61A Lecture 5

Announcements

Office Hours: You Should Go!

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You are not alone!

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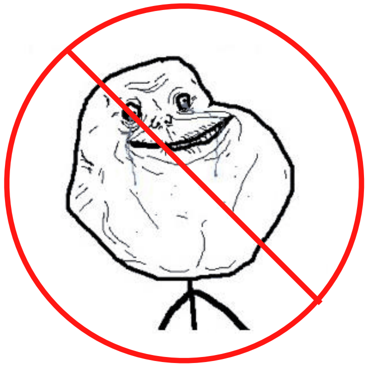

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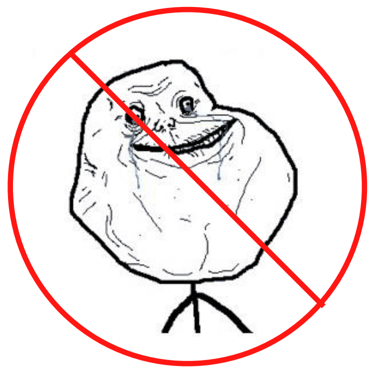

http://cs61a.org/office-hours.html

Environments for Higher-Order Functions

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Environment diagrams describe how higher-order functions work!

(Demo)

Names can be Bound to Functional Arguments

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def apply_twice(f, x):
    return f(f(x))

def square(x):
    return x * x

result = apply_twice(square, 2)
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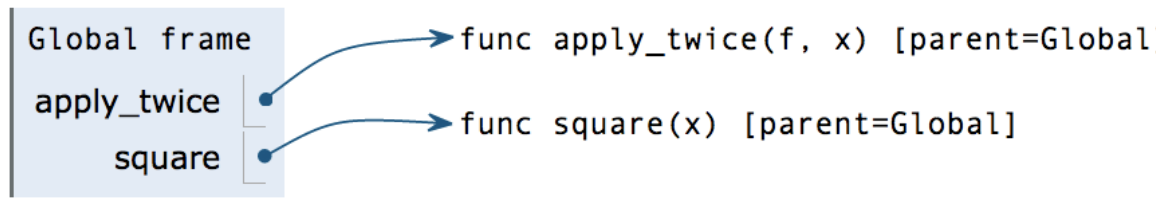

Interactive Diagram

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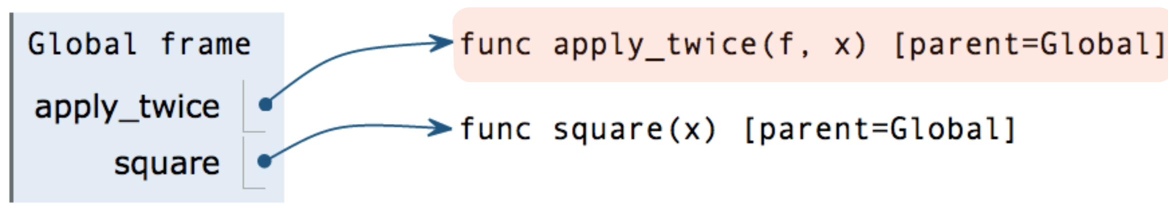

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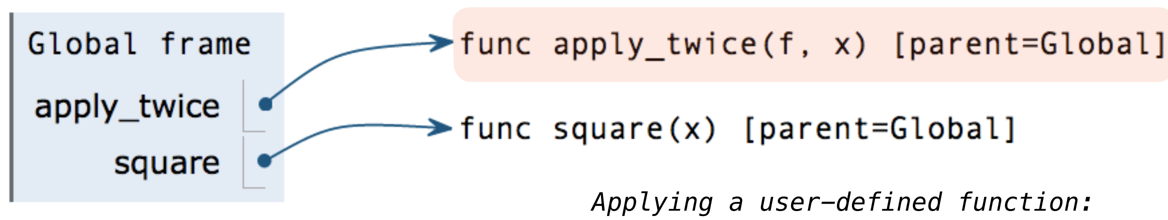

Applying a user-defined function:

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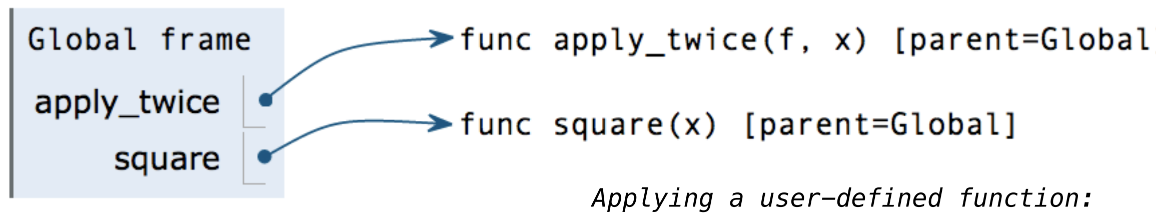

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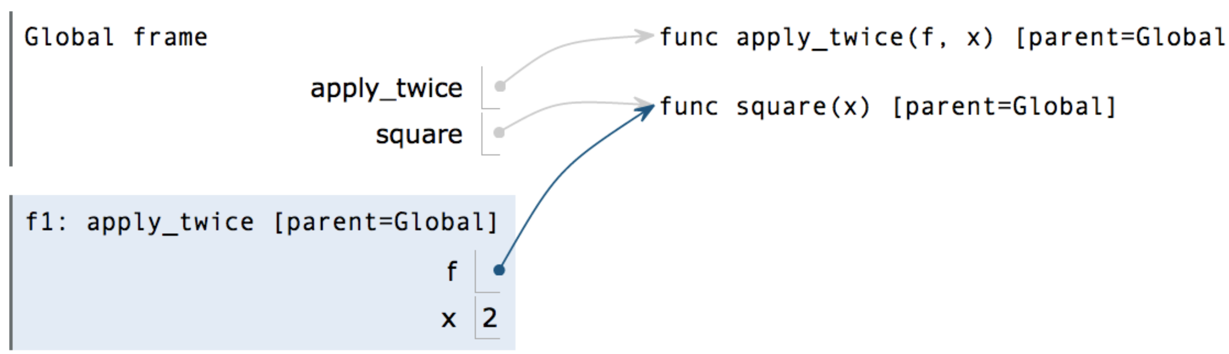

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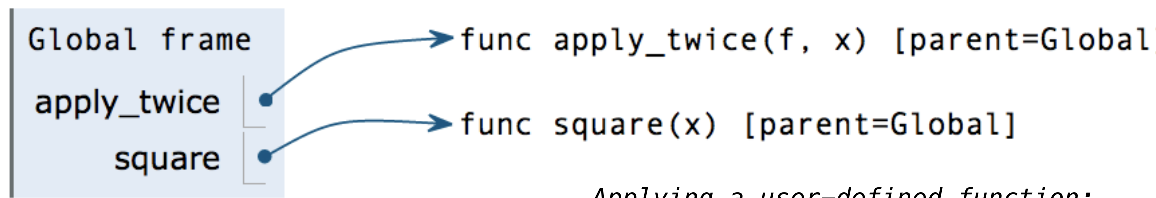

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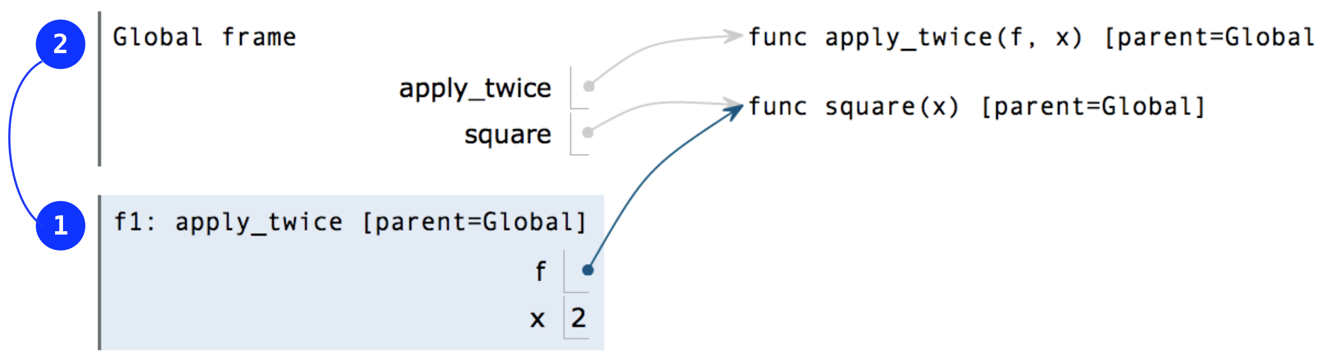

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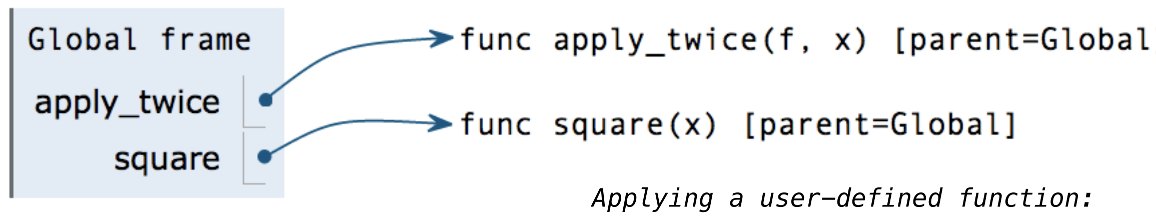

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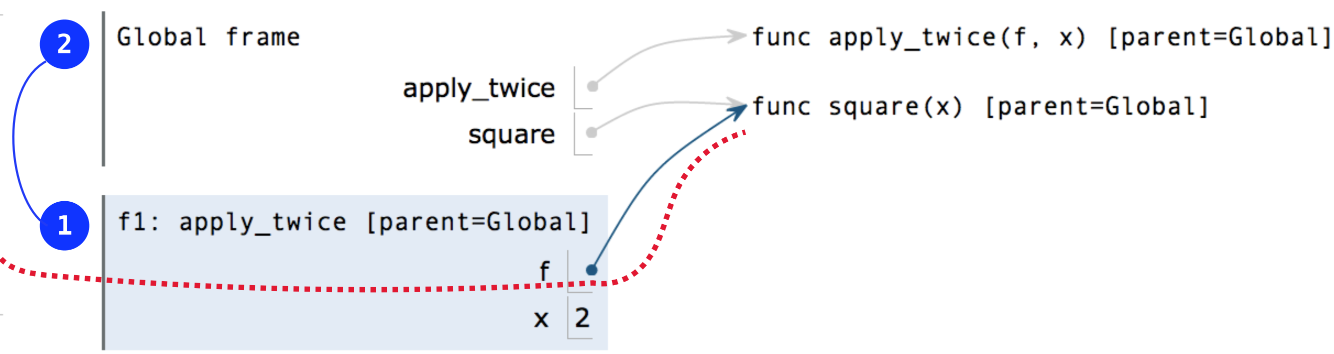

Interactive Diagram

Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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def make_adder(n):
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add_three = make_adder(3)
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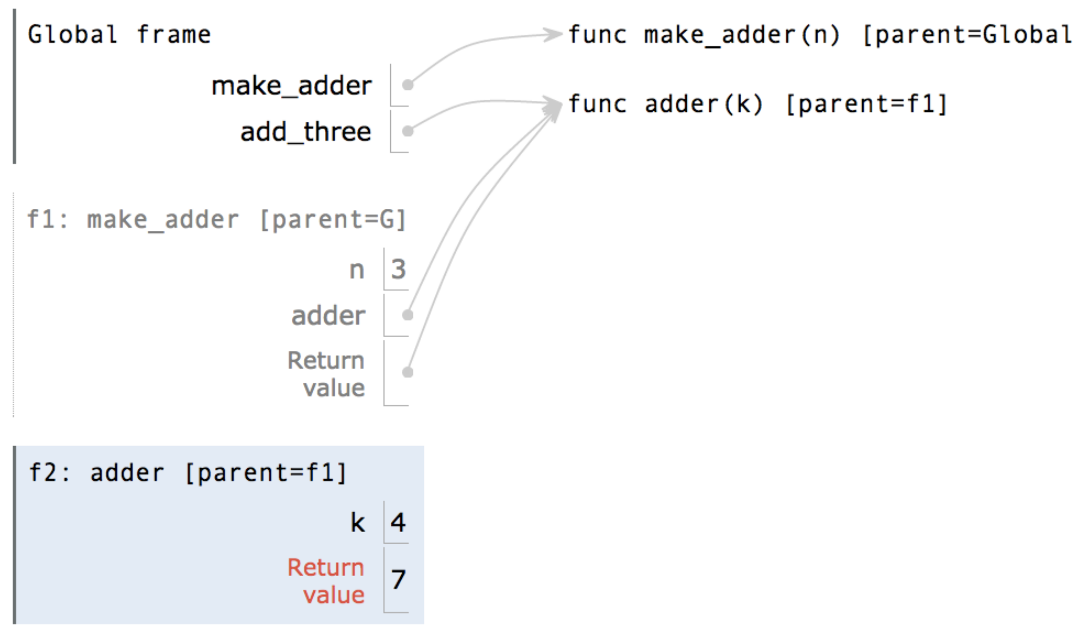

