61A Lecture 6

Announcements

Recursive Functions

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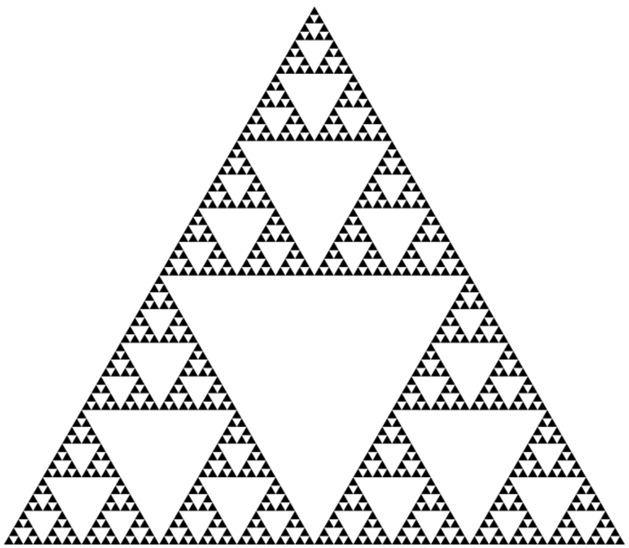

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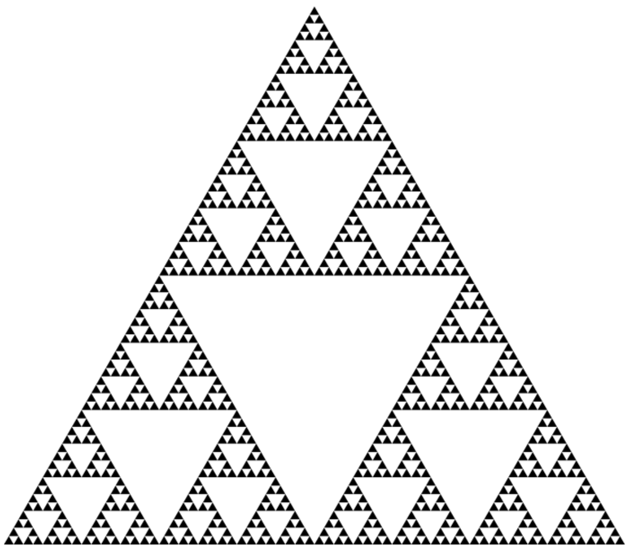

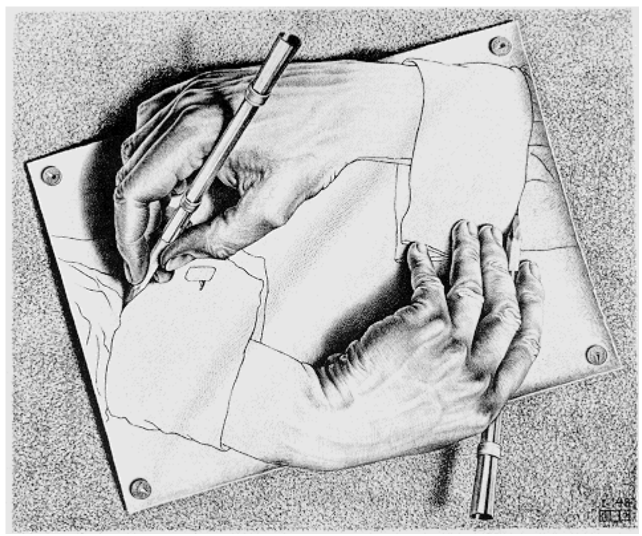

Drawing Hands, by M. C. Escher (lithograph, 1948)

