61A Lecture 6

Friday, September 13

Announcements

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  - There will be a web form for students who cannot attend due to a conflict

Lambda Expressions

(Demo)

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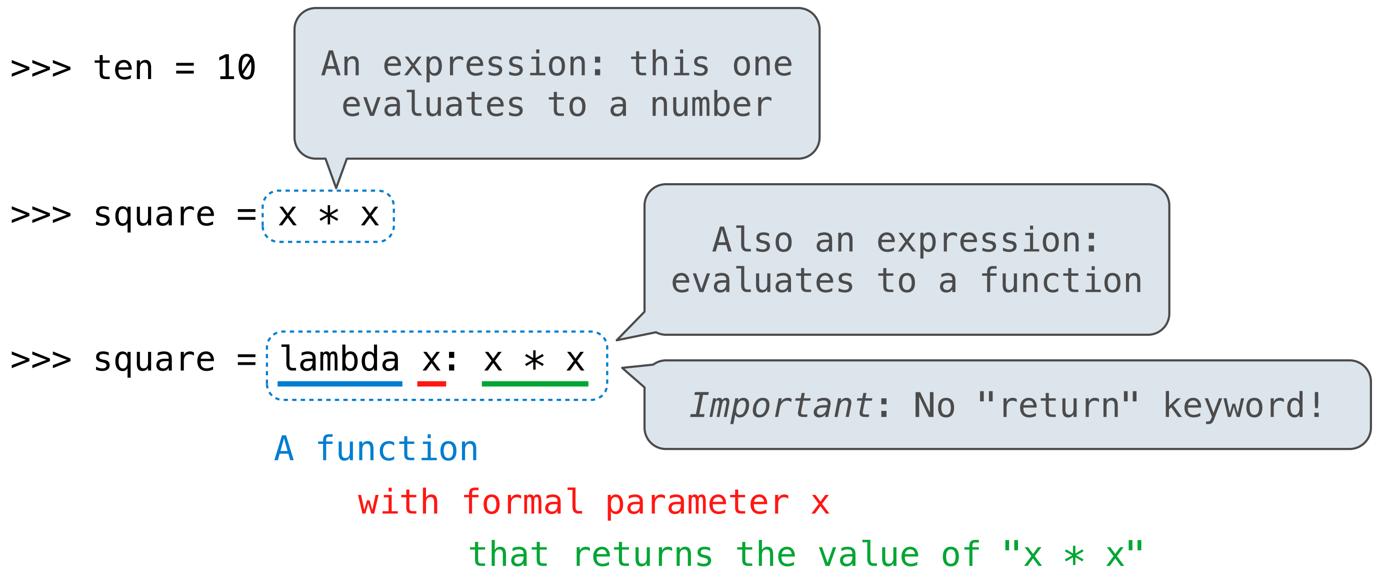

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Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

Example: http://goo.gl/XH54uE

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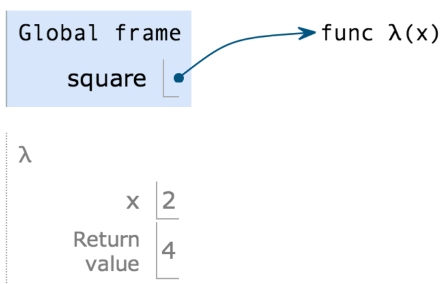

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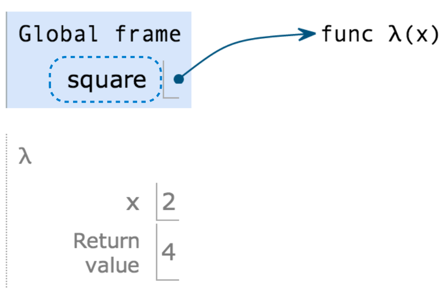

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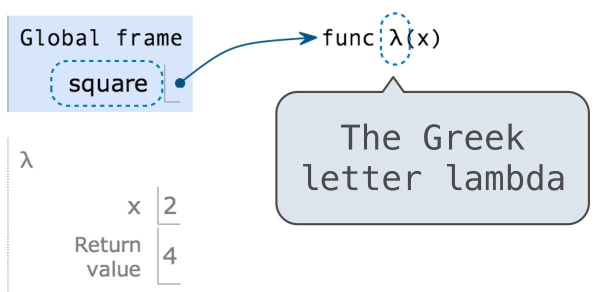

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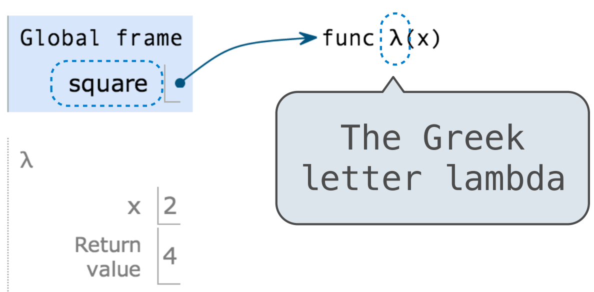

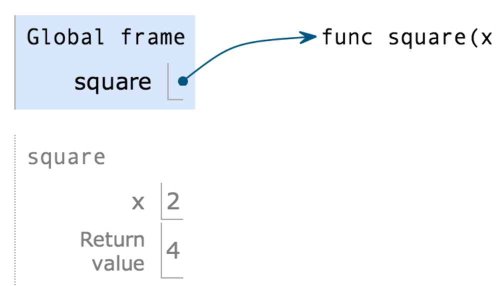

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Schönfinkeling?

Newton's Method

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Quickly finds accurate approximations to zeroes of differentiable functions!

