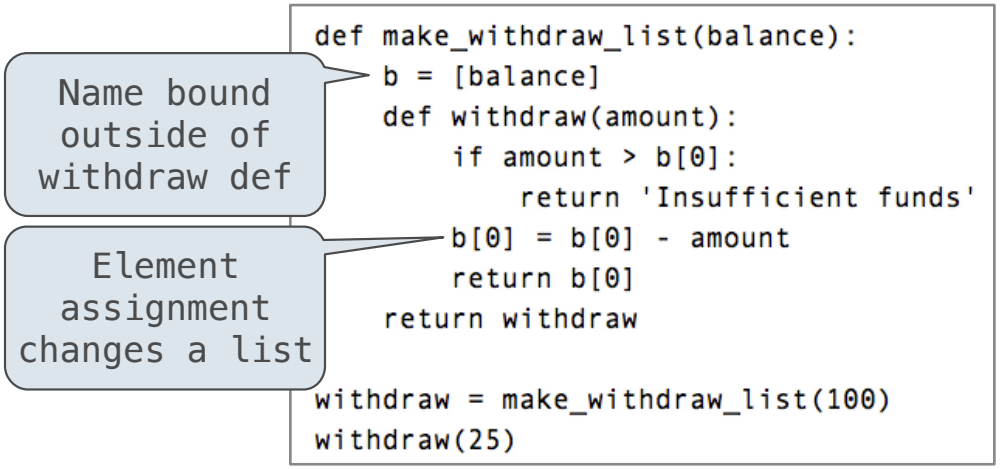

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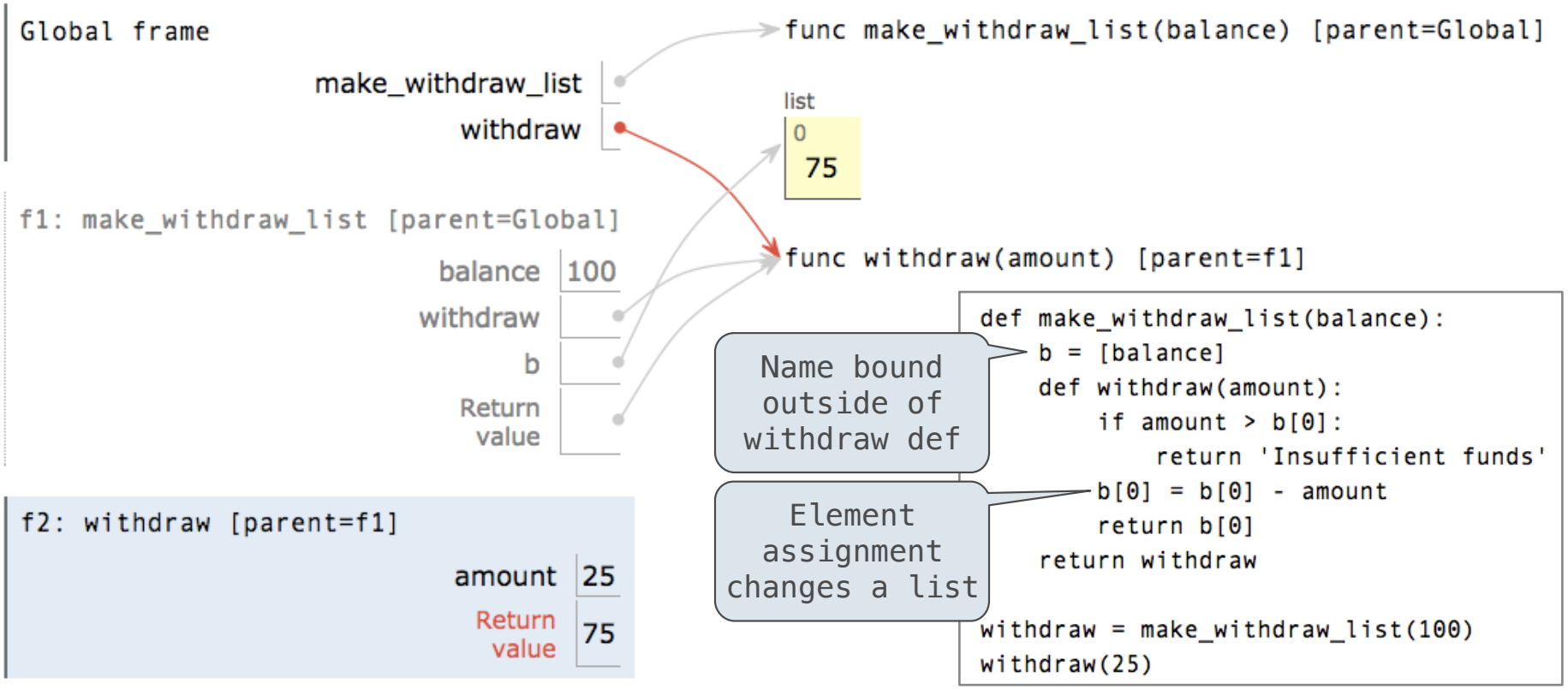