Digit Sums

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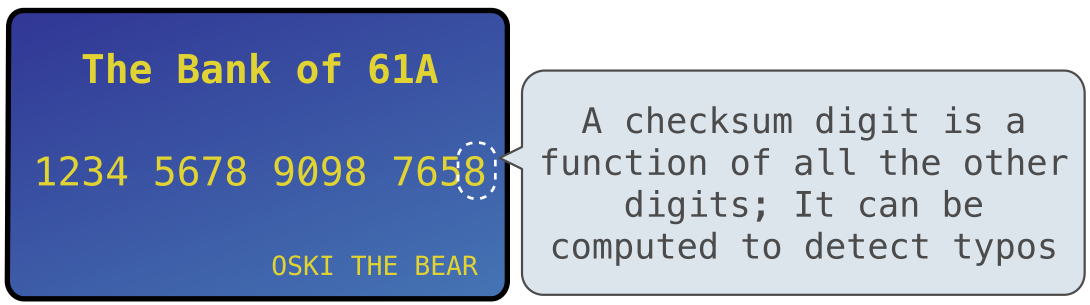

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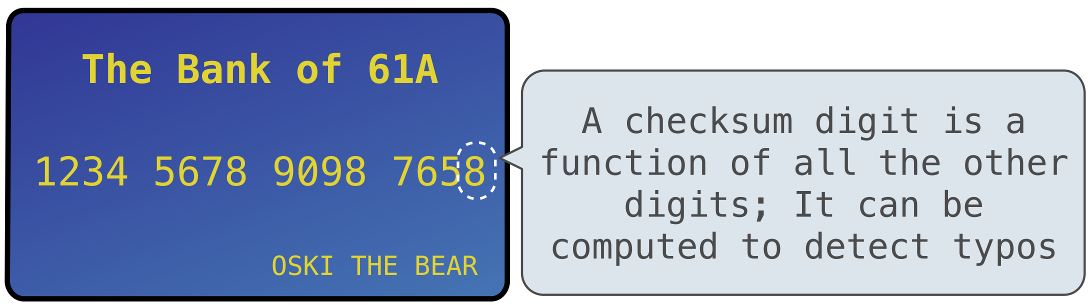

- Credit cards actually use the Luhn algorithm, which we'll implement after digit_sum

Sum Digits Without a While Statement

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(Demo)

Recursion in Environment Diagrams

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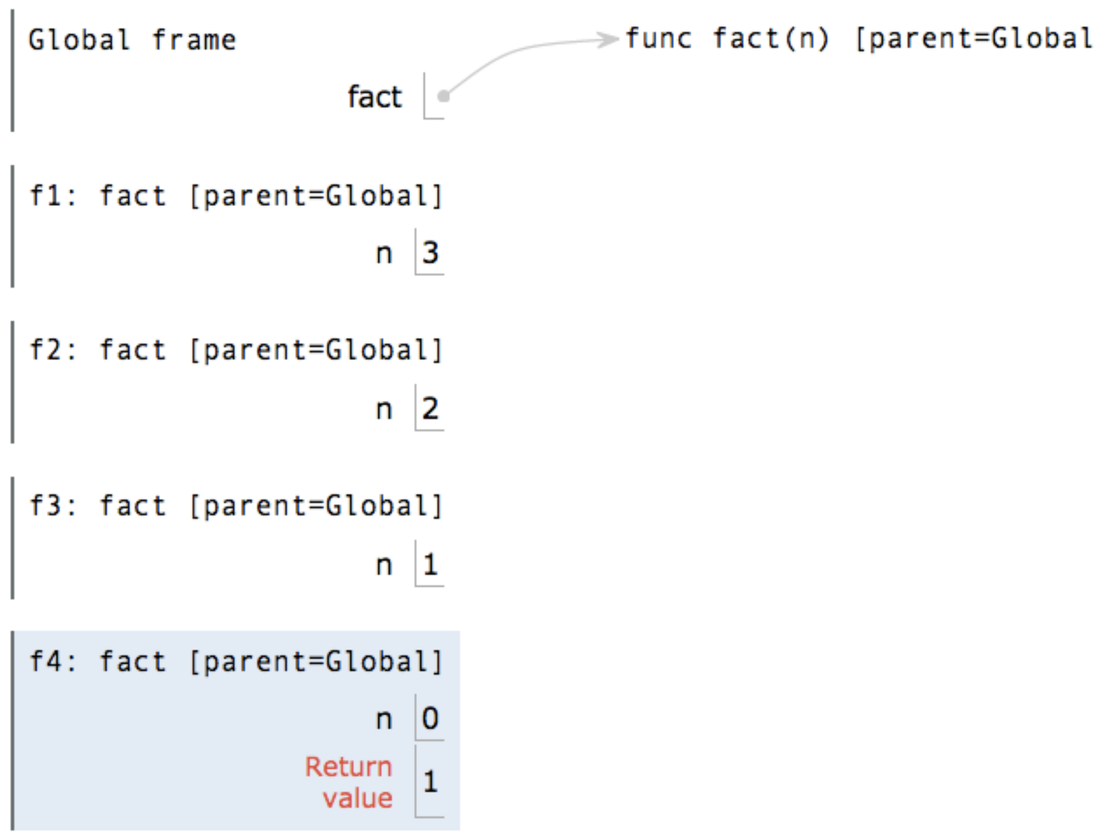