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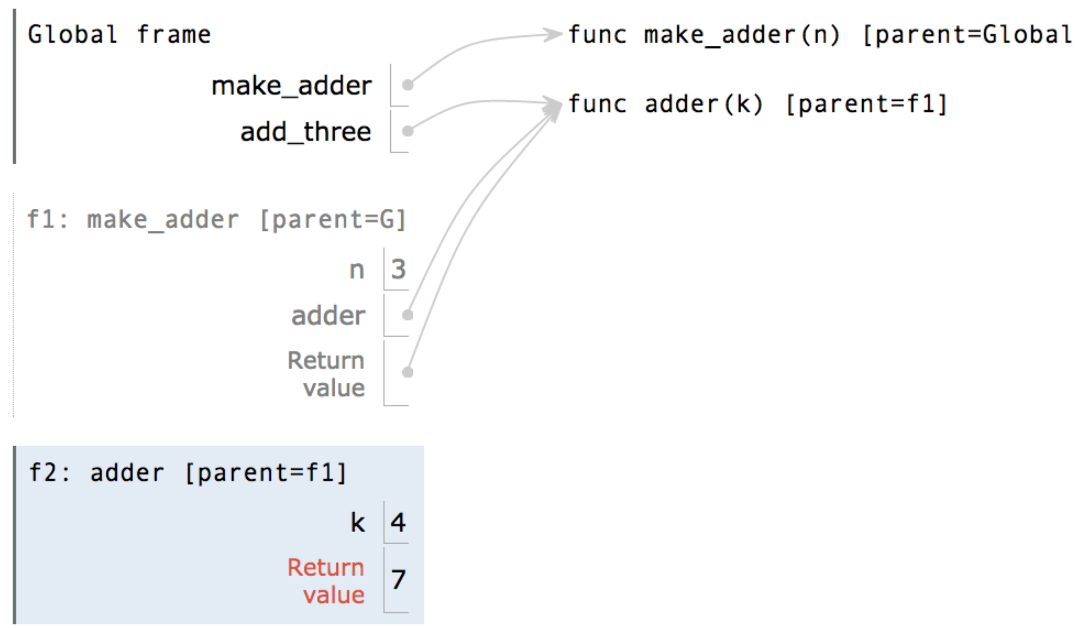

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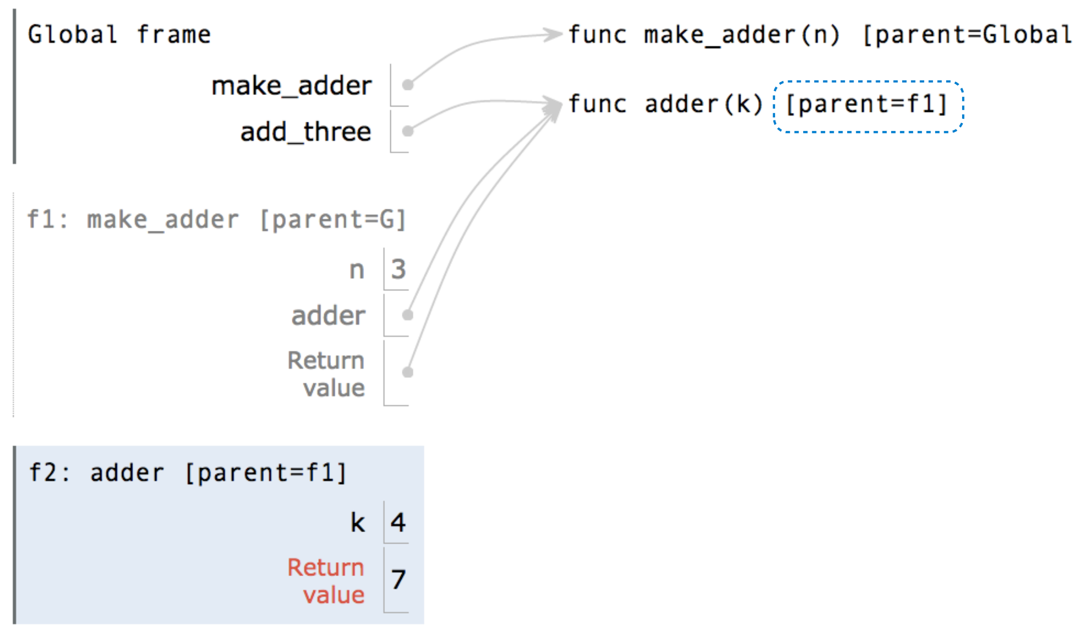

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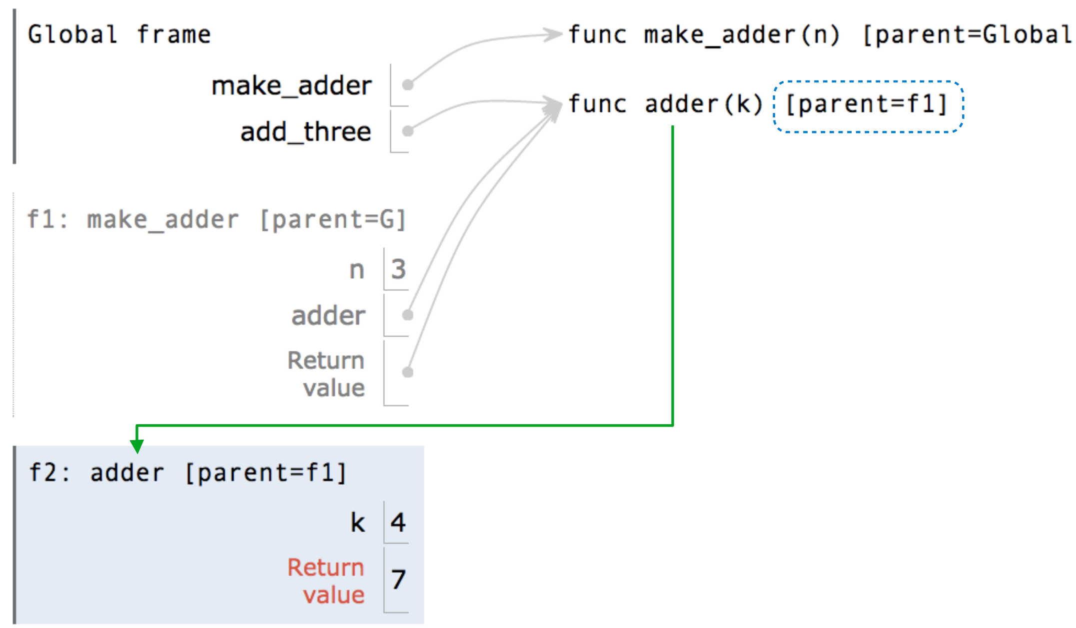

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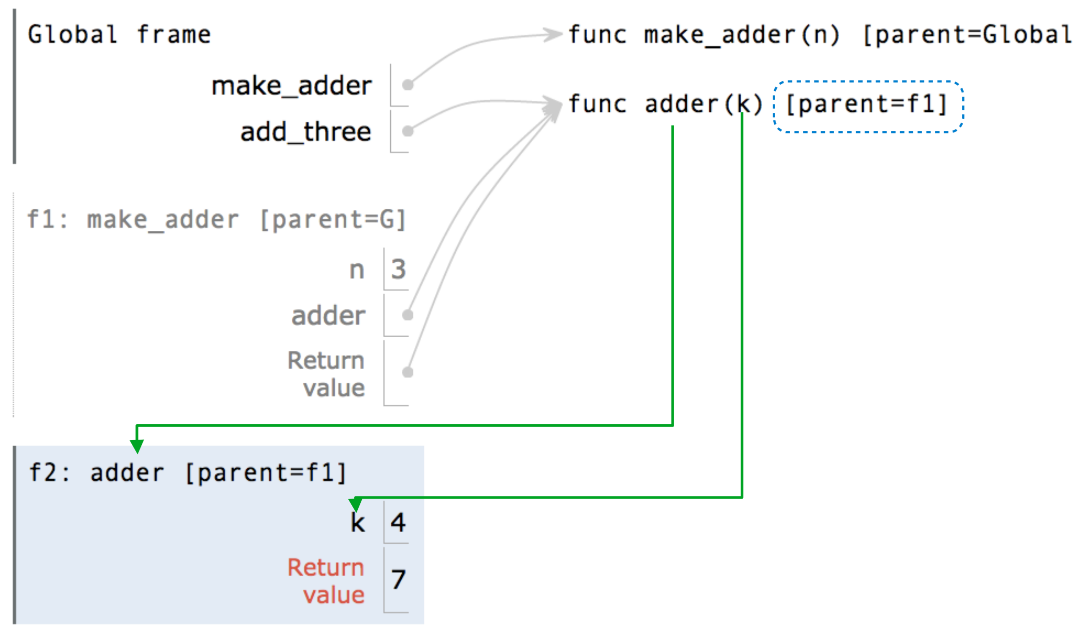

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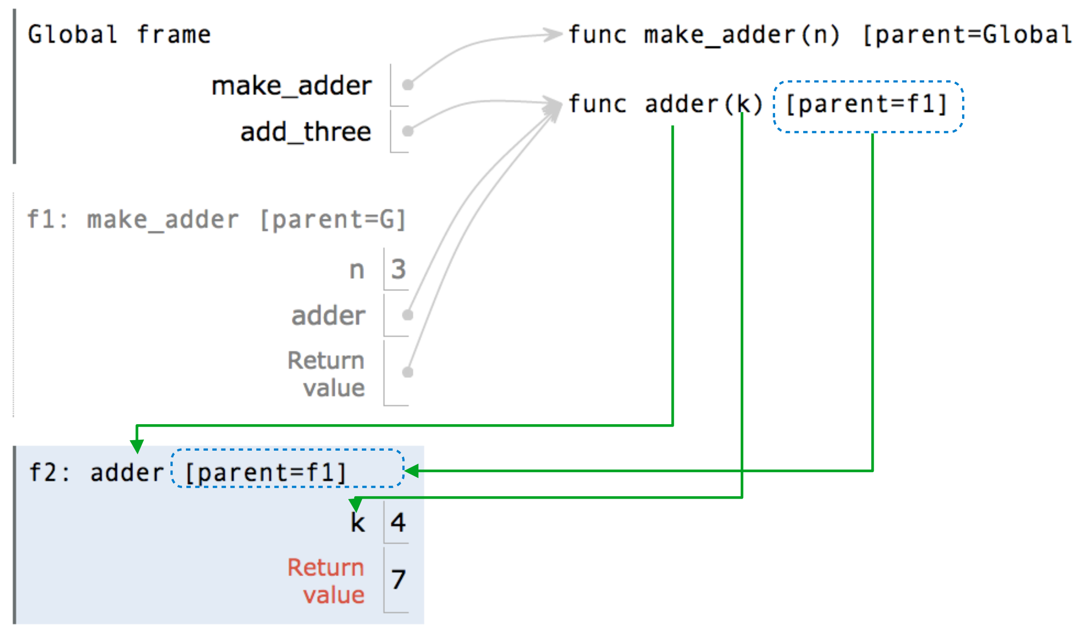

