61A Lecture 23

Wednesday, October 30

Announcements

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  - Mean: 30
  - Solutions will be posted and exams distributed soon.

Scheme

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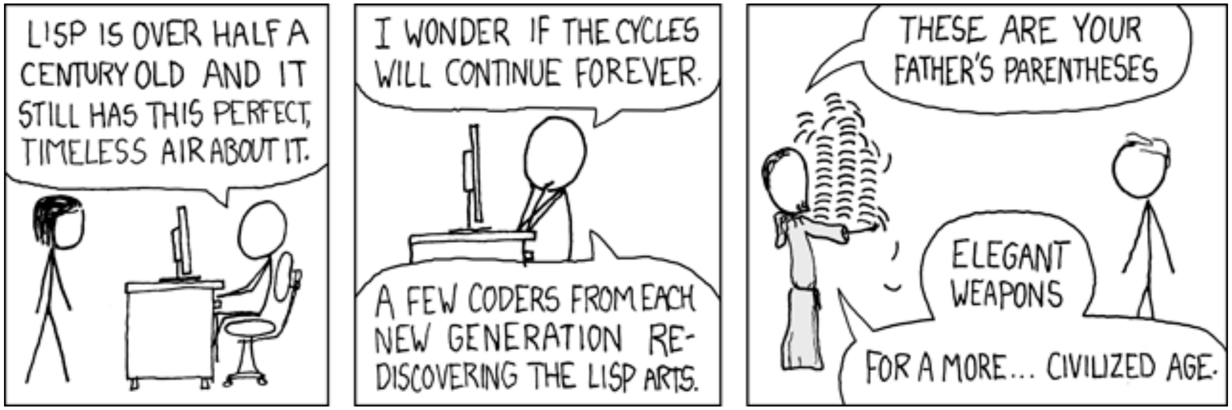

http://imgs.xkcd.com/comics/lisp_cycles.png

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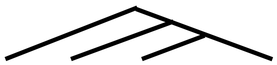

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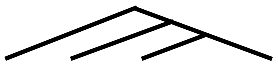

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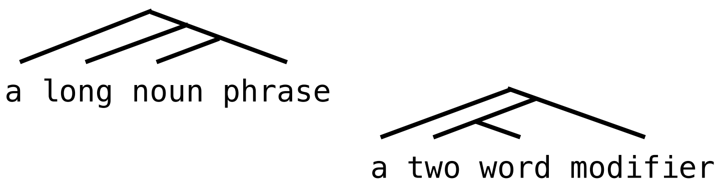

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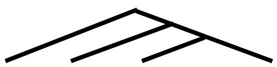

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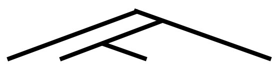

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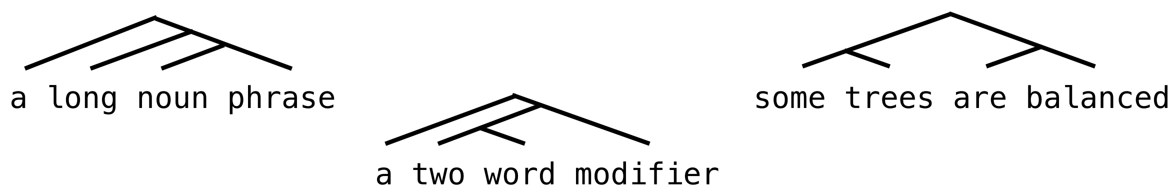

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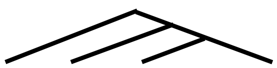

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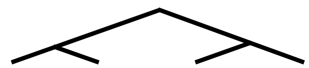

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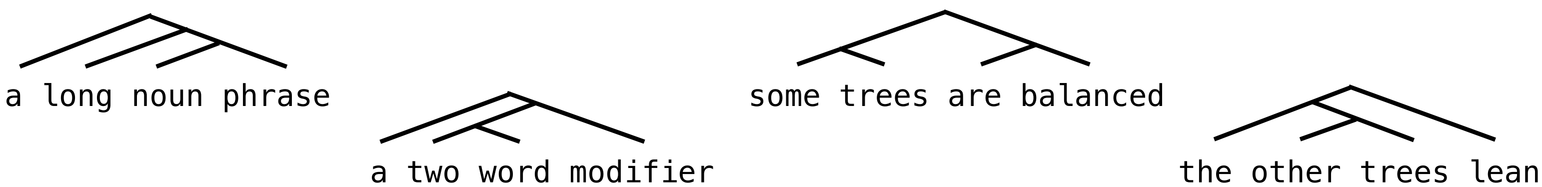