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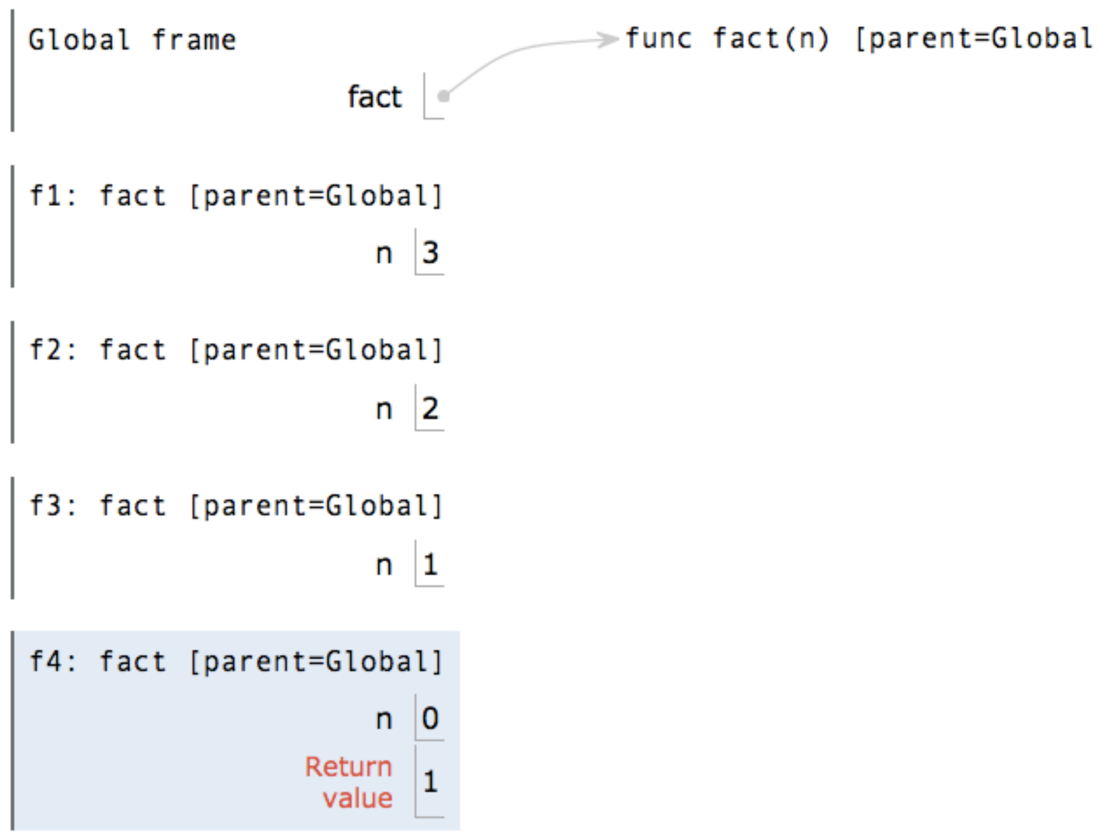

