# 61A Lecture 9

# Announcements

# Data Abstraction

# Data Abstraction

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- Compound values combine other values together

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- Compound values combine other values together
  - A date: a year, a month, and a day

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  - A geographic position: latitude and longitude

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All Programmers

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All Programmers

Great Programmers

# Rational Numbers

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$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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Exact representation of fractions

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- rational(n, d) returns a rational number x

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- numer(x) returns the numerator of x

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- rational(n, d) returns a rational number x

Selectors

- numer(x) returns the numerator of x
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# Rational Number Arithmetic

**Example**

**General Form**

# Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

**Example**

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# Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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**Example**

$$\frac{nx}{dx} * \frac{ny}{dy}$$

**General Form**

# Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

**Example**

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

**General Form**

## Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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**General Form**

# Rational Number Arithmetic Implementation

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`
- `denom(x)` returns the denominator of `x`

# Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

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These functions implement an abstract representation for rational numbers

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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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def add_rational(x, y):
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    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

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def add_rational(x, y):
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    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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# Pairs

## Representing Pairs Using Lists

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```
>>> pair = [1, 2]
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A list literal:
Comma-separated expressions in brackets

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```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

```
A list literal:
Comma-separated expressions in brackets
```

# Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

```
A list literal:
Comma-separated expressions in brackets
```

# Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

# Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
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>>> x, y = pair
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A list literal:
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"Unpacking" a list

# Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

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>>> pair = [1, 2]
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A list literal:
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>>> pair[0]
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```

Element selection using the selection operator

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>>> pair[0]
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```

Element selection using the selection operator

```
>>> from operator import getitem
```

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A list literal:
Comma-separated expressions in brackets

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>>> x, y = pair
>>> x
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>>> y
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```

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```
>>> pair[0]
1
>>> pair[1]
2
```

Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
```

# Representing Pairs Using Lists

```
>>> pair = [1, 2]
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Element selection function

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Element selection function

More lists next lecture

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Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
```

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Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
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def denom(x):
    """Return the denominator of rational number X."""
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```

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Select item from a list

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Select item from a list

(Demo)

# Reducing to Lowest Terms

**Example:**

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$$\frac{3}{2} * \frac{5}{3}$$

## Reducing to Lowest Terms

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$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

# Reducing to Lowest Terms

**Example:**

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

# Reducing to Lowest Terms

**Example:**

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}} \qquad \frac{2}{5} + \frac{1}{10}$$

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```
from fractions import gcd
```

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def rational(n, d):
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from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
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def rational(n, d):
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    g = gcd(n, d)
```

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Greatest common divisor

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# Abstraction Barriers

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| Parts of the program that... | Treat rationals as... | Using... |
| --- | --- | --- |

# Abstraction Barriers

| Parts of the program that... | Treat rationals as... | Using... |
| --- | --- | --- |
| Use rational numbers to perform computation | | |

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| Create rationals or implement rational operations | | |

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| Parts of the program that... | Treat rationals as... | Using... |
|---|---|---|
| Use rational numbers to perform computation | whole data values | `add_rational`, `mul_rational` `rationals_are_equal`, `print_rational` |
| Create rationals or implement rational operations | numerators and denominators | `rational`, `numer`, `denom` |

## Abstraction Barriers

| Parts of the program that... | Treat rationals as... | Using... |
| --- | --- | --- |
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## Violating Abstraction Barriers

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# Data Representations

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(Demo)

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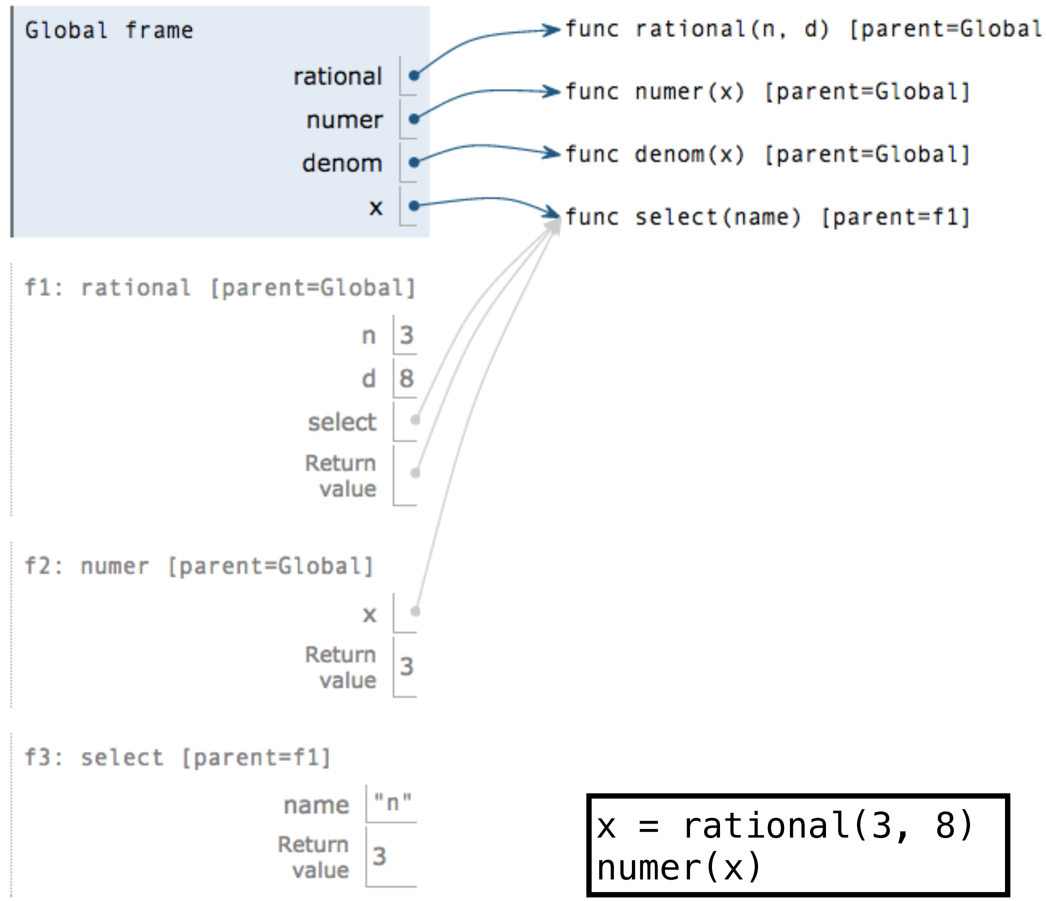


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Interactive Diagram