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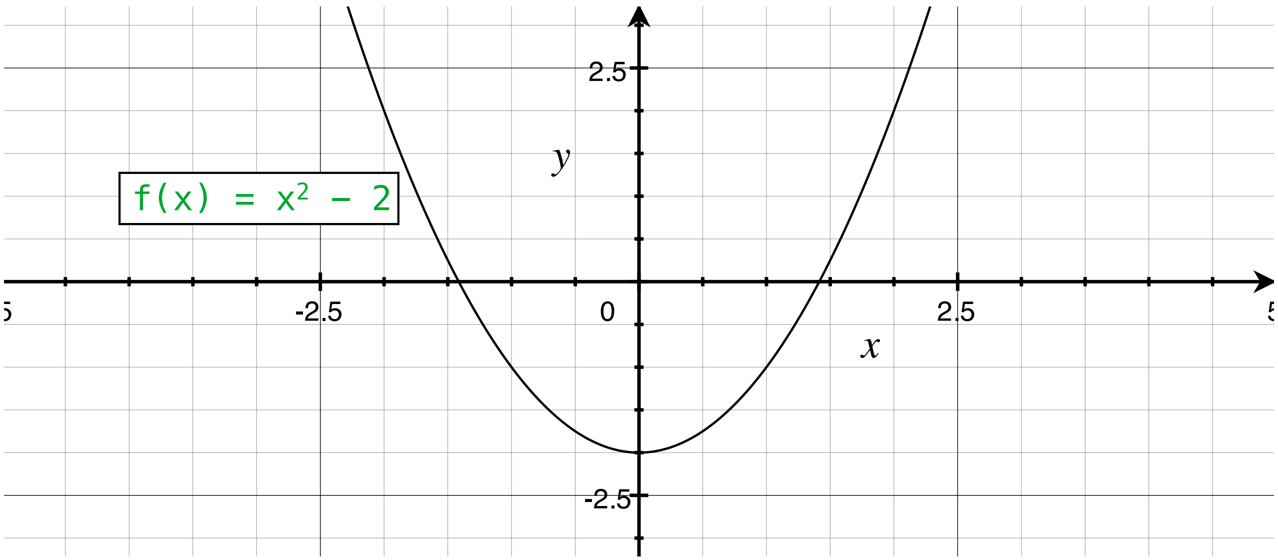

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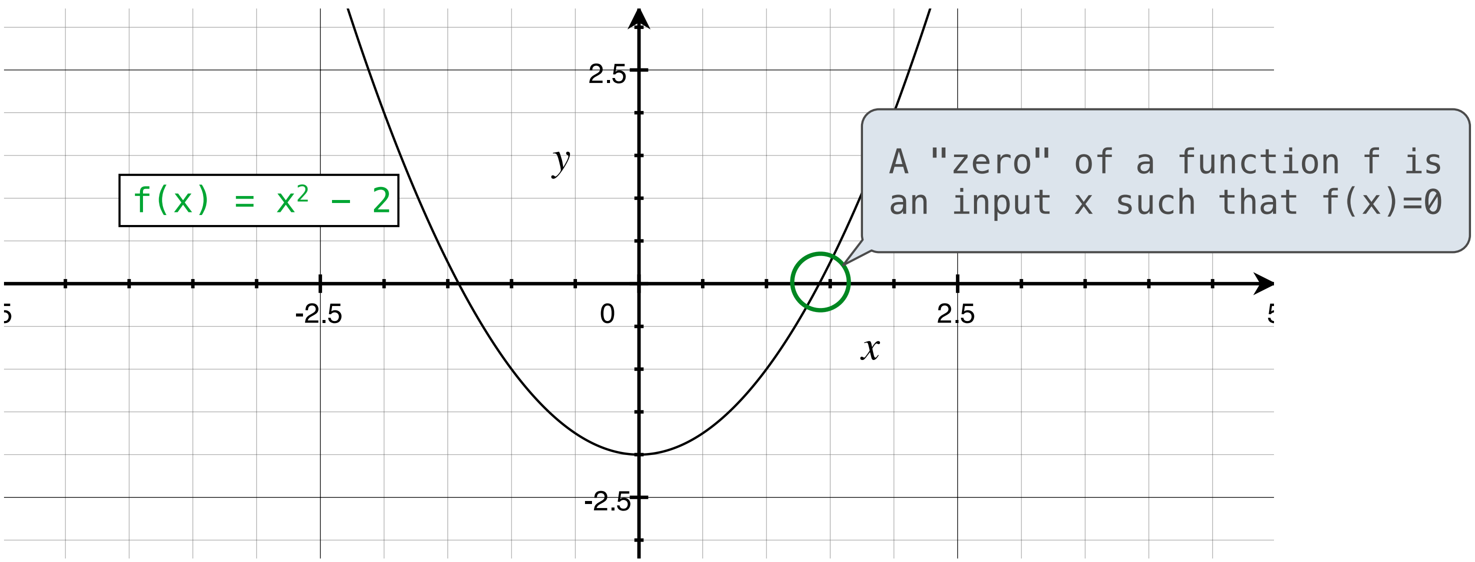

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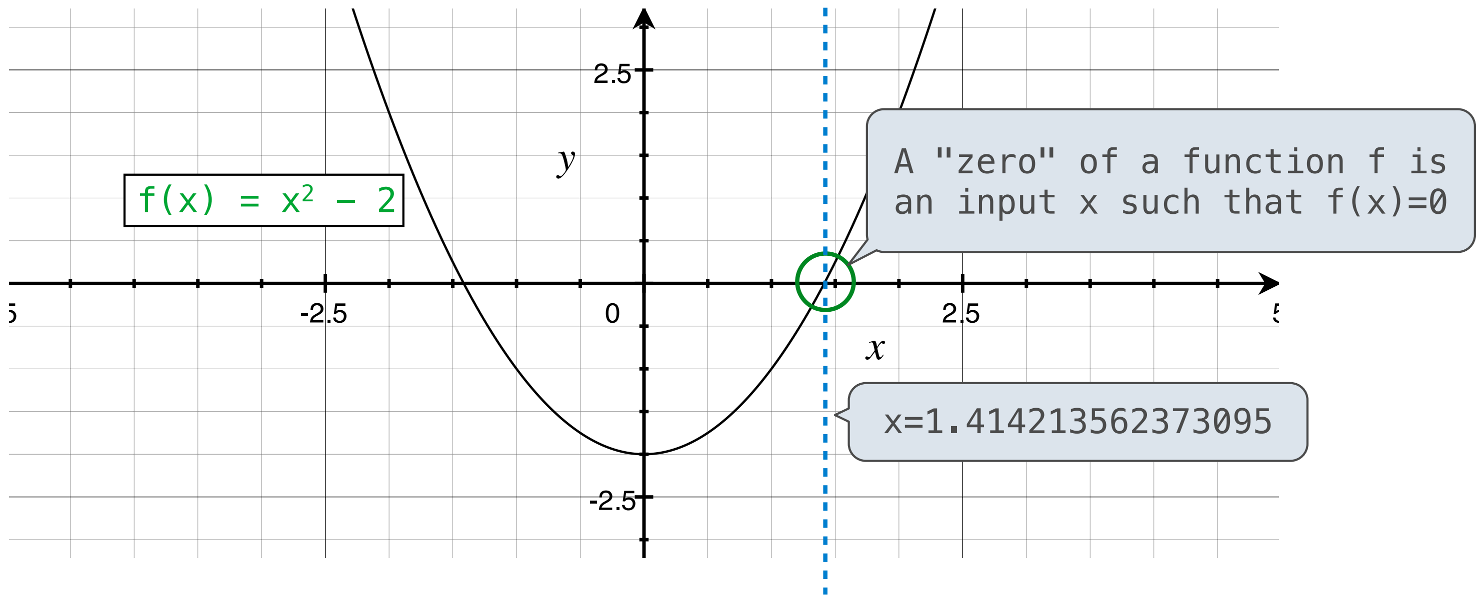