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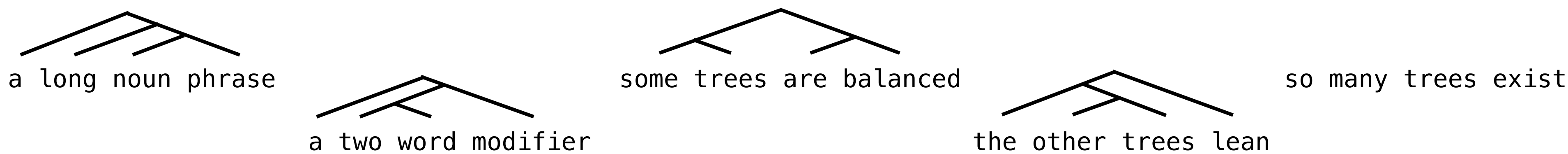

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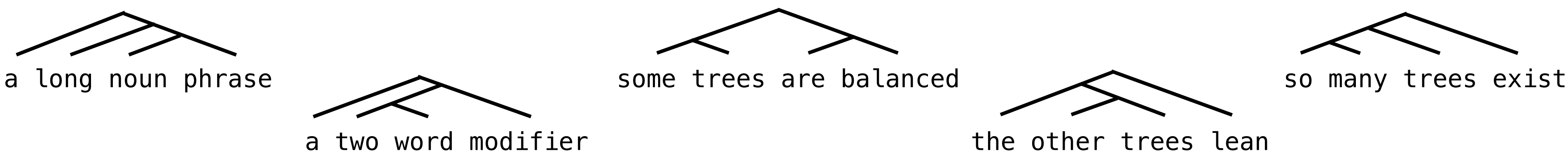

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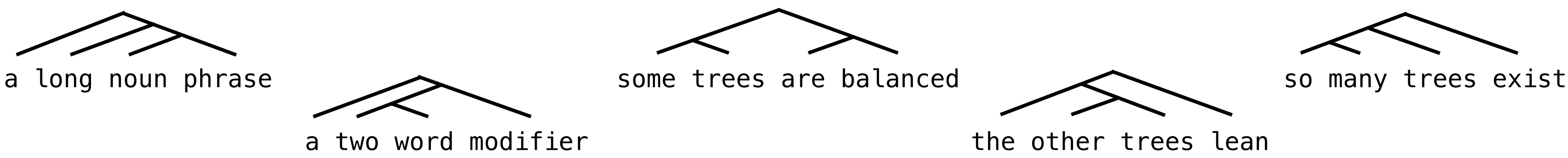

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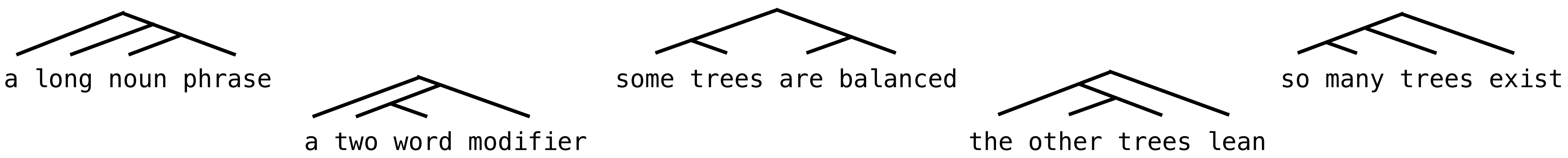