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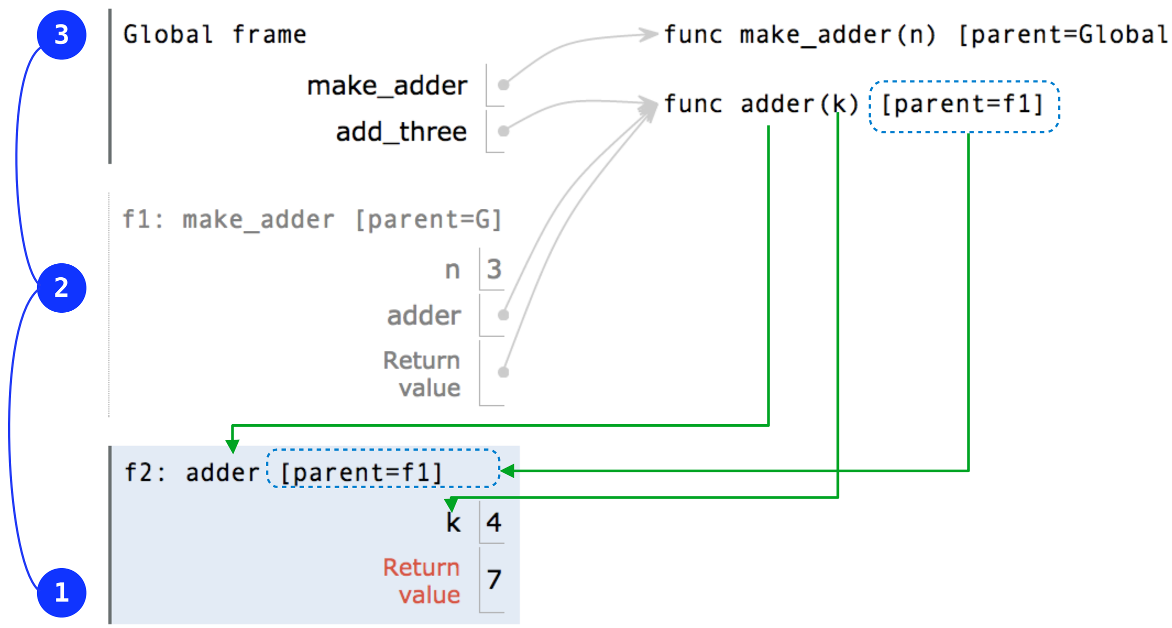

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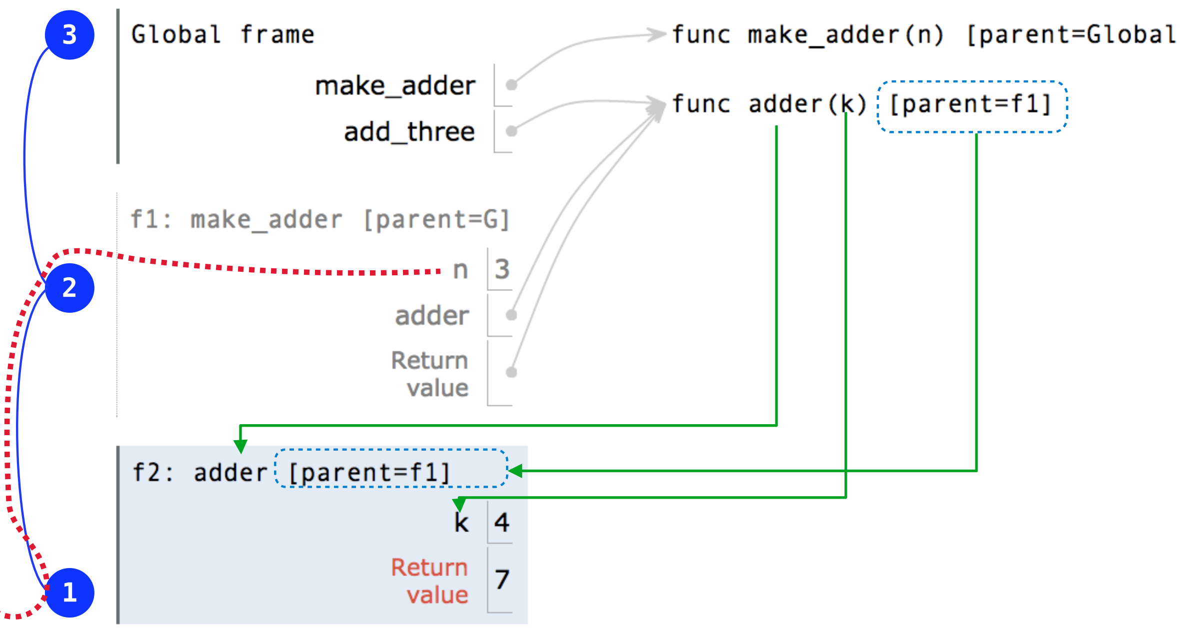

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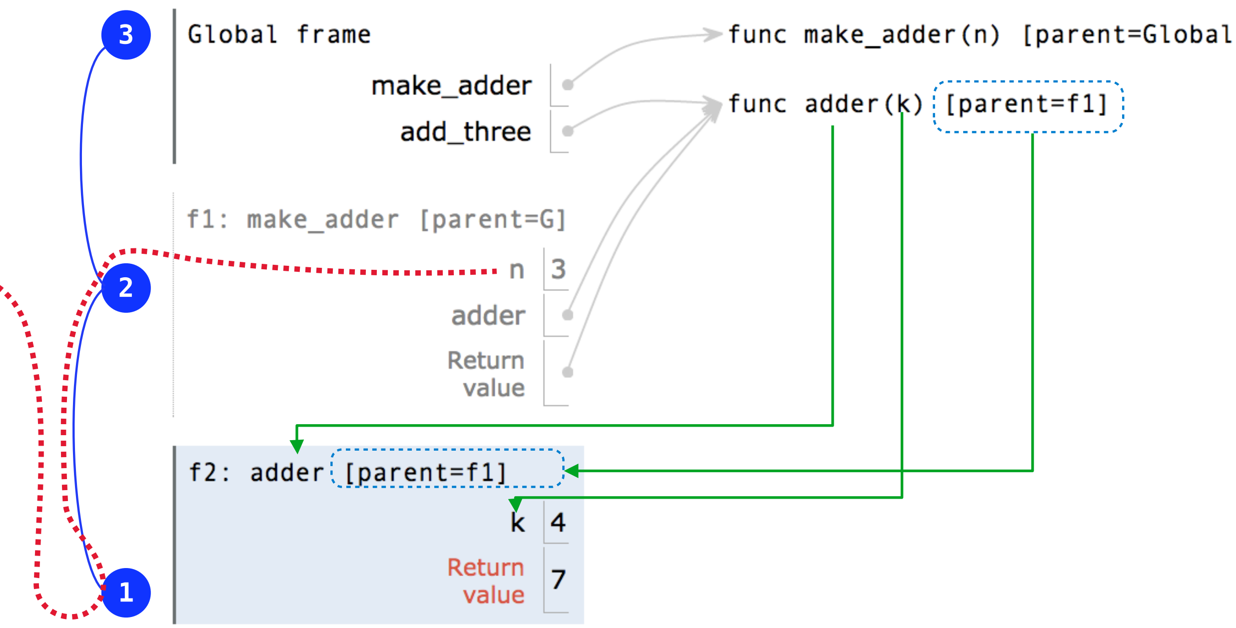

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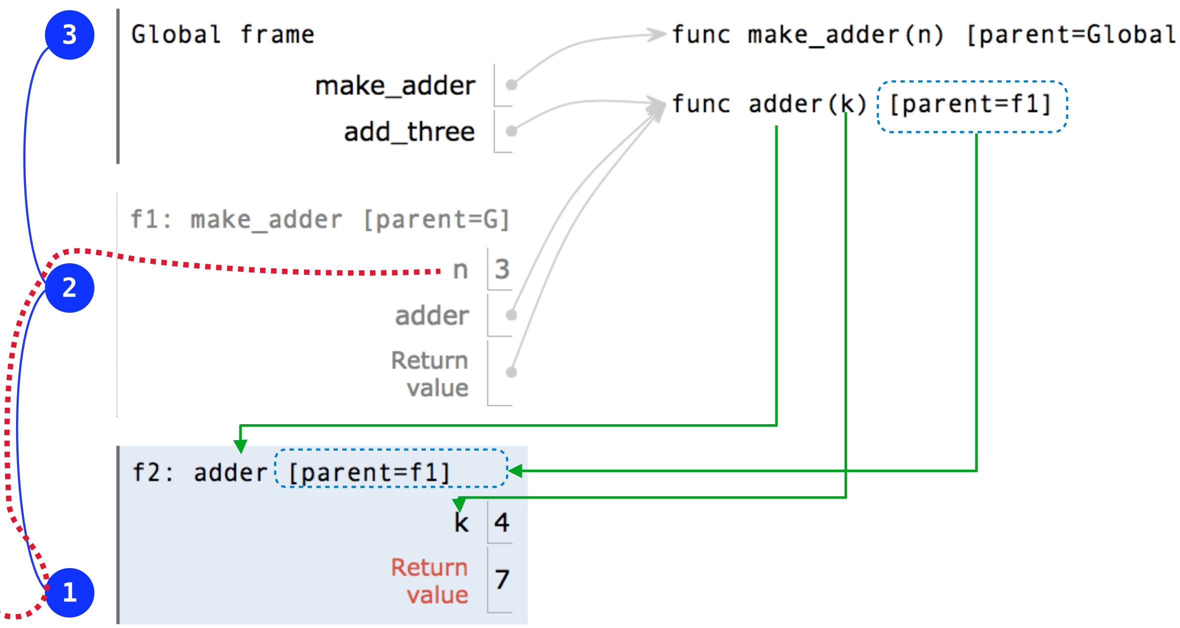

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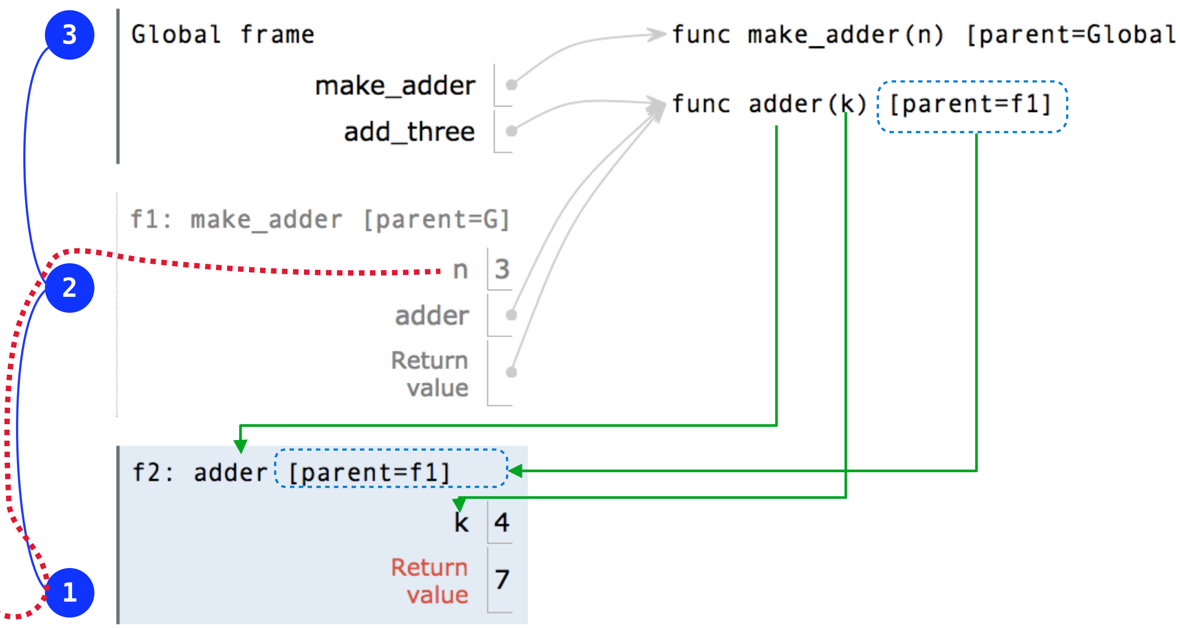