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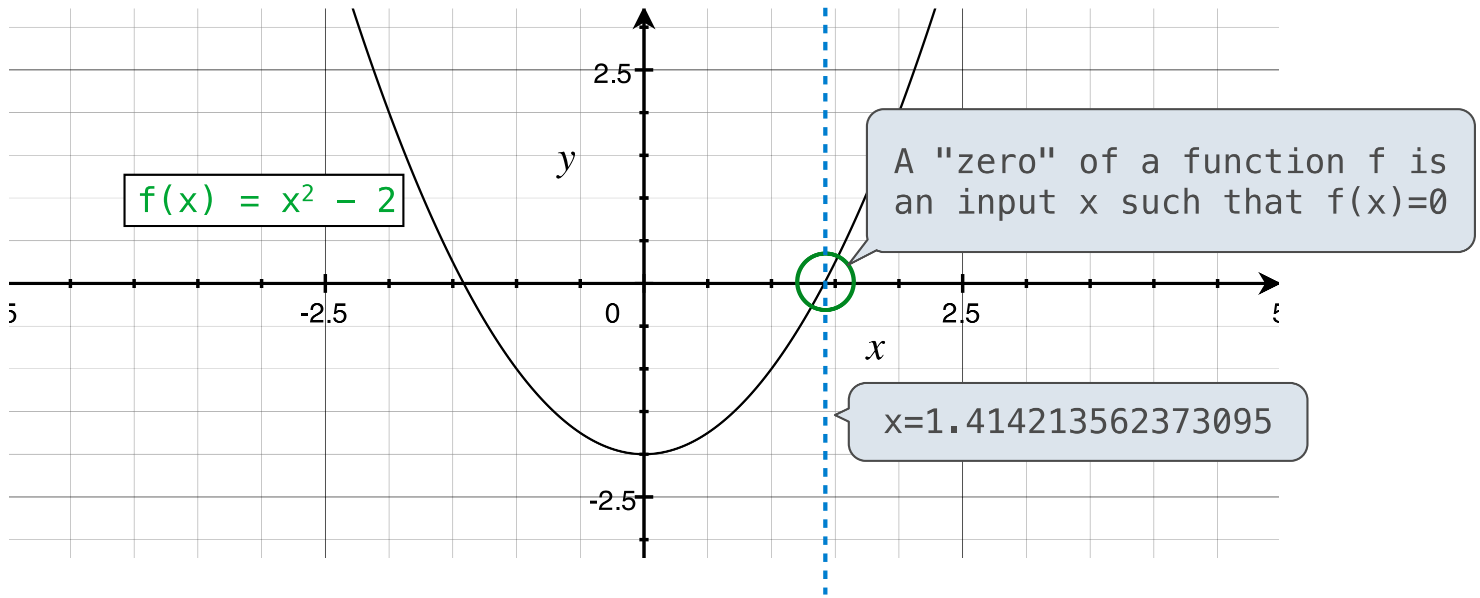

Application: a method for computing square roots, cube roots, etc.

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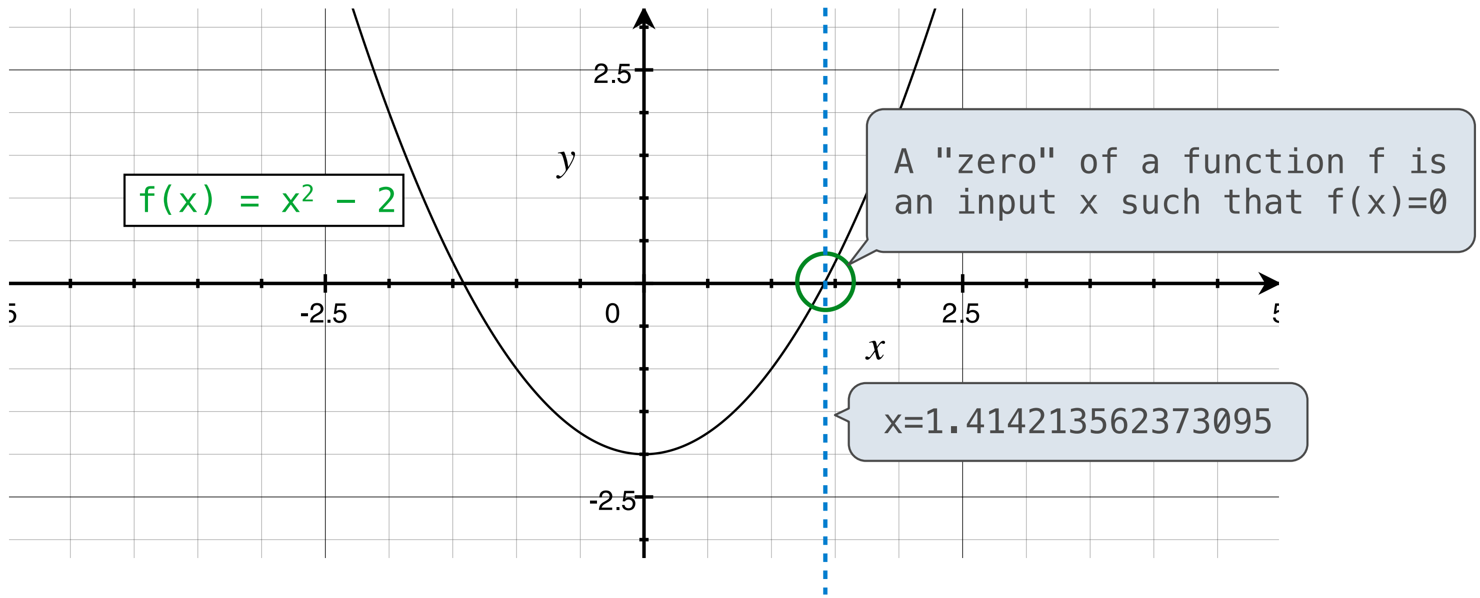