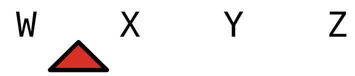

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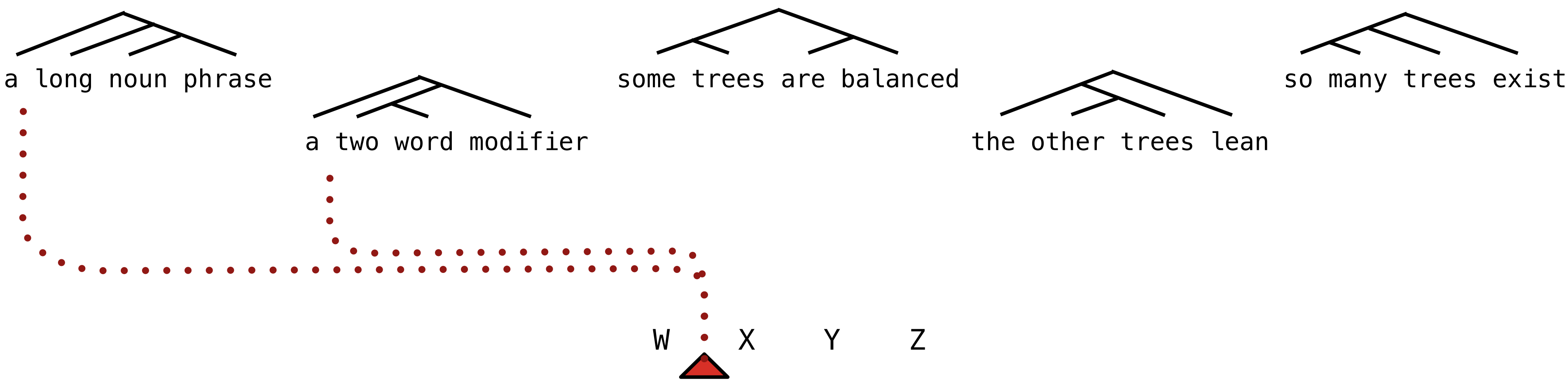

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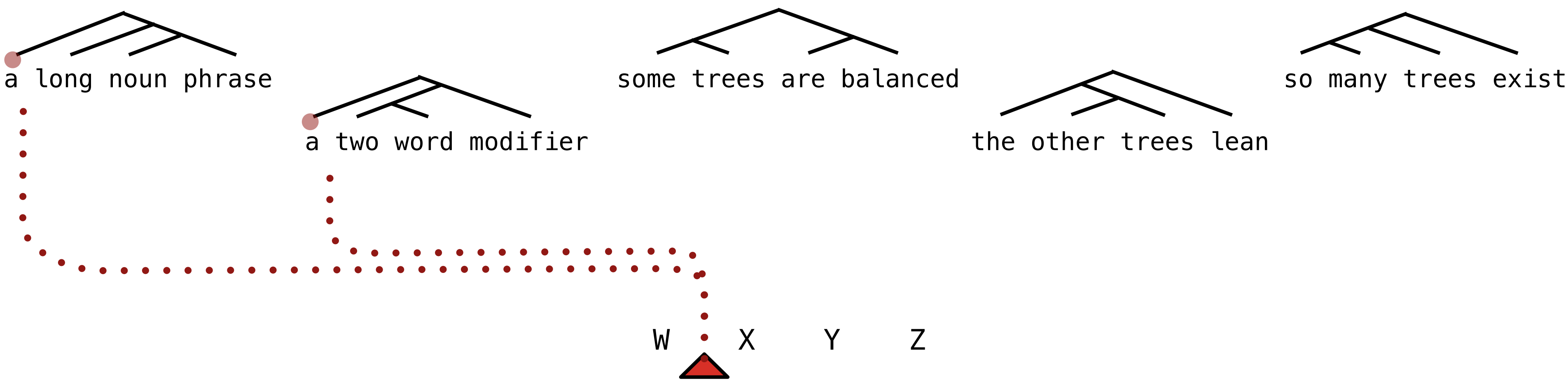