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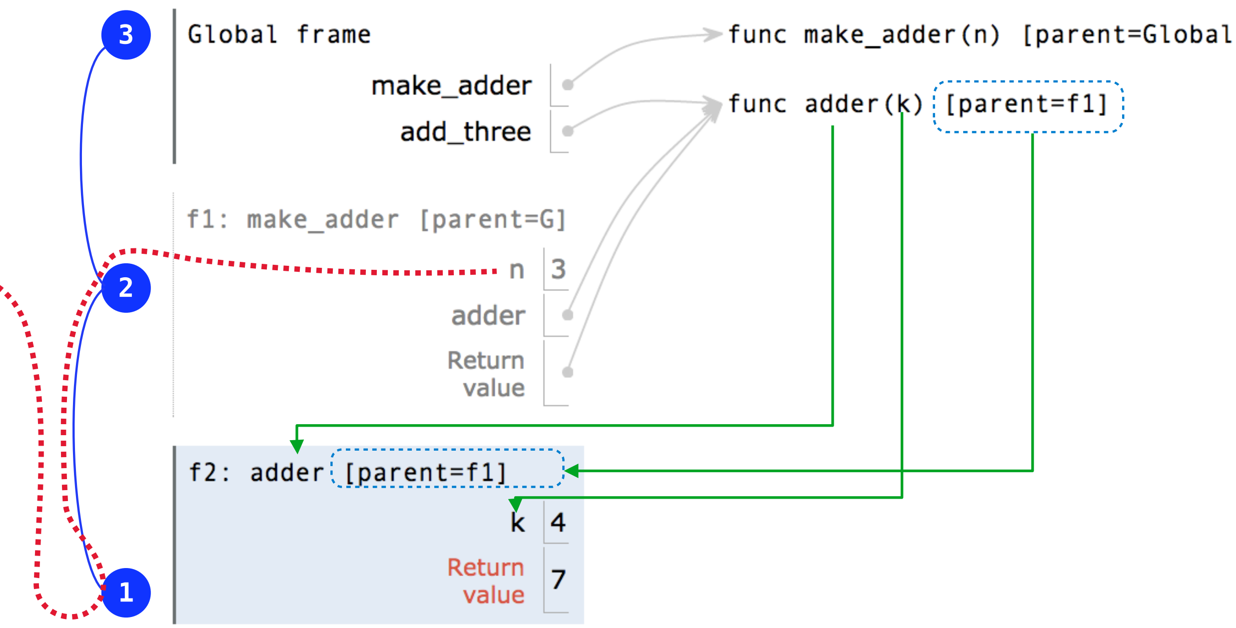

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Interactive Diagram

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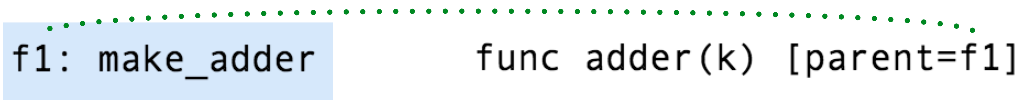

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3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions

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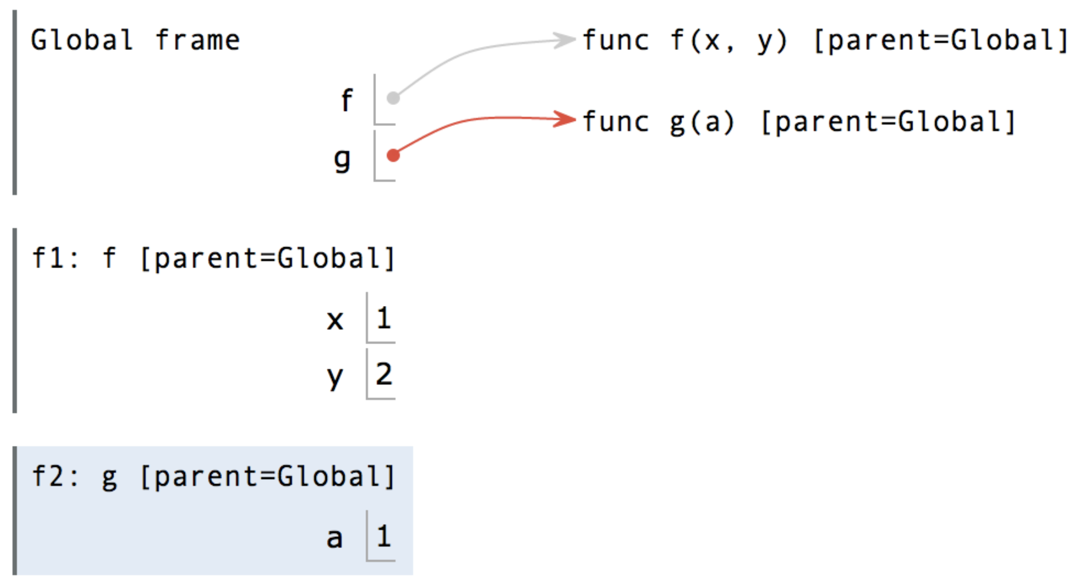
def f(x, y):
    return g(x)