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

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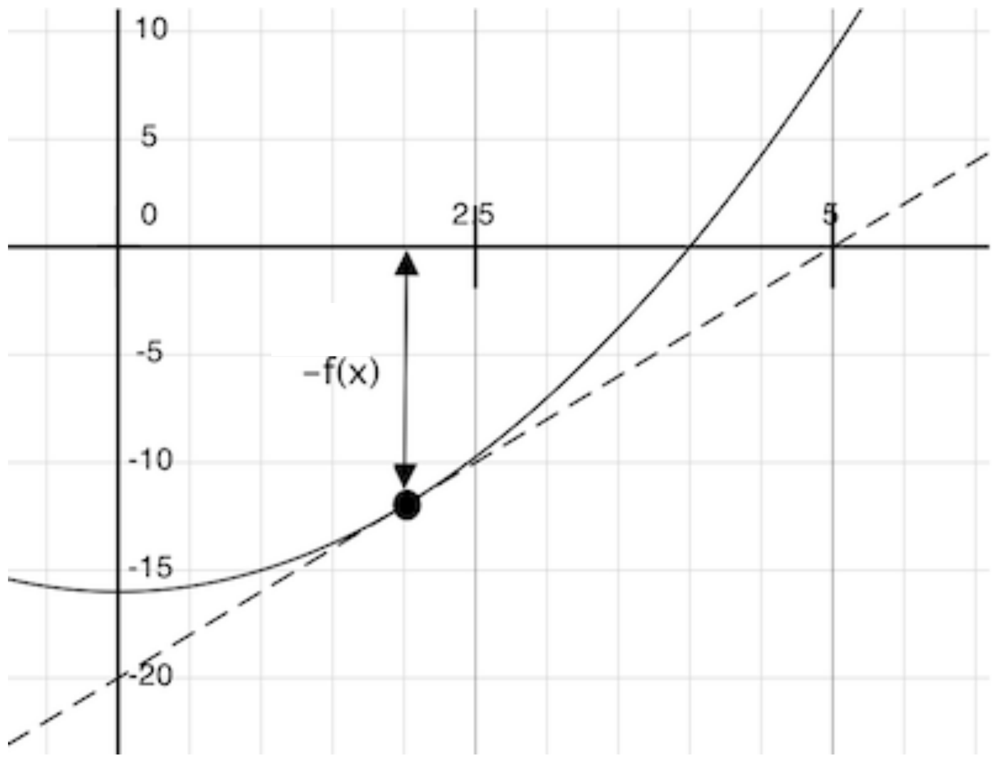

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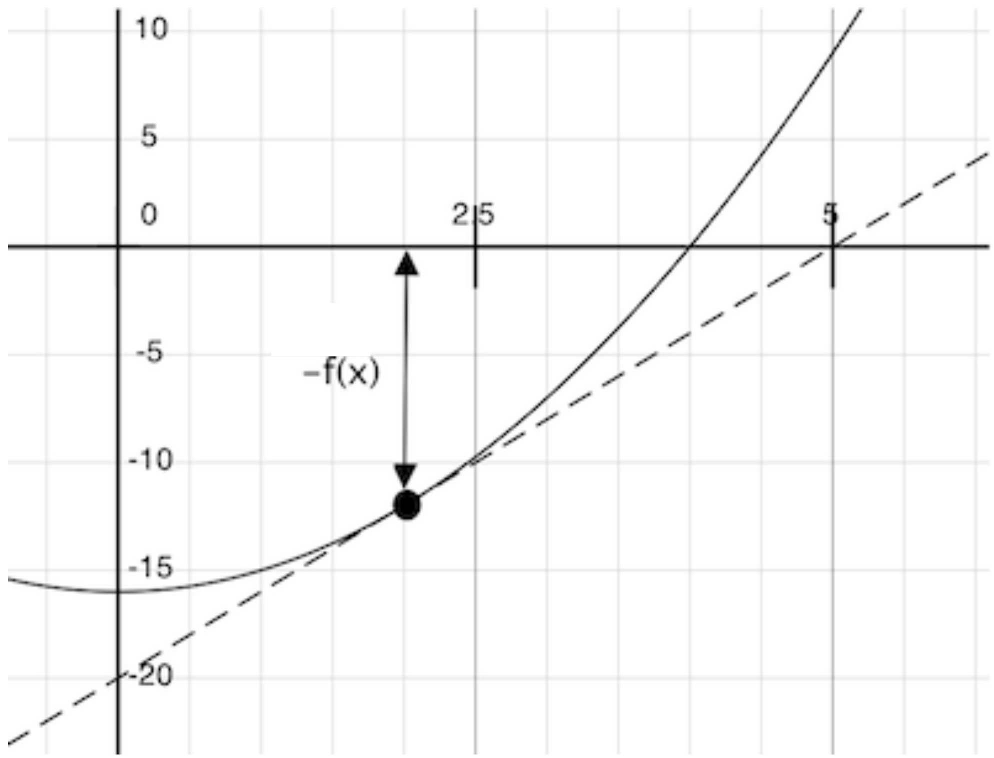