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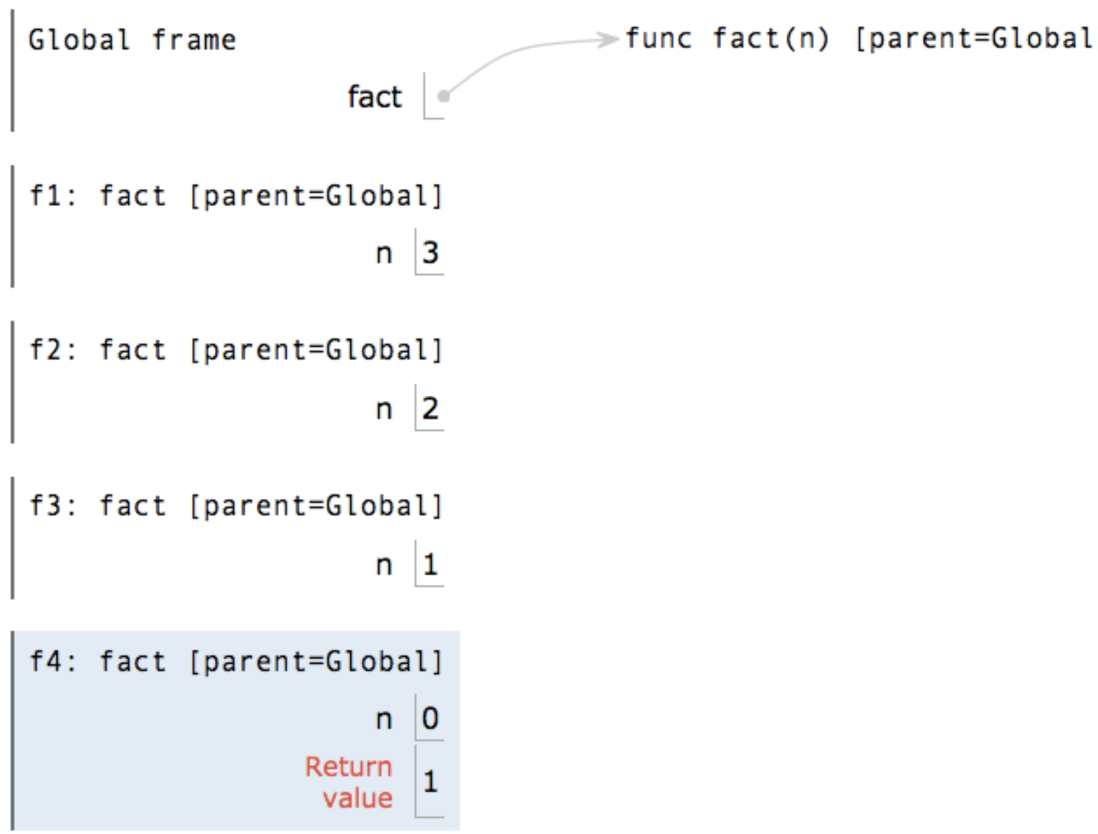