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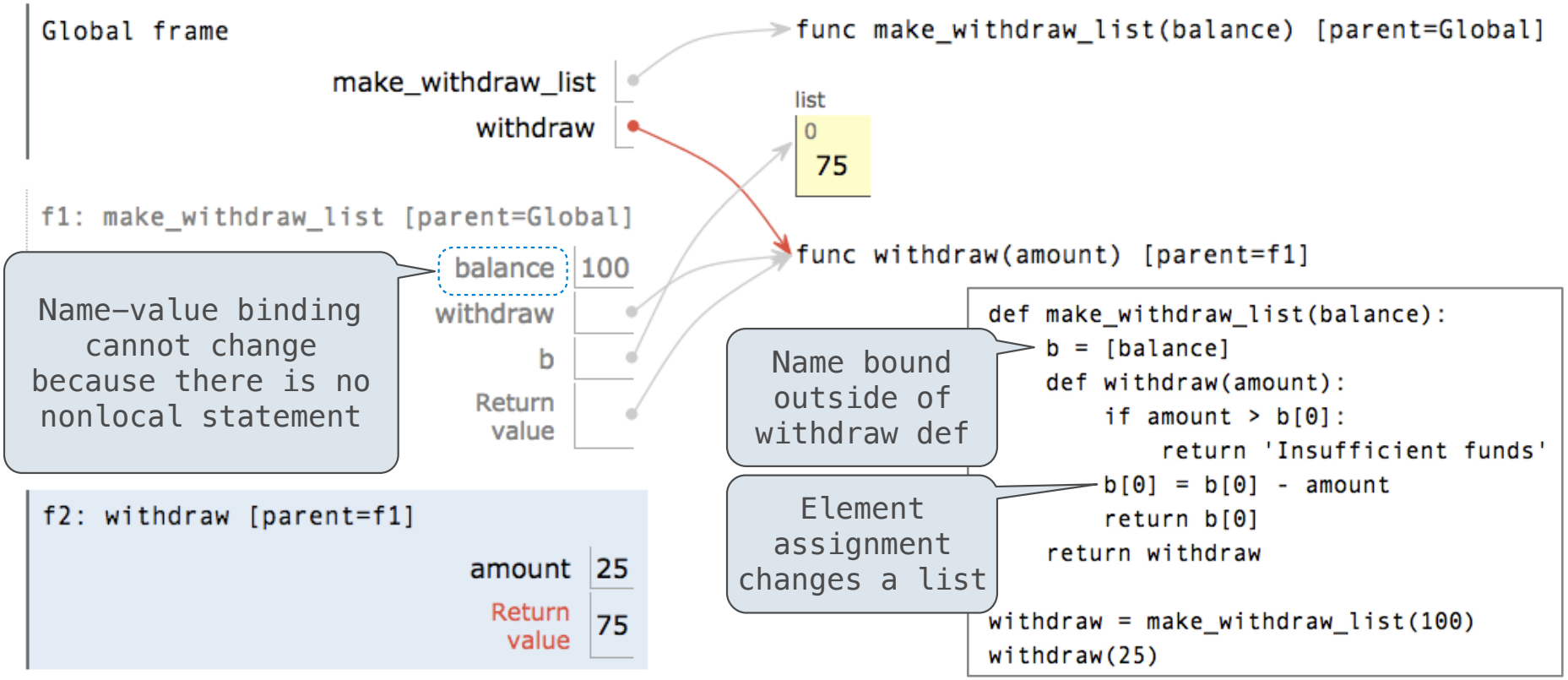