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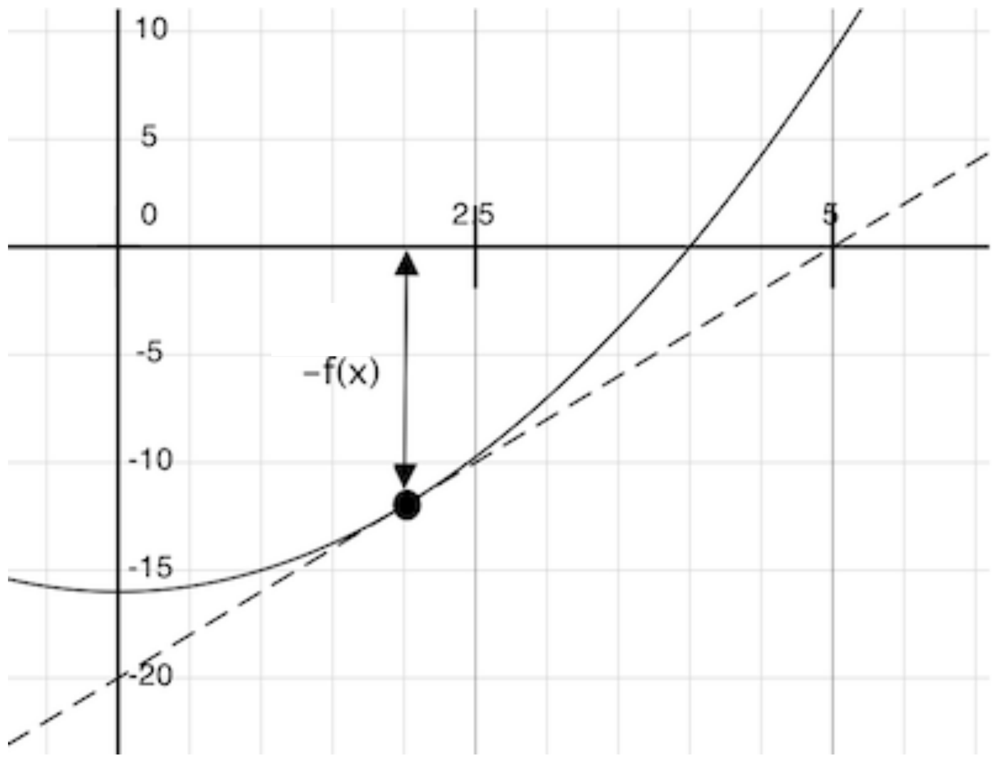

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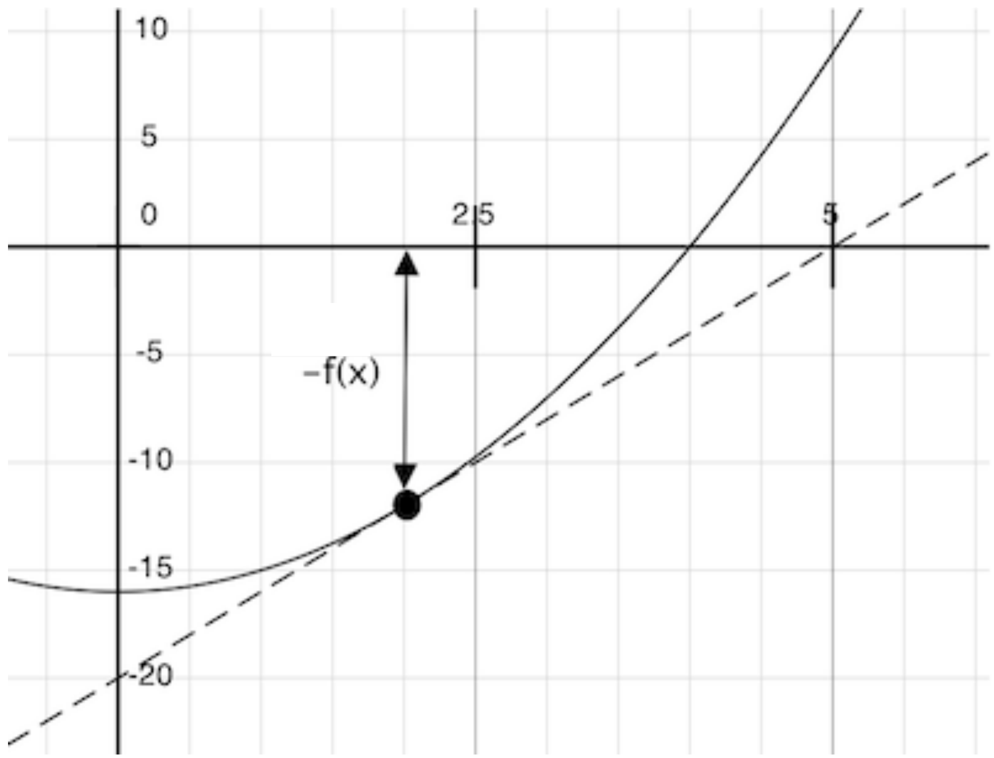