def g(a):
    return a + y

result = f(1, 2)
```



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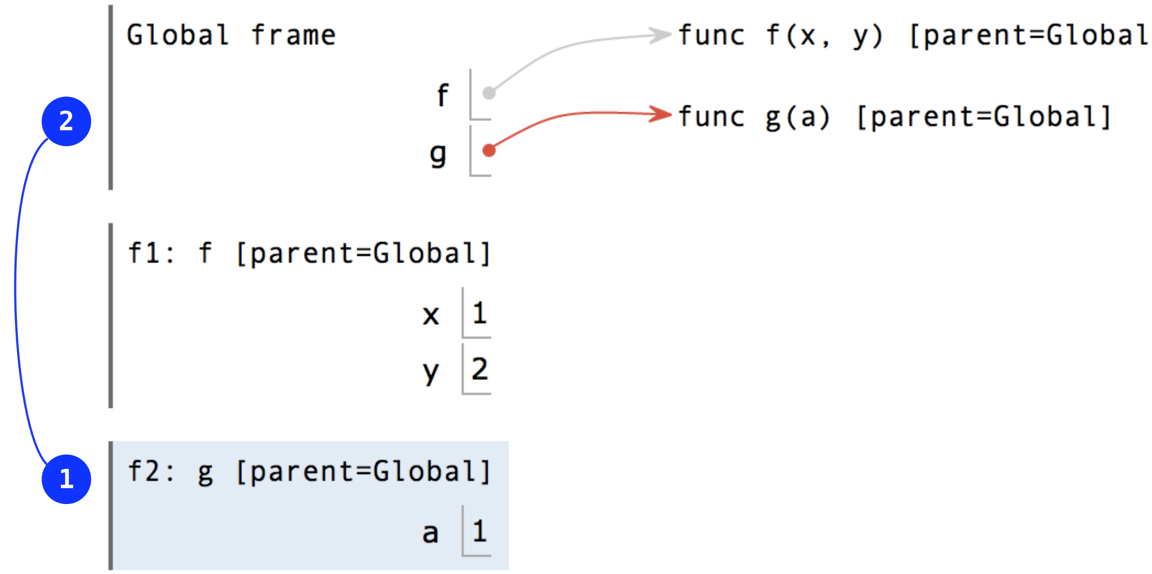

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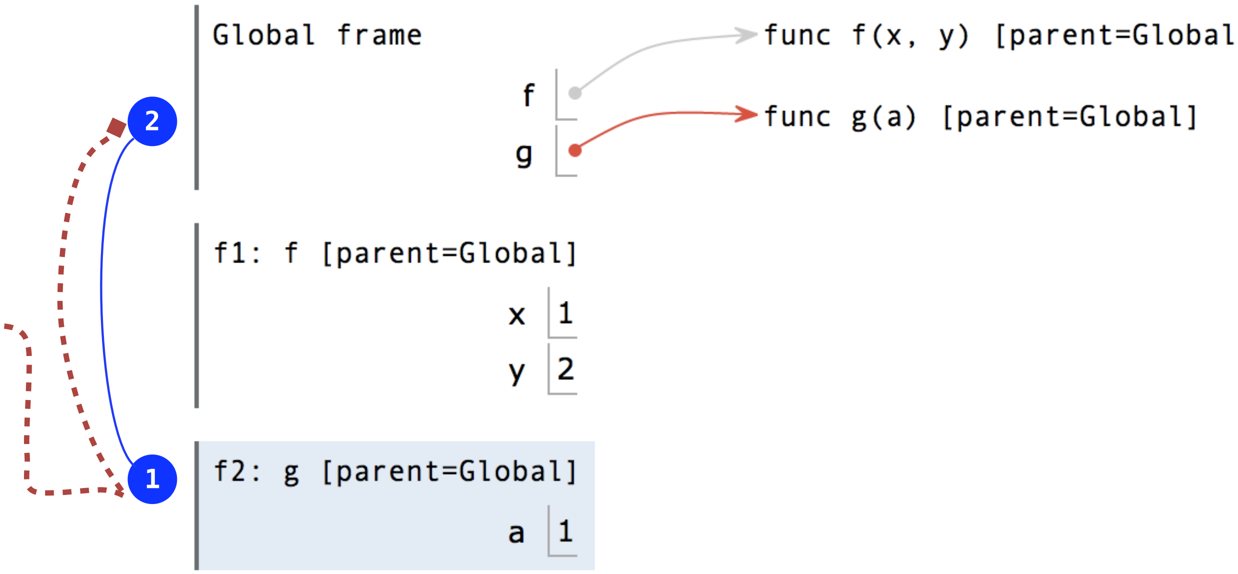

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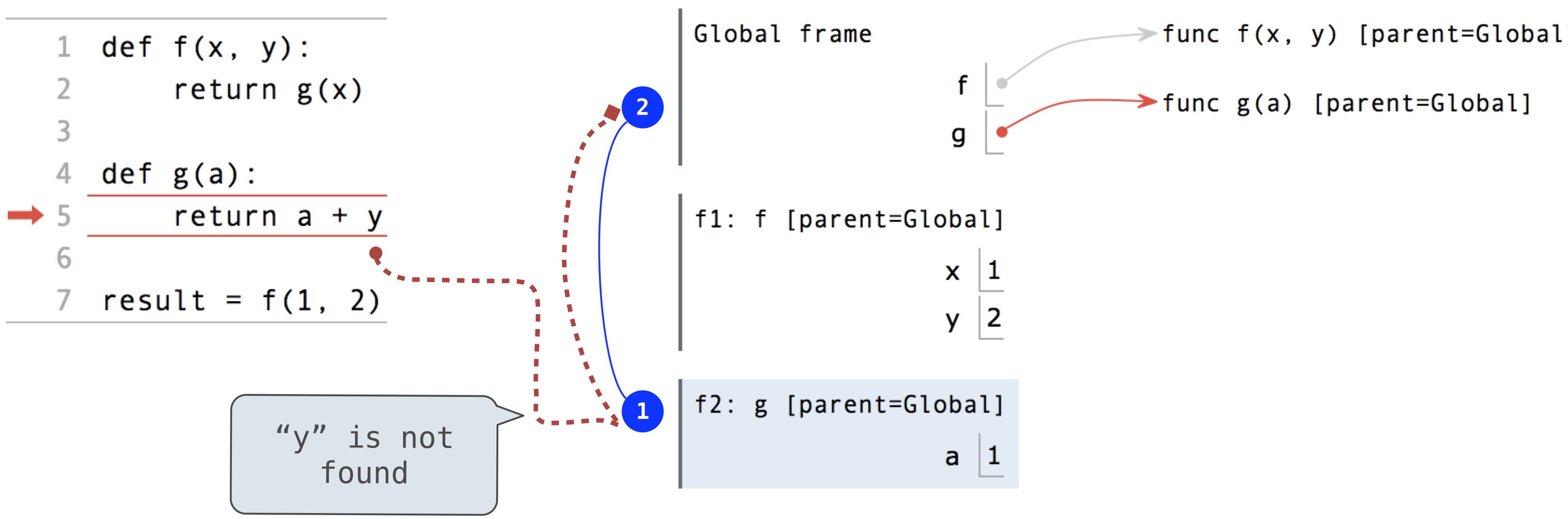

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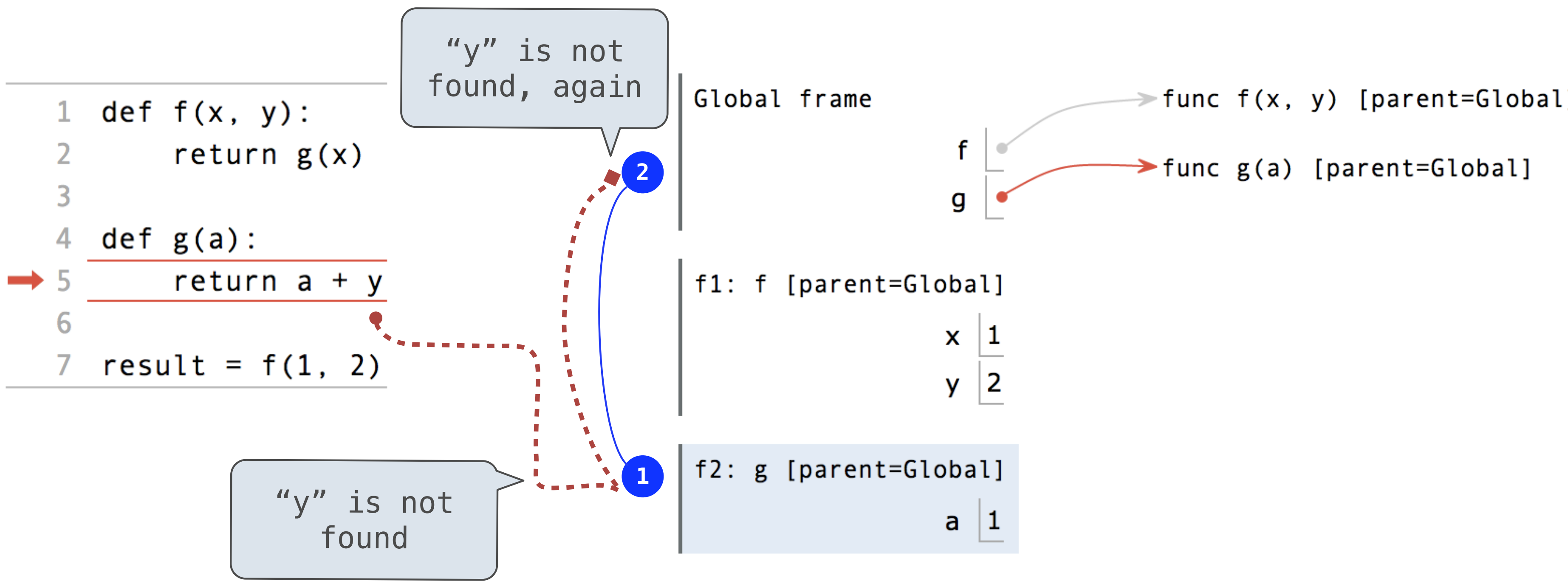

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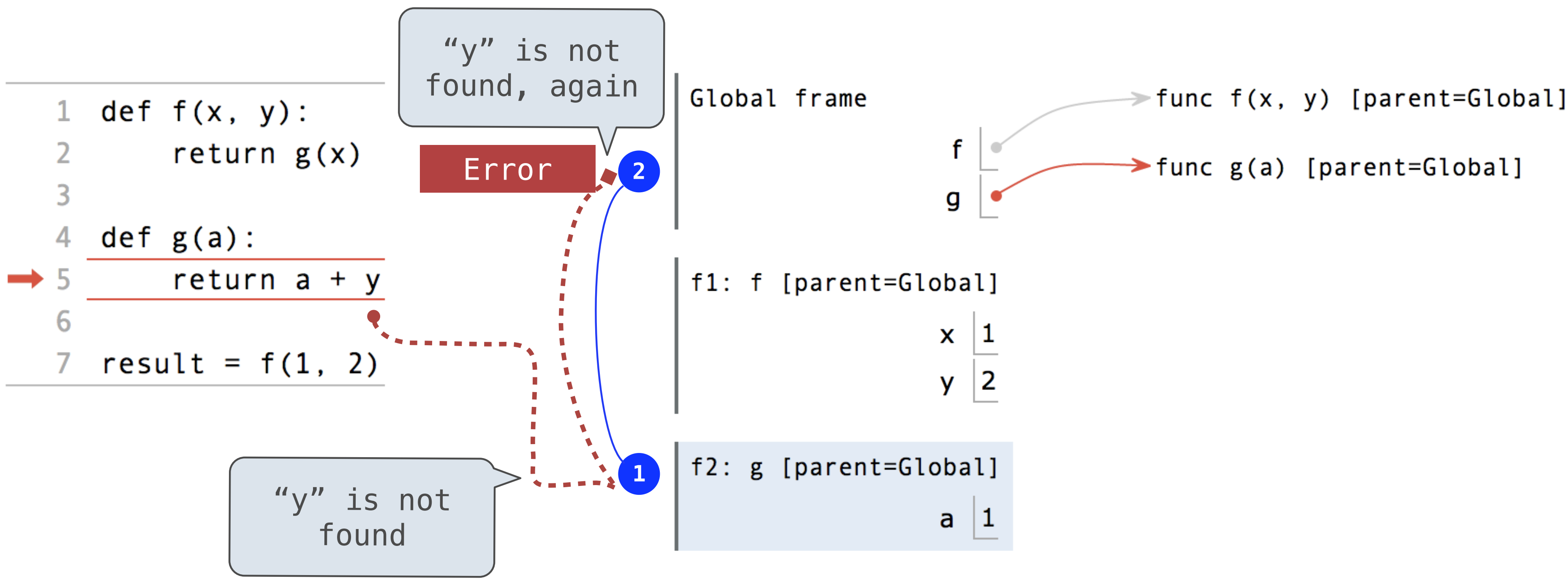

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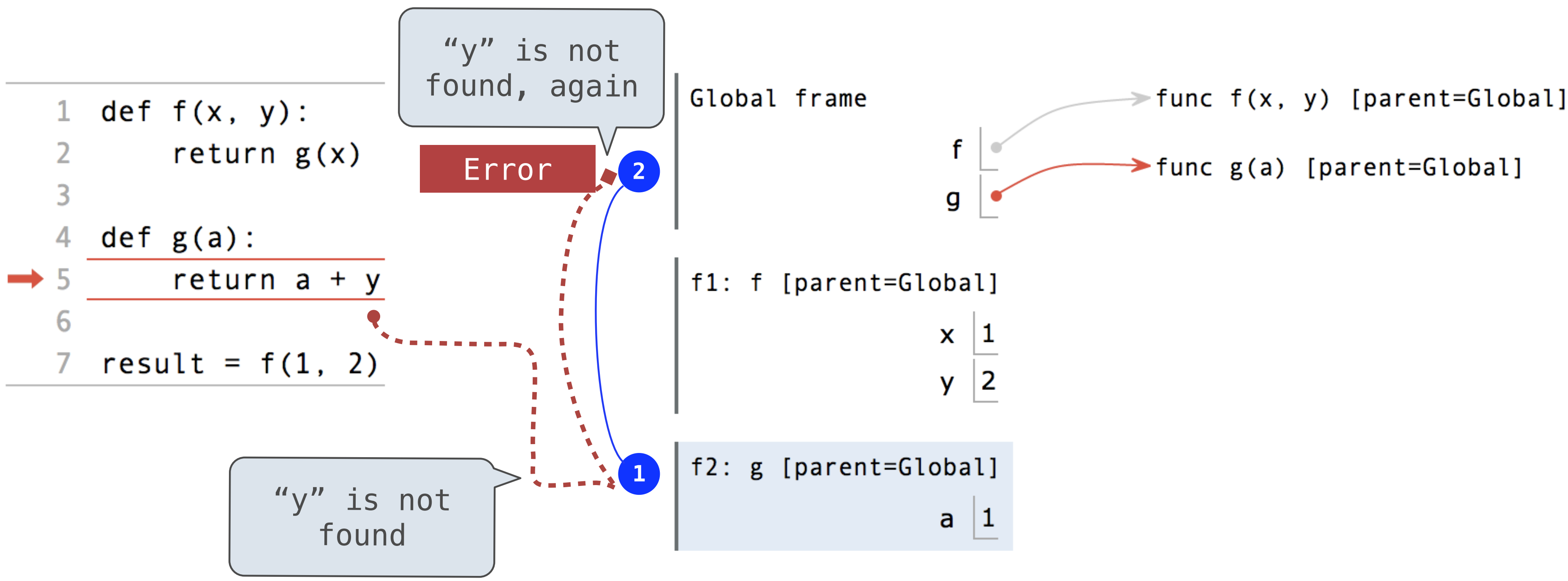

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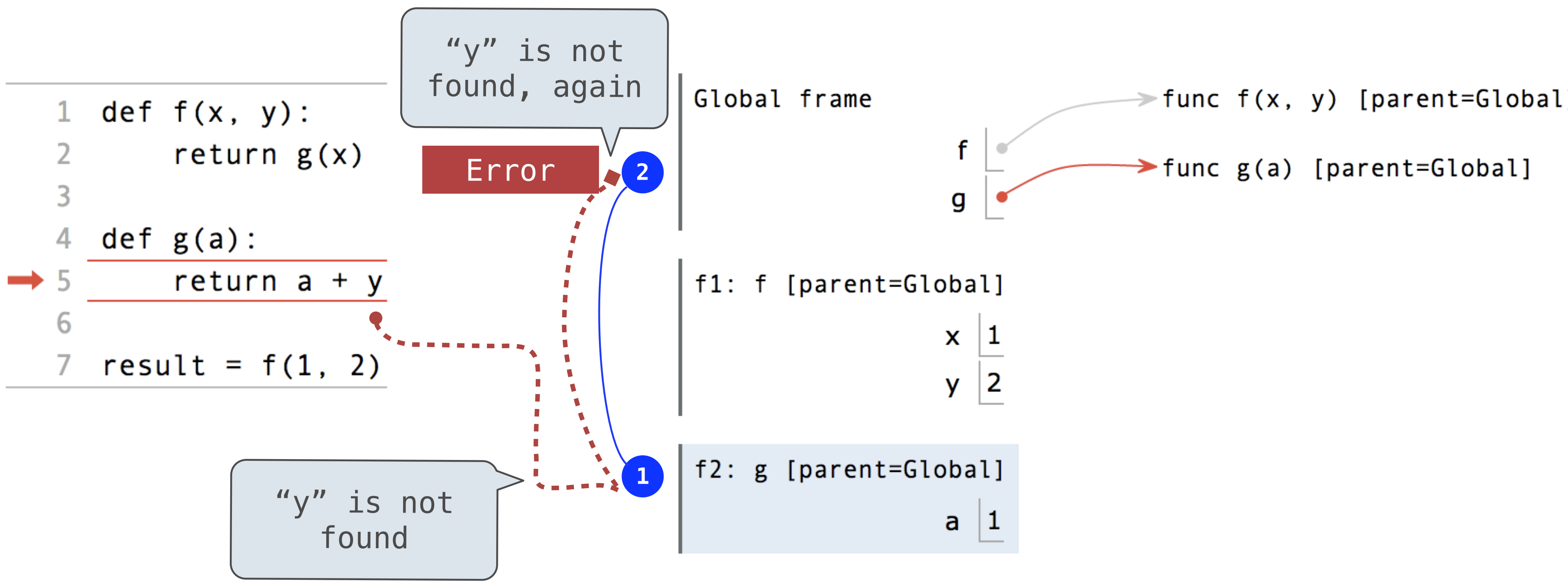

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

Interactive Diagram

Function Composition

(Demo)

The Environment Diagram for Function Composition