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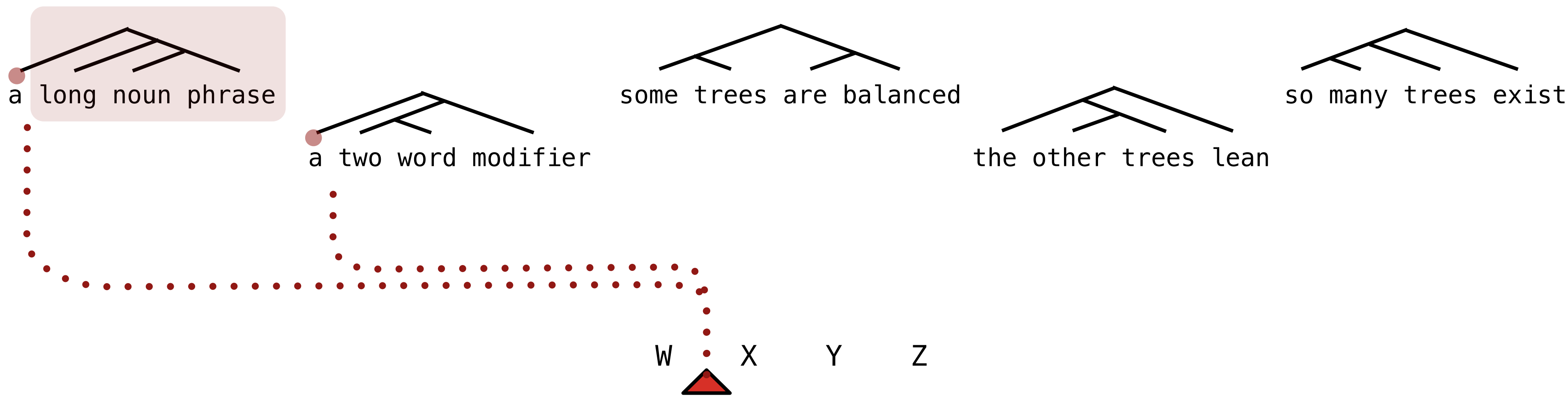

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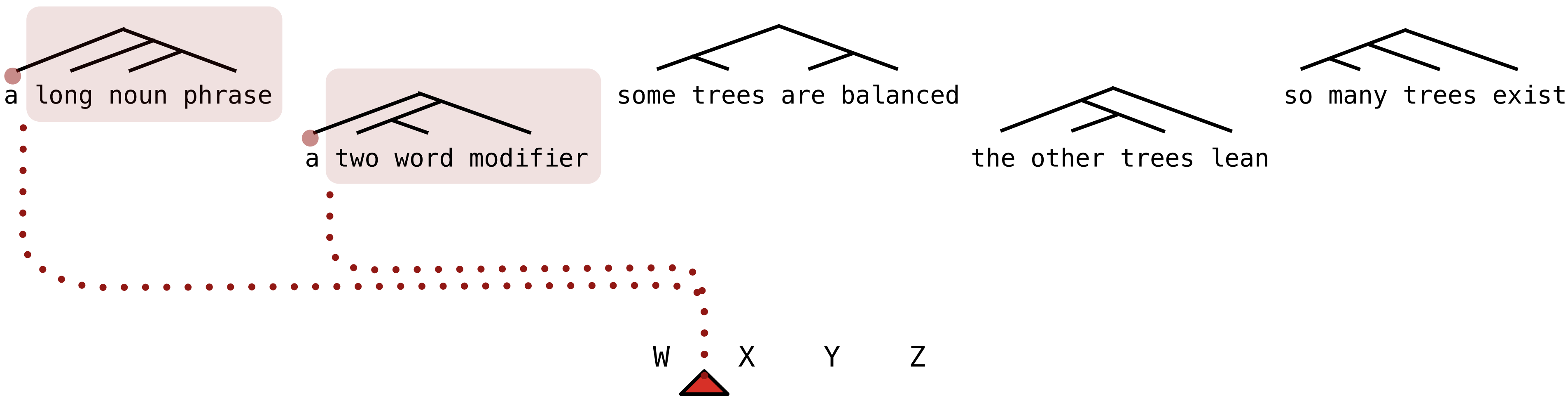