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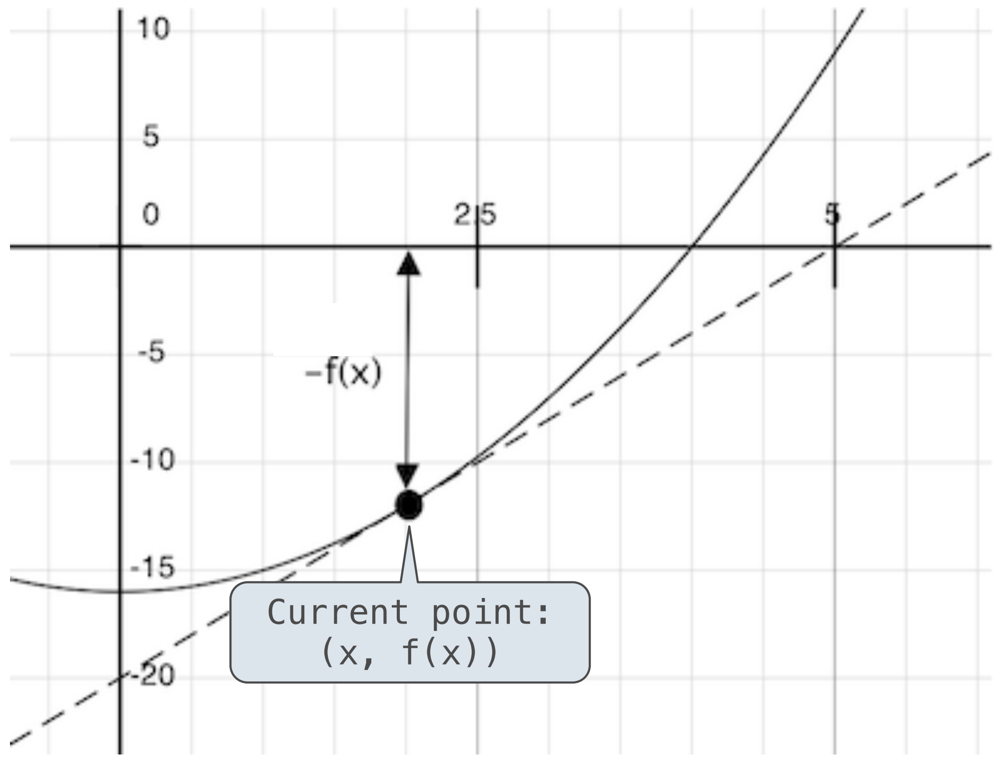

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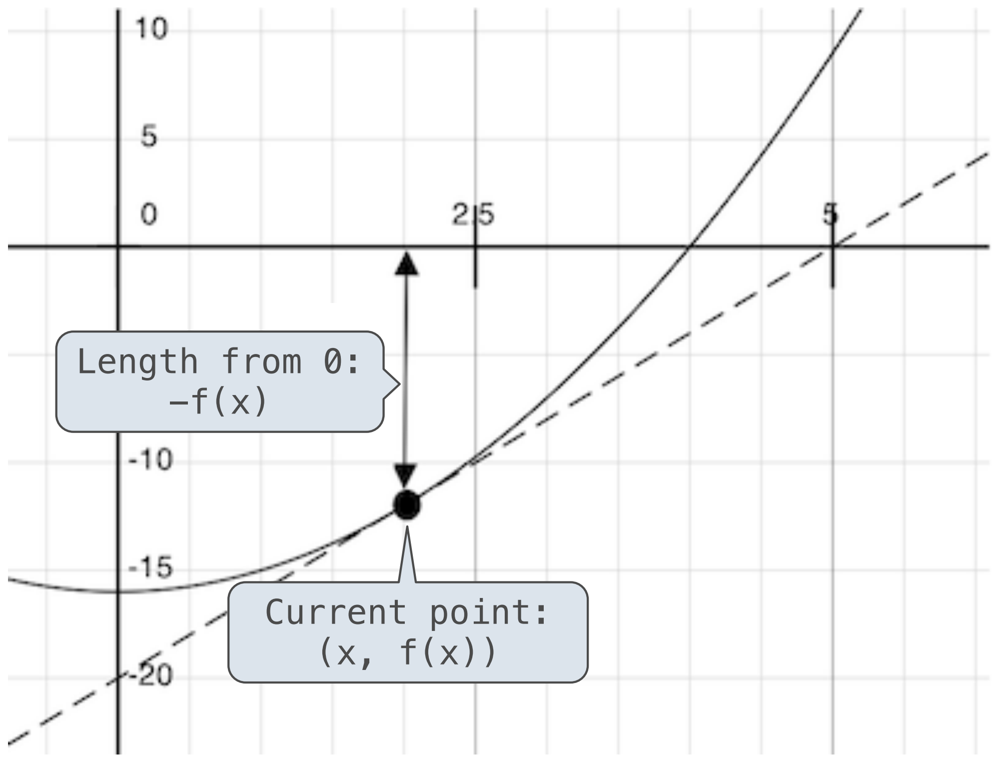

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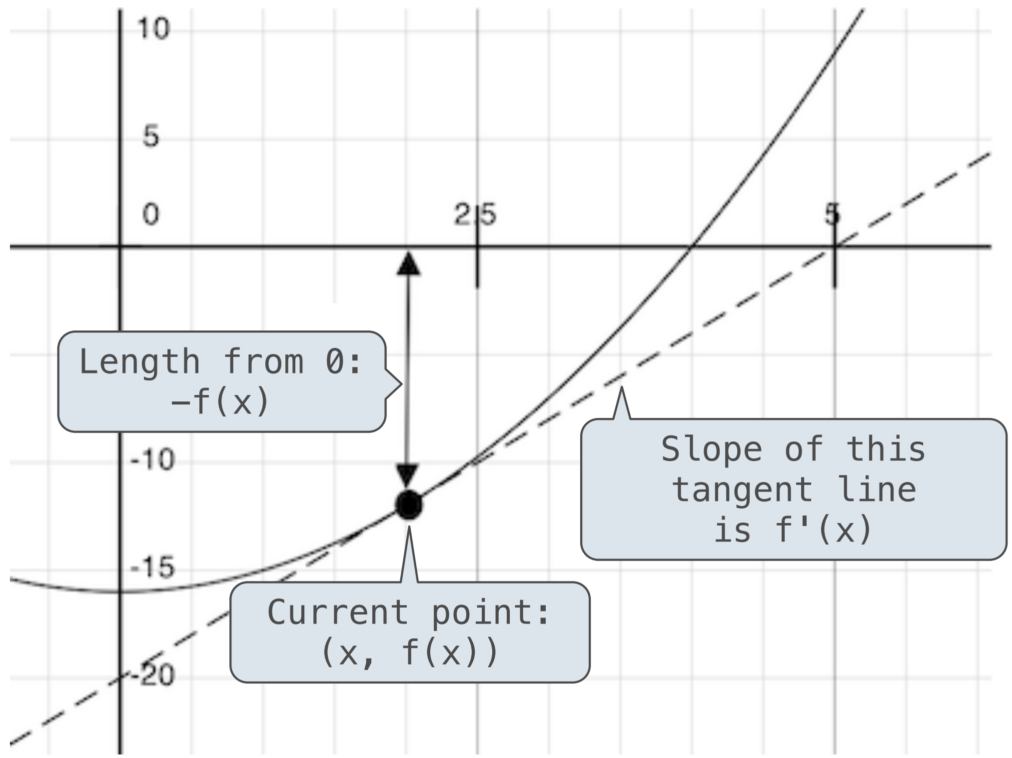