```
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
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def compose1(f, g):
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compose1(square, make_adder(2))(3)
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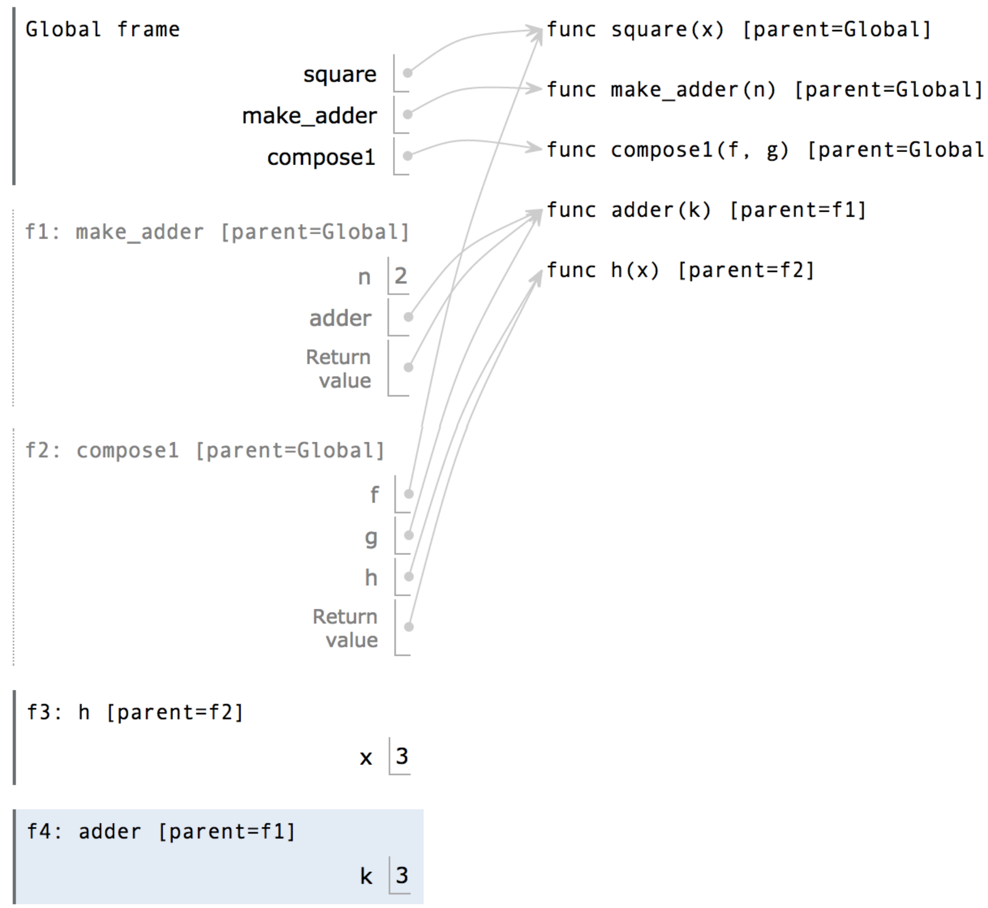

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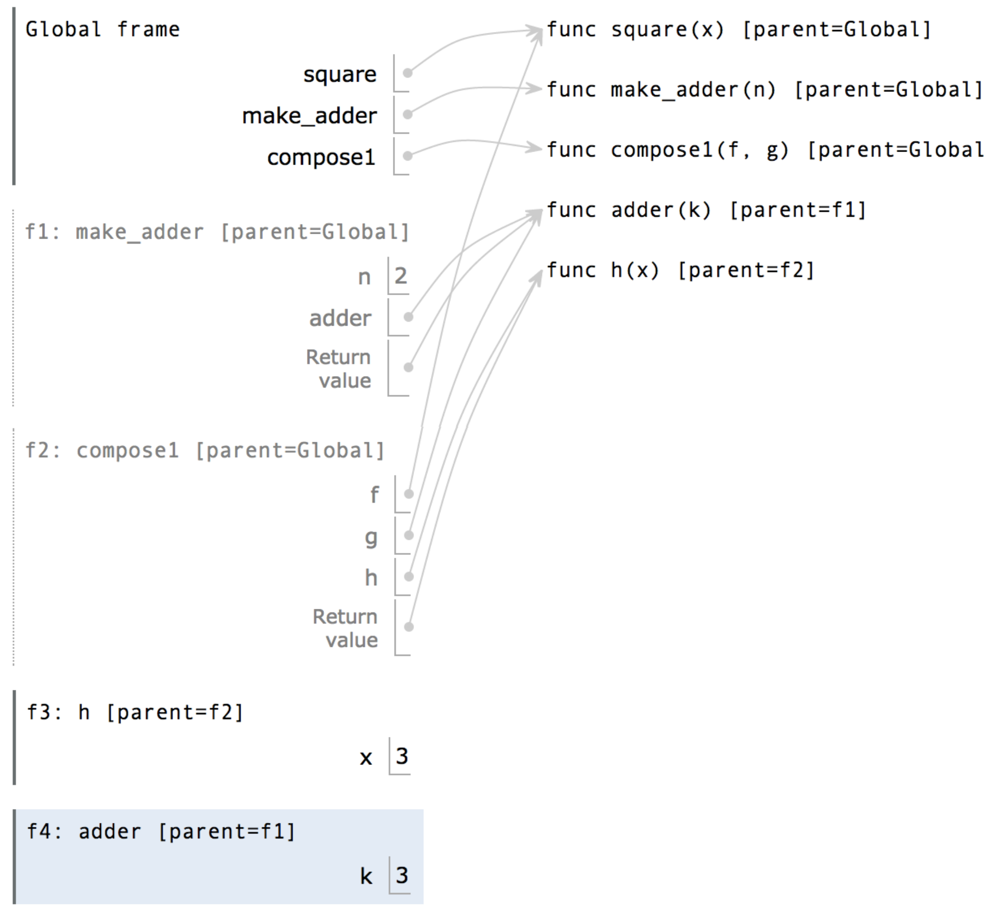

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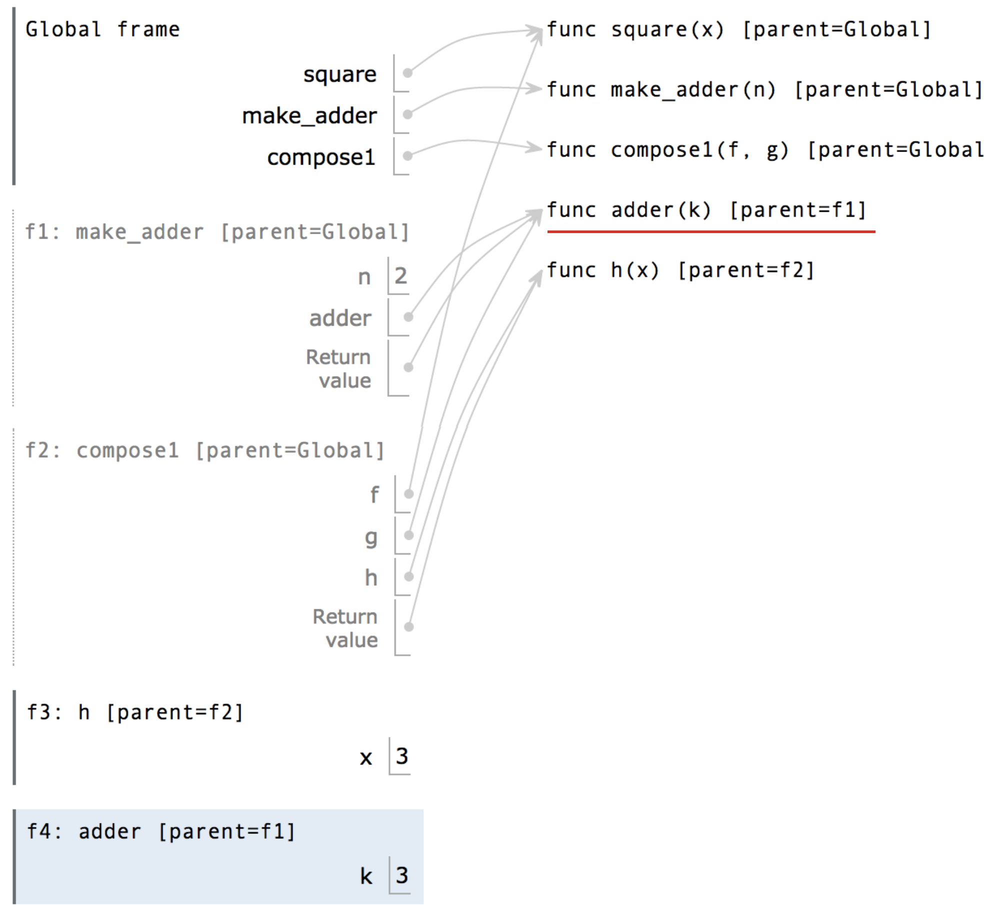

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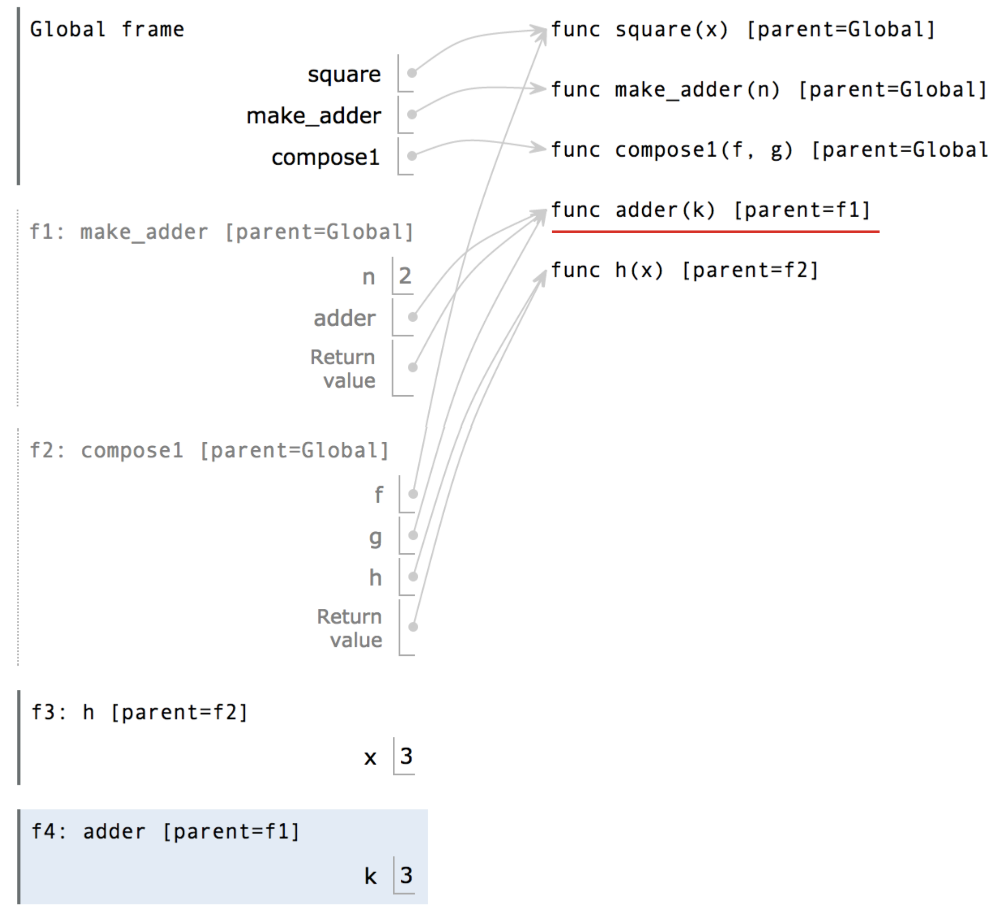

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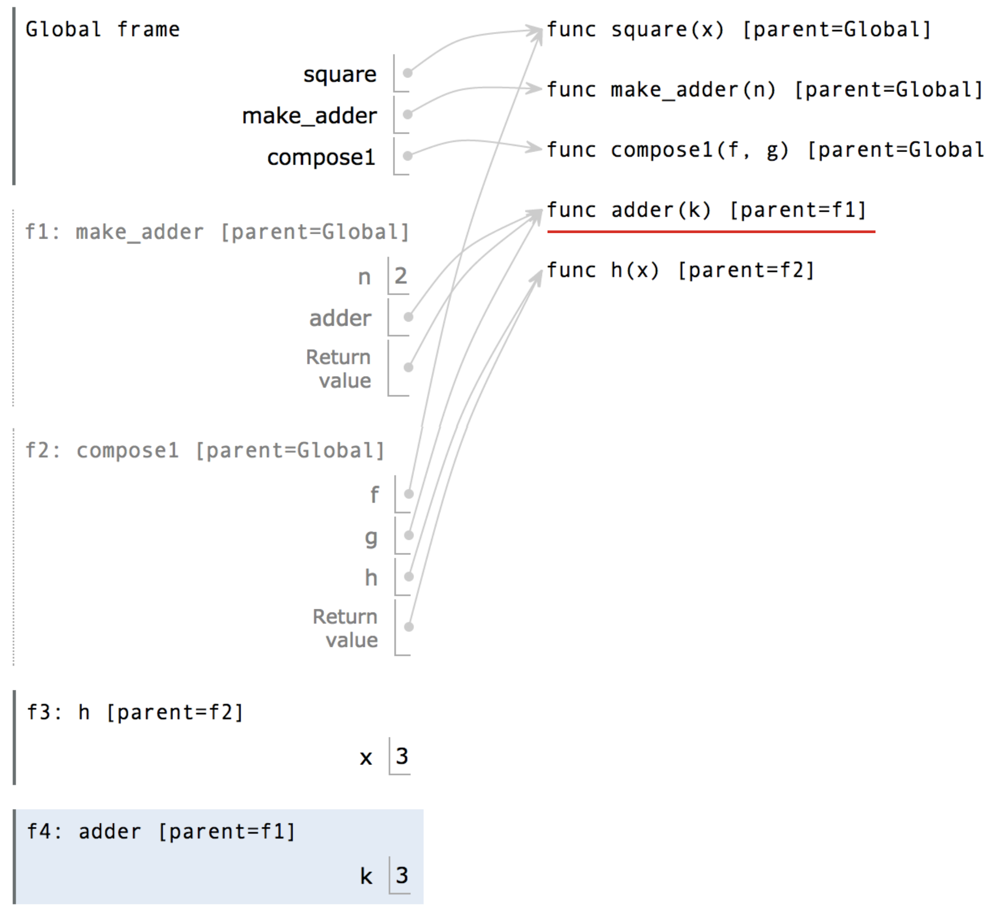