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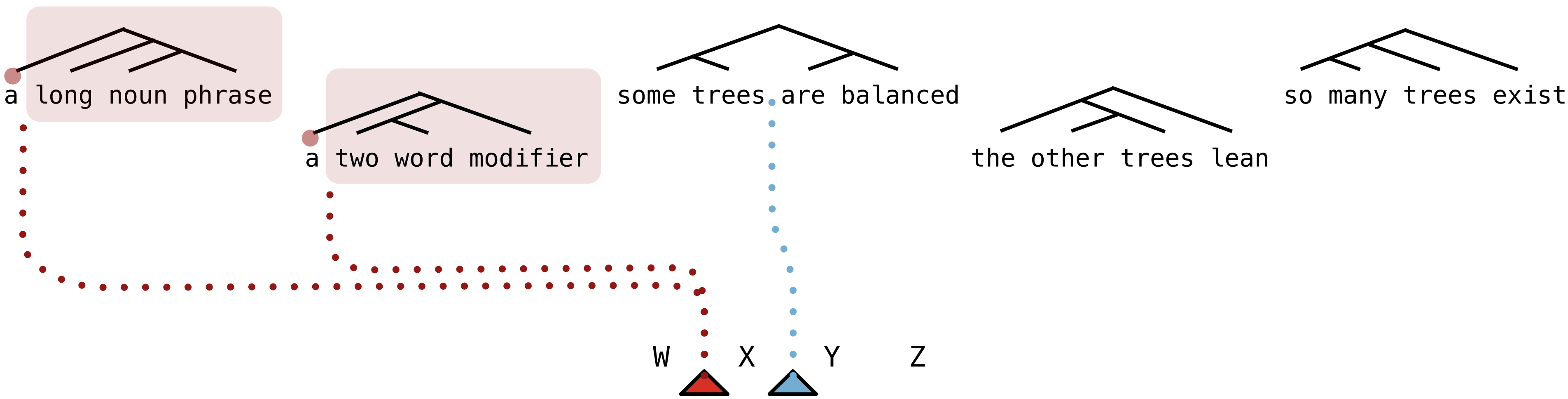

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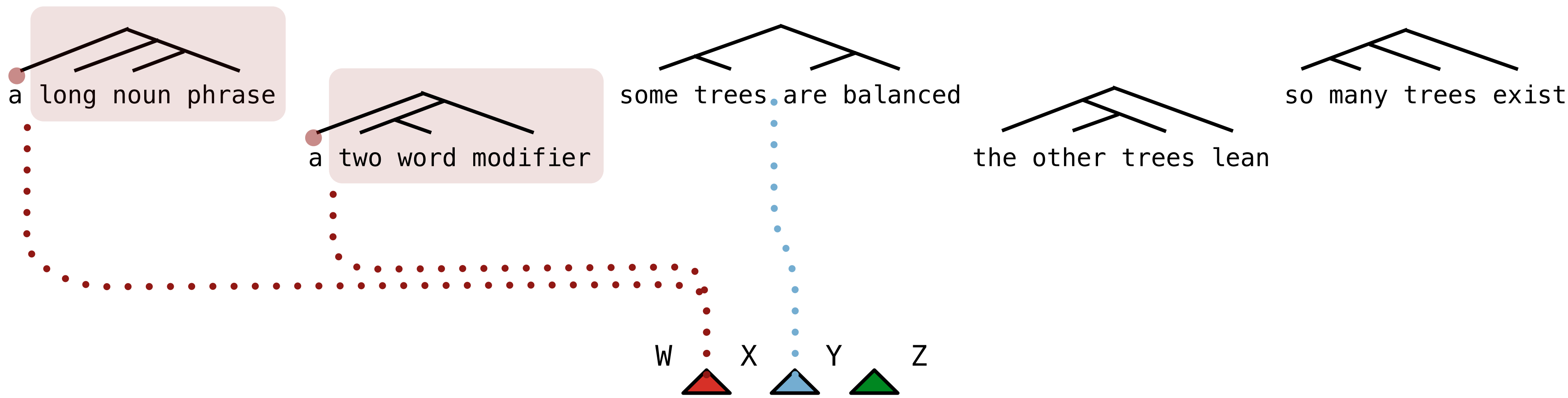