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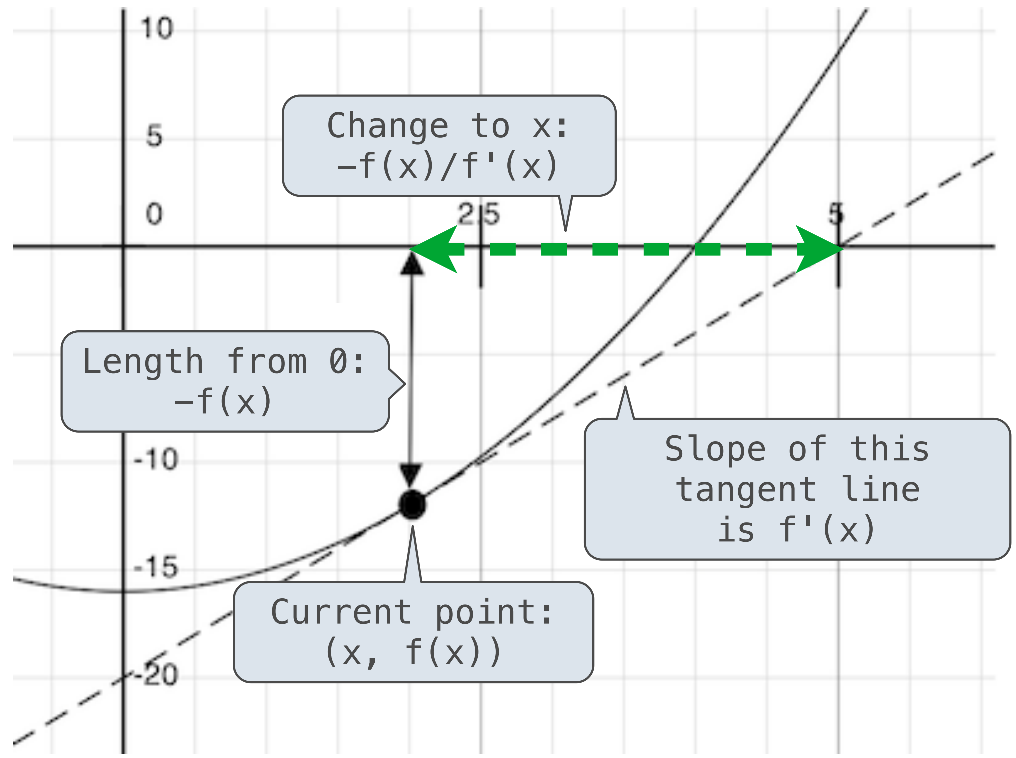

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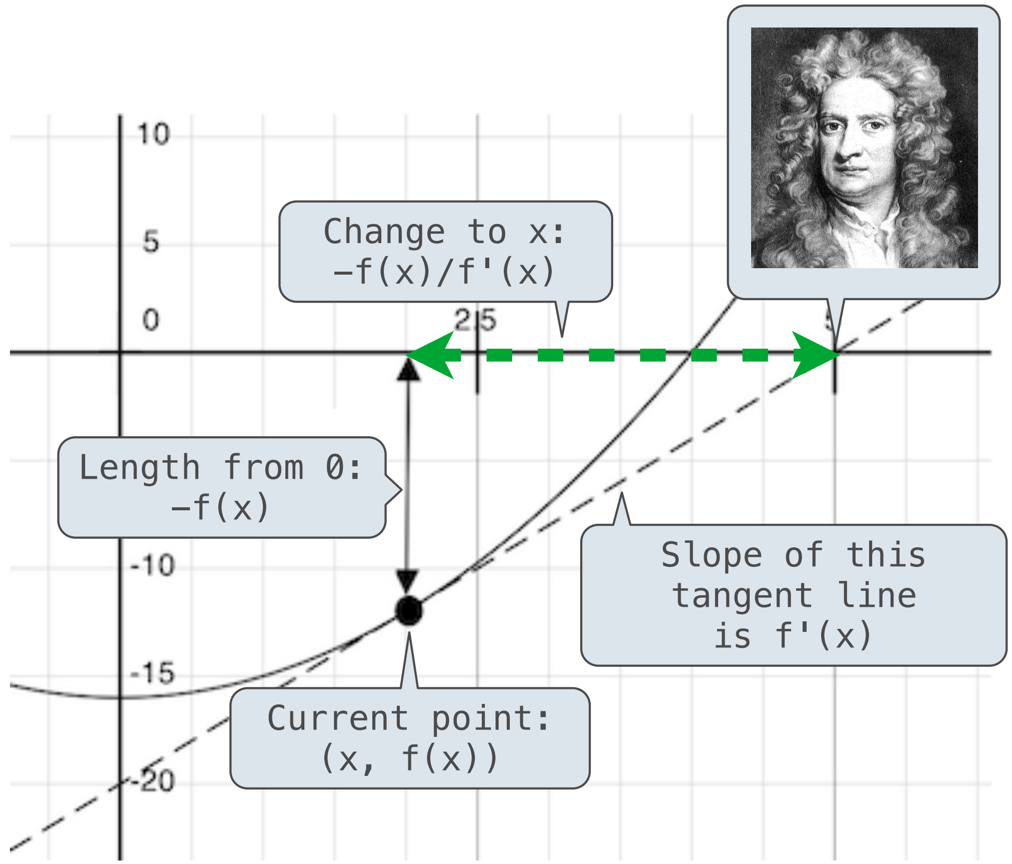