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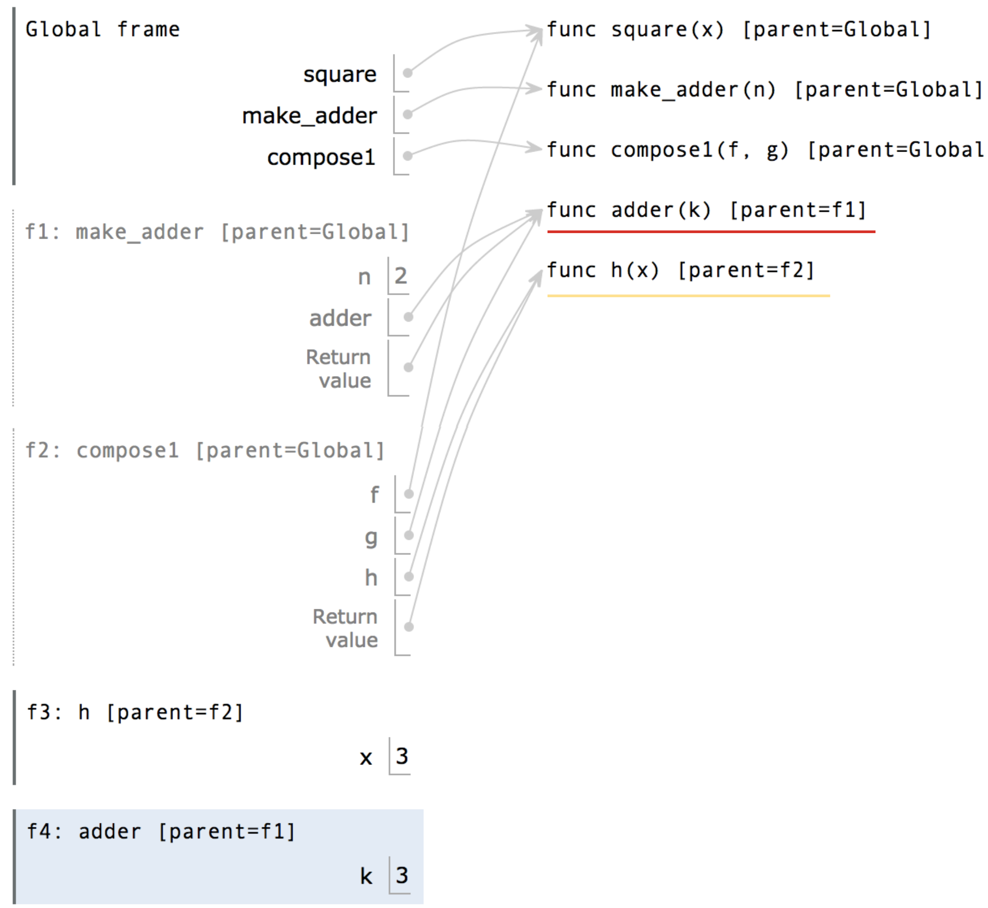