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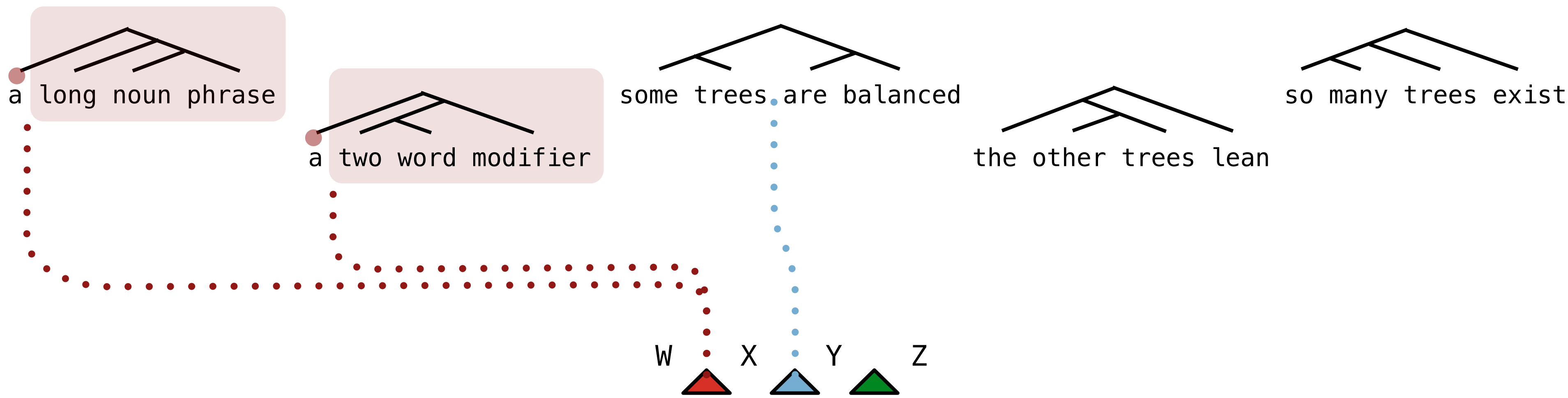

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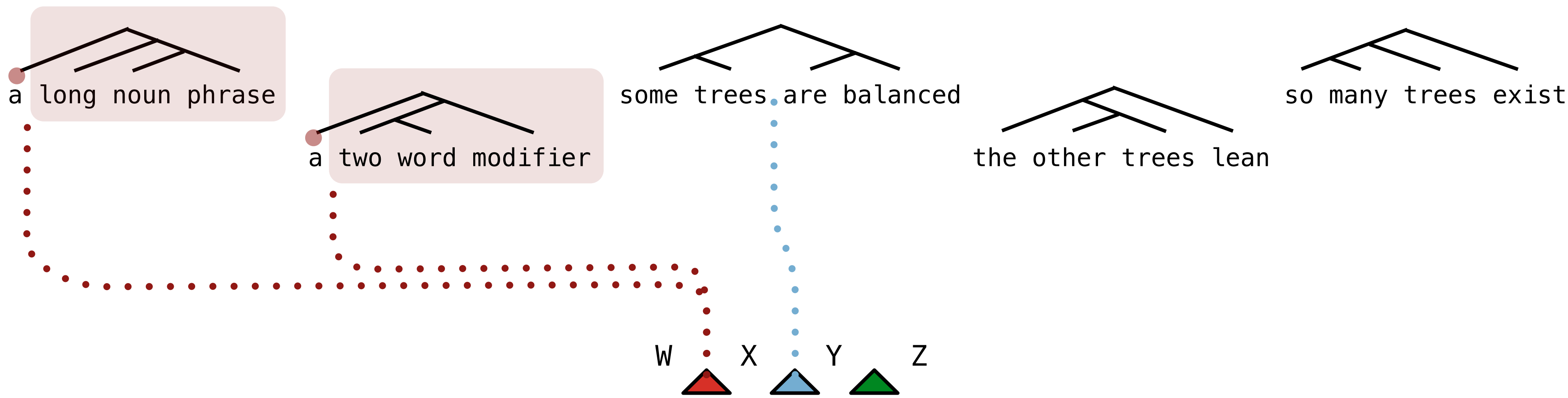