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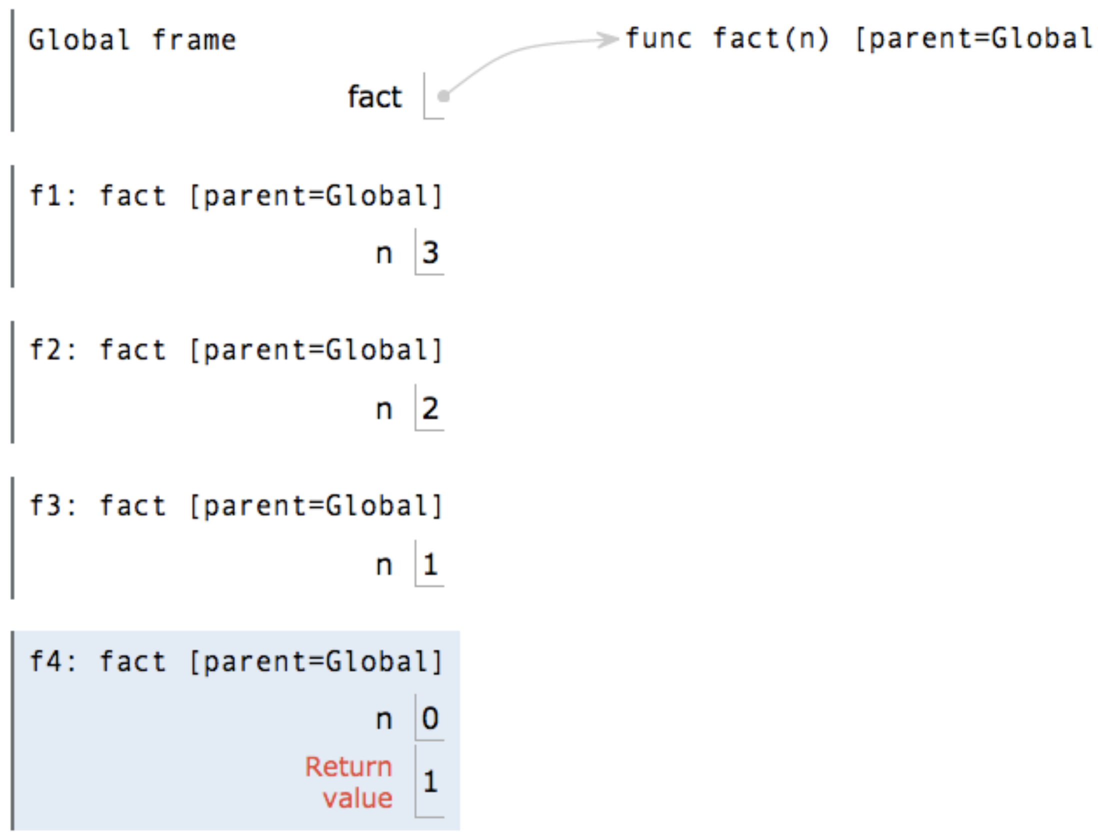

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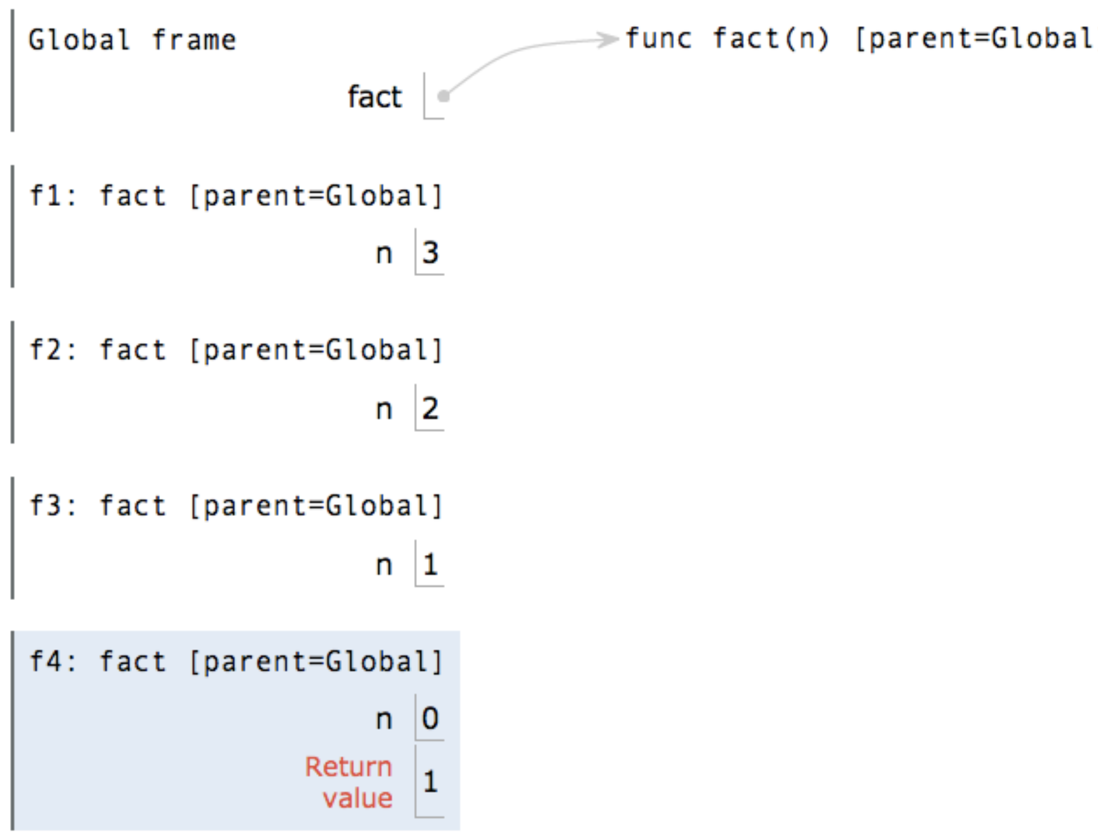