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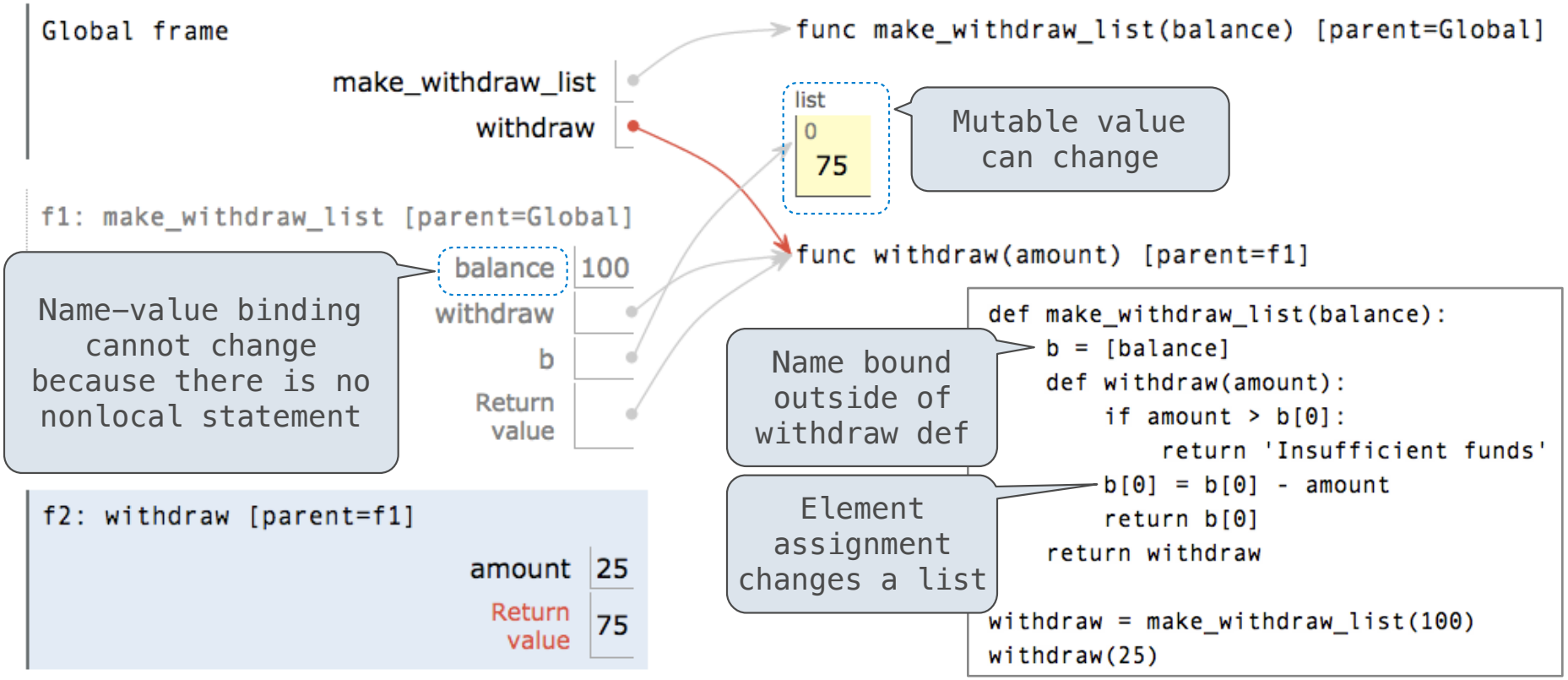

Interactive Diagram

Multiple Mutable Functions

(Demo)

Referential Transparency, Lost

Interactive Diagram

Referential Transparency, Lost

- Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

Interactive Diagram

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