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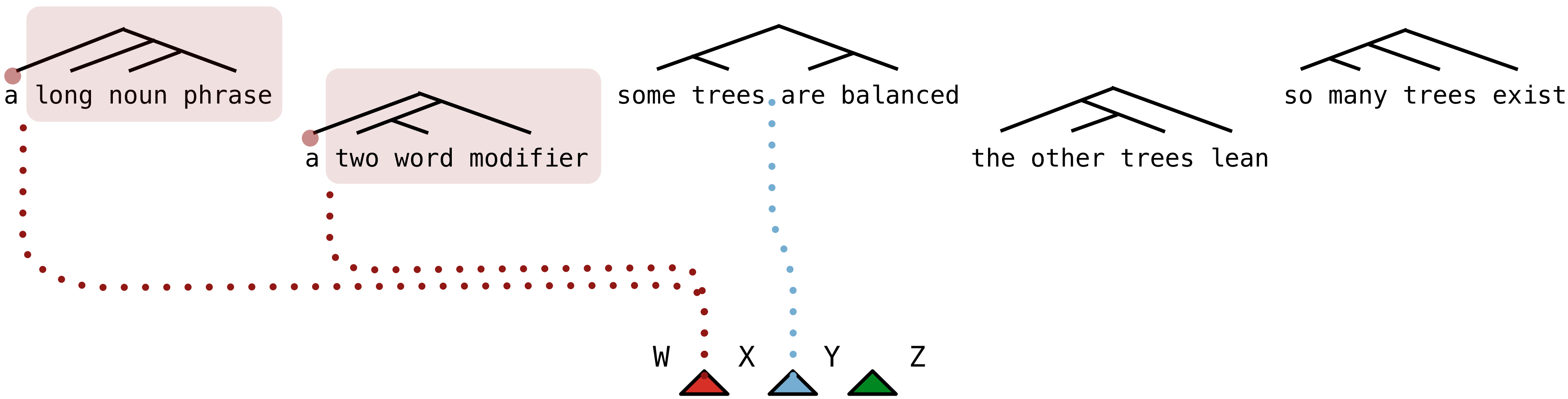

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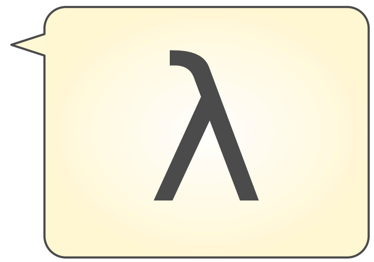

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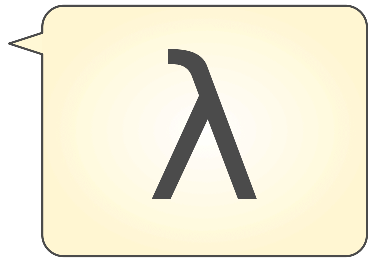

Two equivalent expressions:

```
(define (plus4 x) (+ x 4))

(define plus4 (lambda (x) (+ x 4)))
```

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Lambda expressions evaluate to anonymous procedures.

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Two equivalent expressions:

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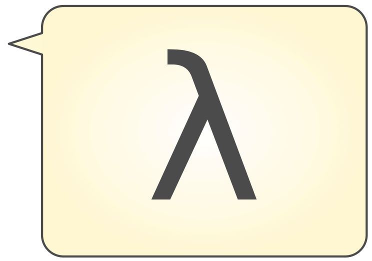

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Evaluates to the *add-**x**-&-**y**-&-**z***^{2} procedure

Pairs and Lists

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(Demo)

Symbolic Programming

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```
> (define a 1)
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> (define a 1)
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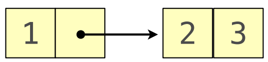

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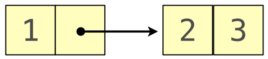

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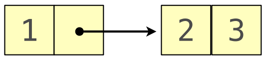

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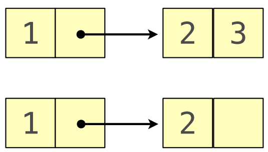

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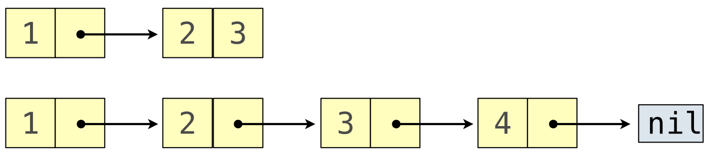