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def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
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def compose1(f, g):
    def h(x):
        return f(g(x))
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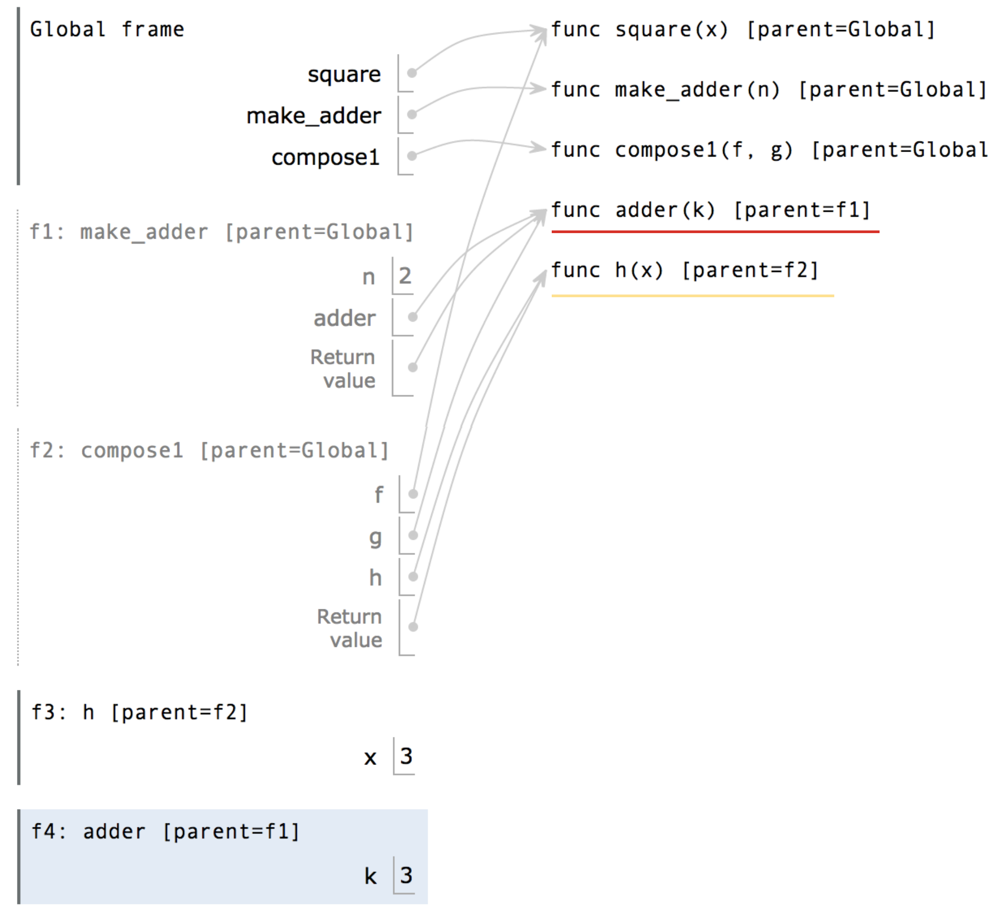
compose1(square, make_adder(2))(3)
```

Return value of make_adder is an argument to compose1



Interactive Diagram

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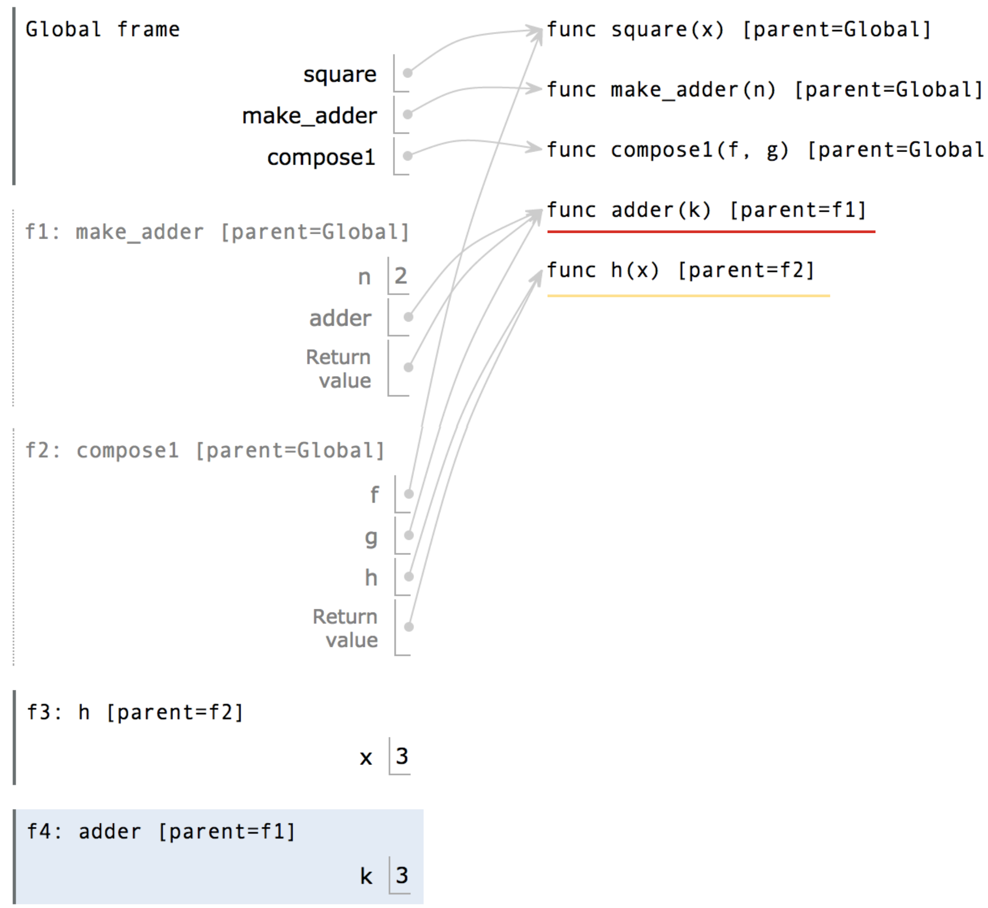

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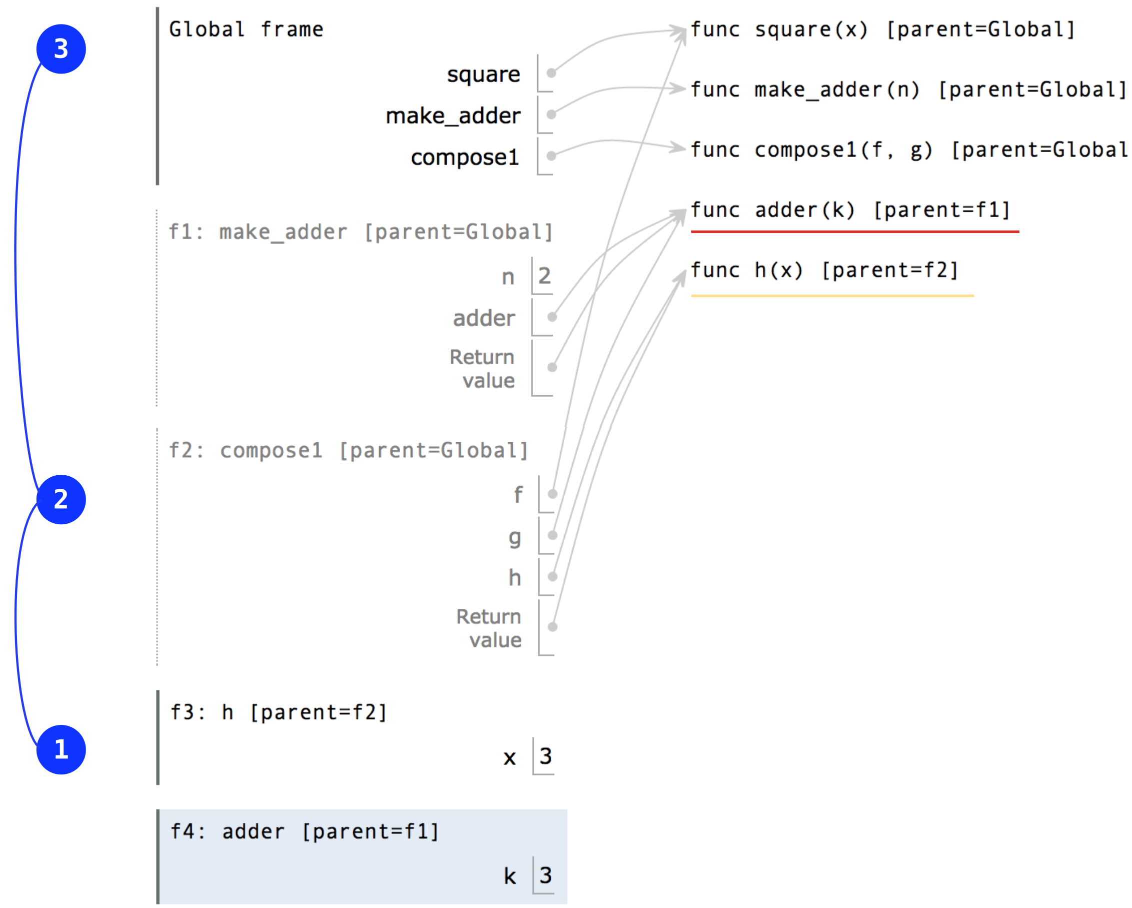

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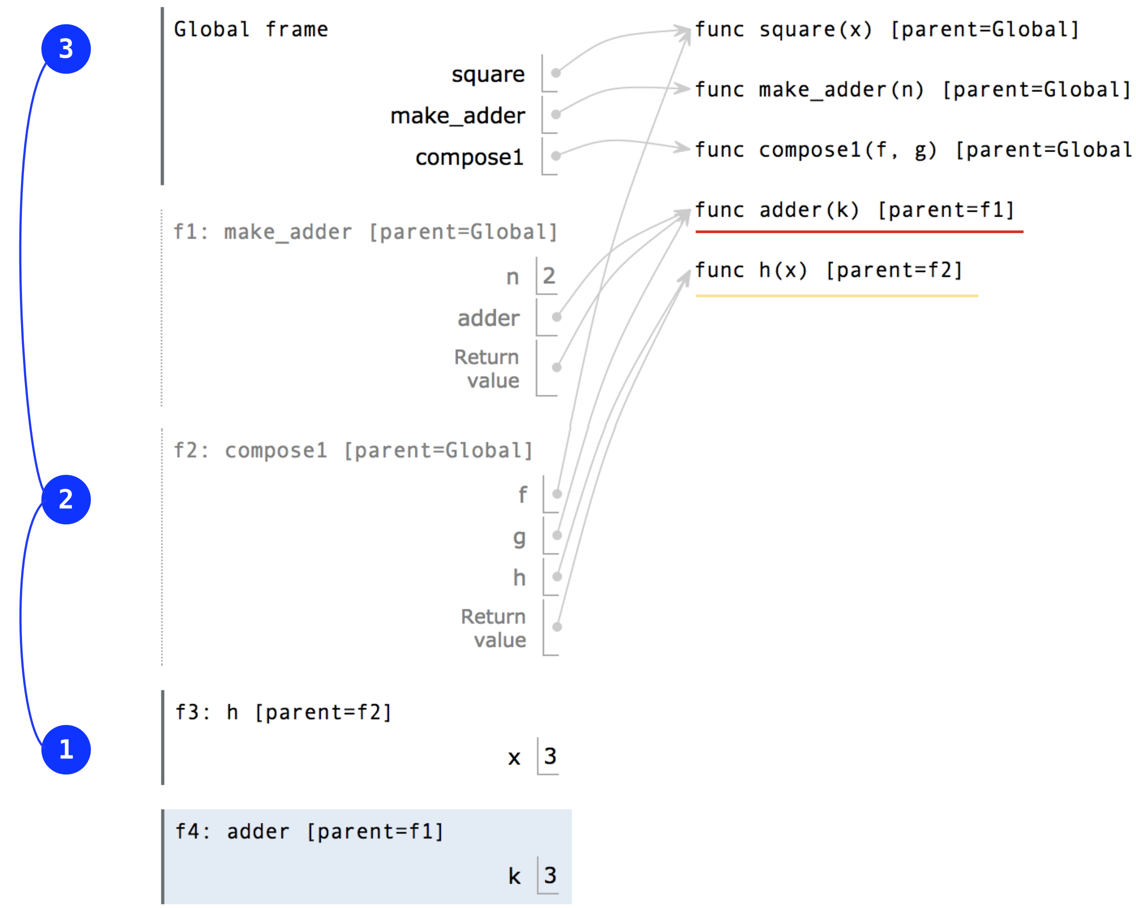

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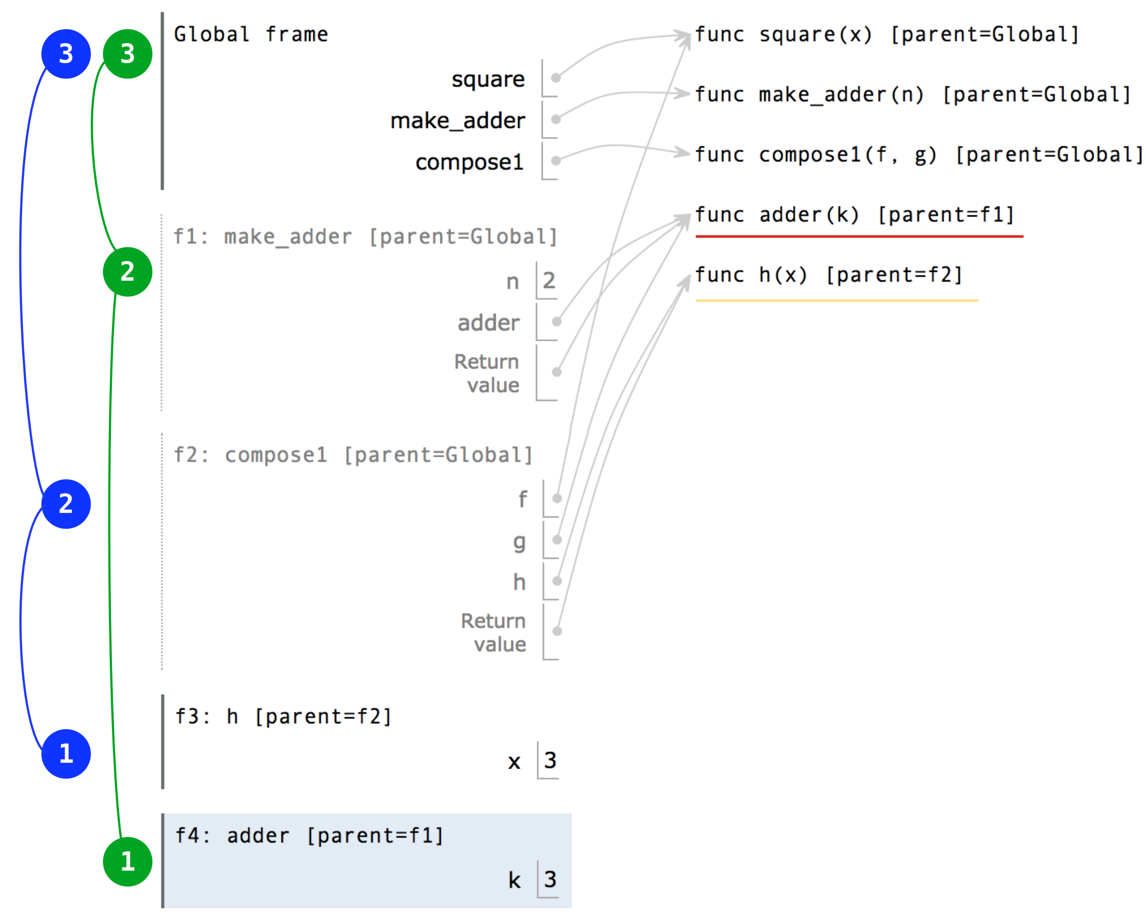

Interactive Diagram

Lambda Expressions

(Demo)

Lambda Expressions

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>>> x = 10
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with formal parameter x

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Lambda expressions are not common in Python, but important in general

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Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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VS

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- Both create a function with the same domain, range, and behavior.

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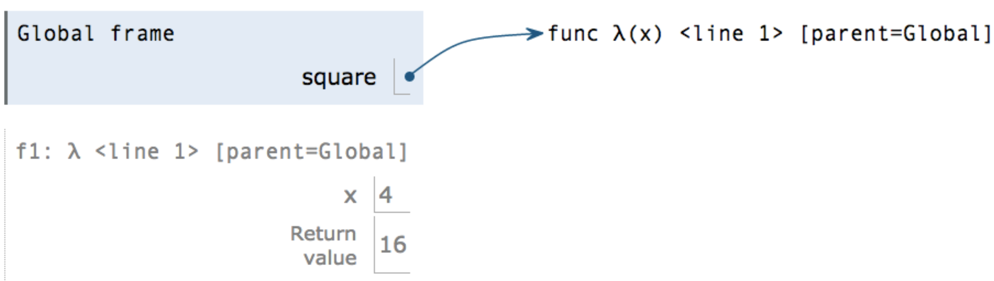

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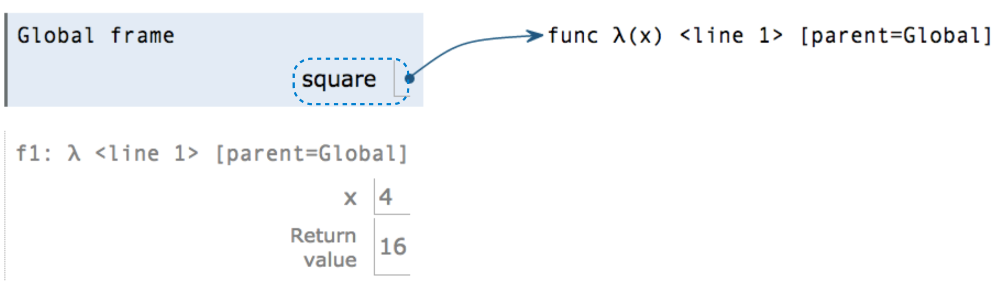

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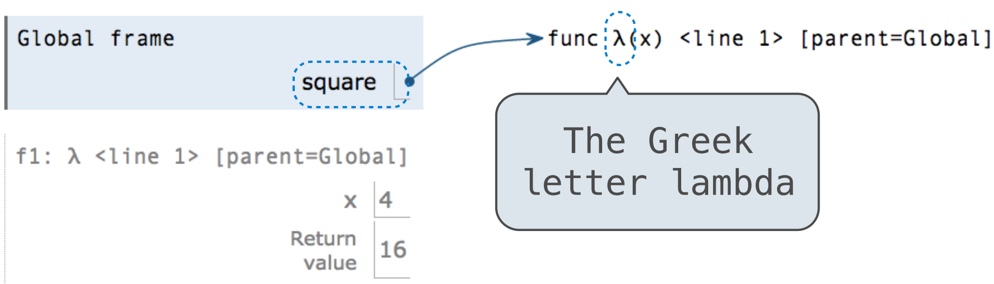

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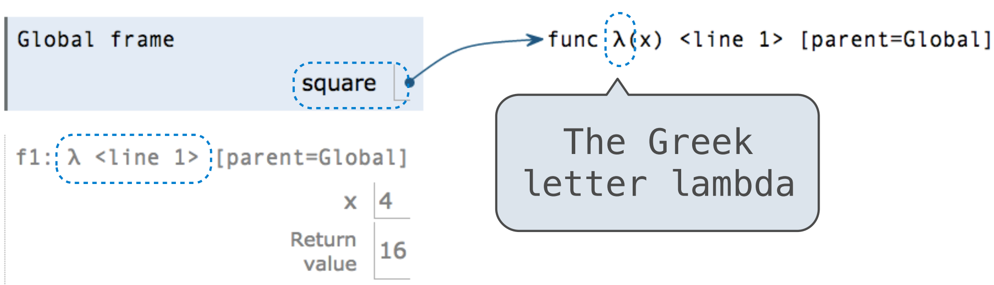

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