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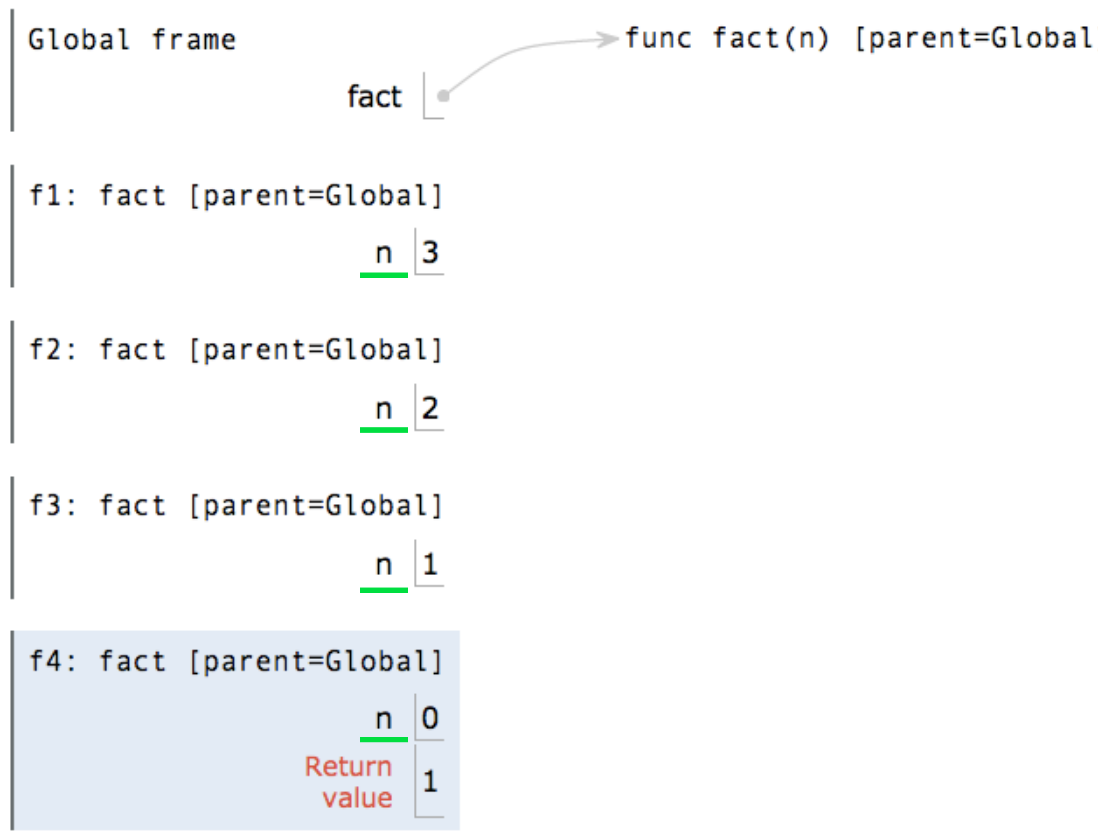