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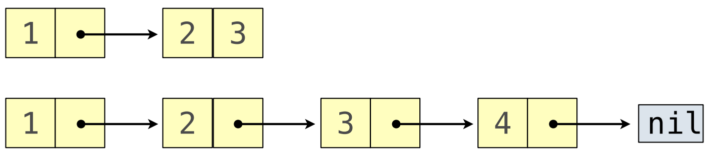

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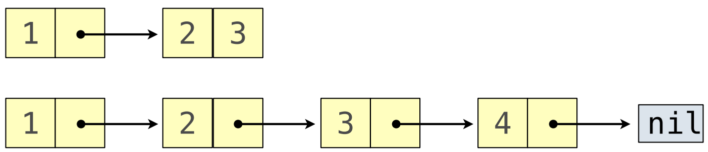

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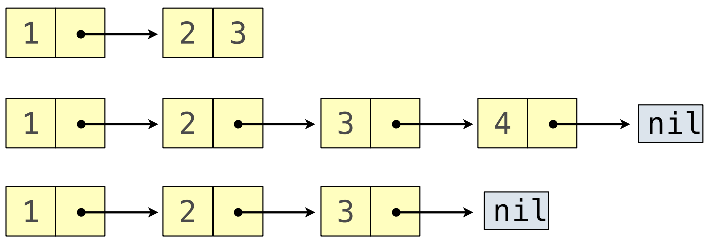

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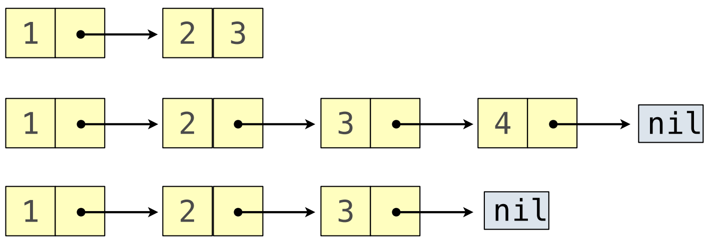

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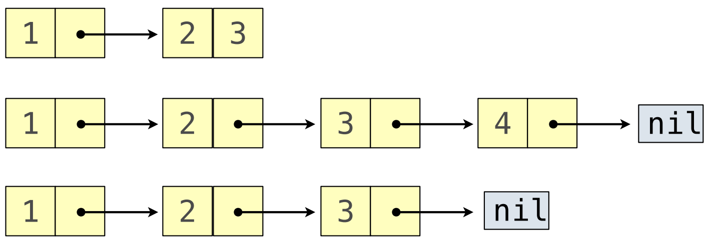

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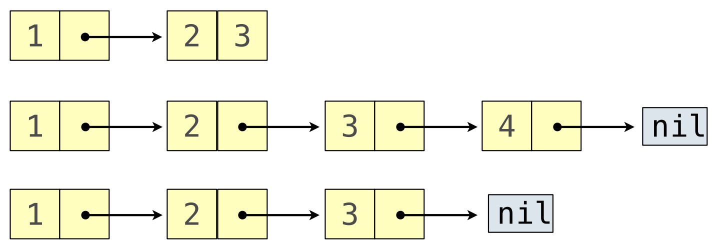

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> (cdr '((1 2) . (3 4 . (5))))
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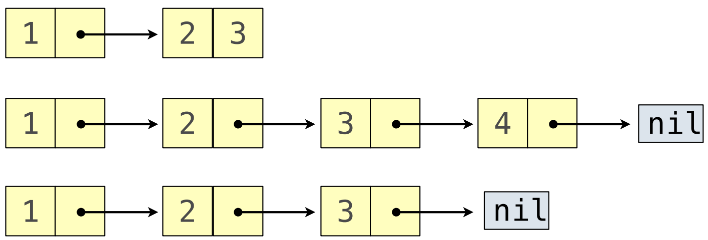
Dots can be used in a quoted list to specify the second element of the final pair.

```
> (cdr (cdr '(1 2 . 3)))
3
```

However, dots appear in the output only of ill-formed lists.

```
> '(1 2 . 3)
(1 2 . 3)
> '(1 2 . (3 4))
(1 2 3 4)
> '(1 2 3 . nil)
(1 2 3)
```



What is the printed result of evaluating this expression?

```
> (cdr '((1 2) . (3 4 . (5))))
(3 4 5)
```

Sierpinski's Triangle

(Demo)