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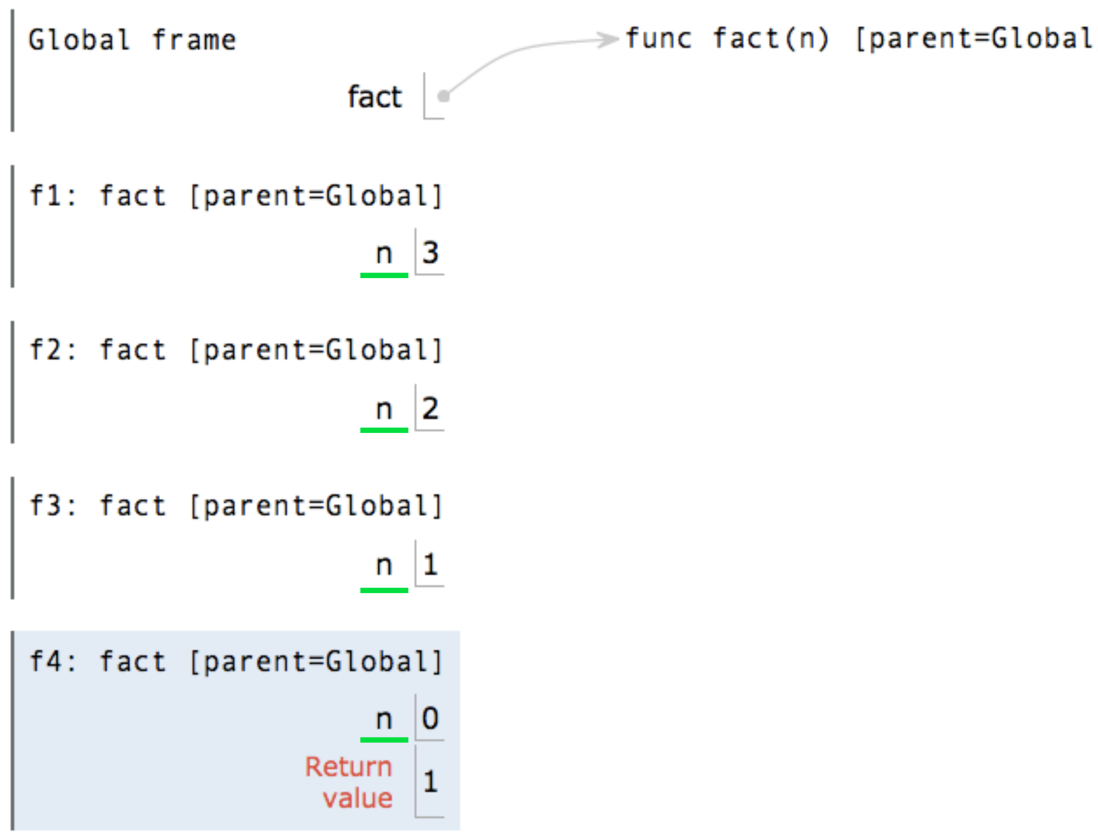

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- Each call to **fact** solves a simpler problem than the last: smaller **n**

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Iteration vs Recursion

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Verifying Recursive Functions

The Recursive Leap of Faith

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Photo by Kevin Lee, Preikestolen, Norway

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3. Assume that **fact(n-1)** is correct
4. Verify that **fact(n)** is correct

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Verifying Digit Sum

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The sum_digits function computes the sum of positive n correctly because:

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sum_digits(n) will add the digit sum of n // 10 to n % 10 **(conclusion)**

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Mutual Recursion

The Luhn Algorithm

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- **Second:** Take the sum of all the digits

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Used to verify credit card numbers

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Recursion and Iteration

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