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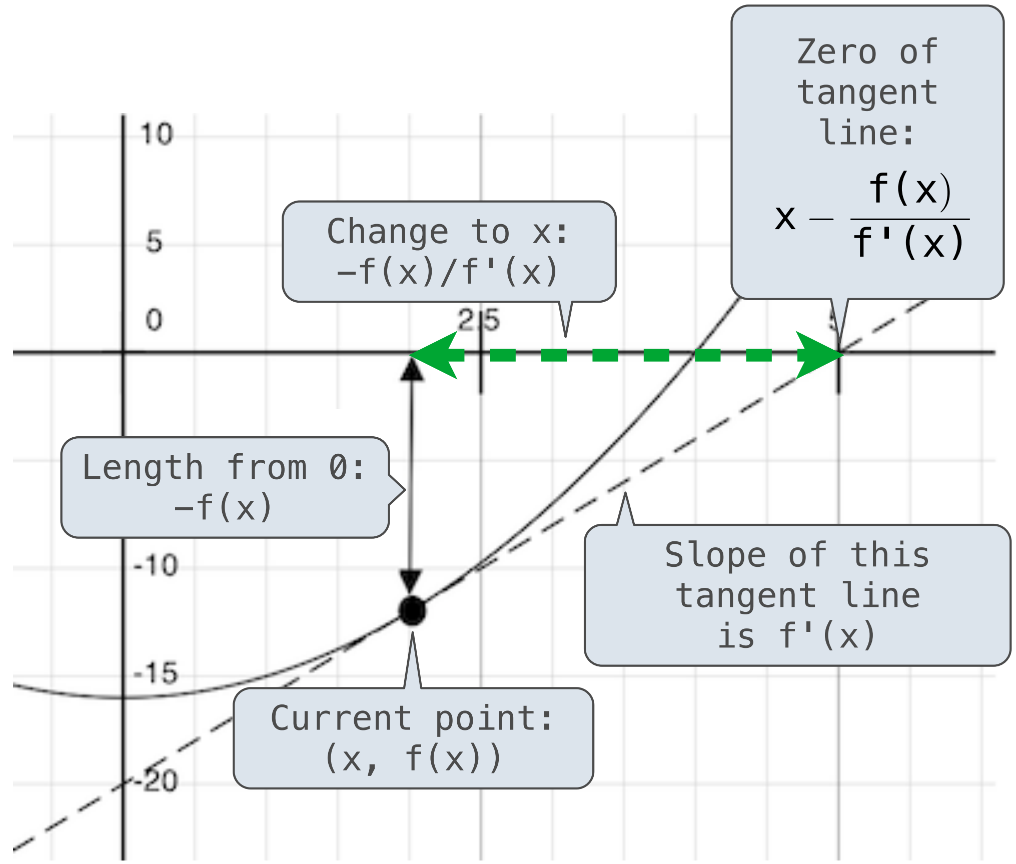

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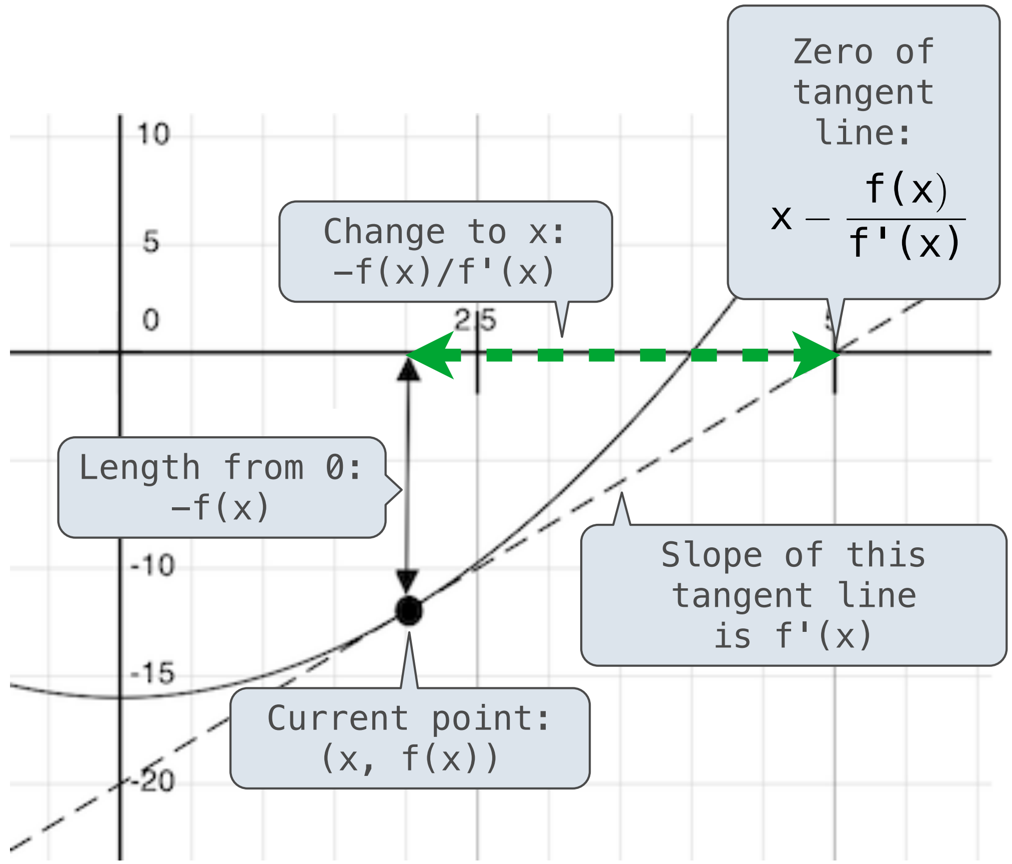
Finish when f(x) = 0 (or close enough)



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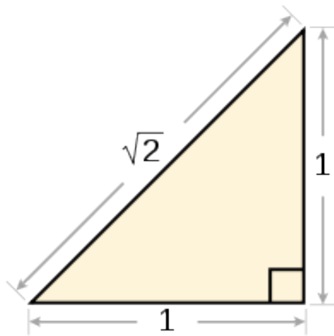


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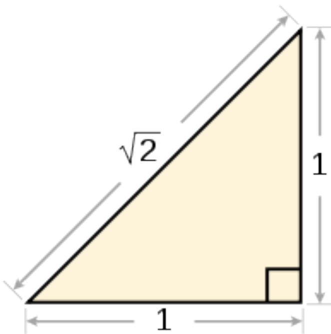


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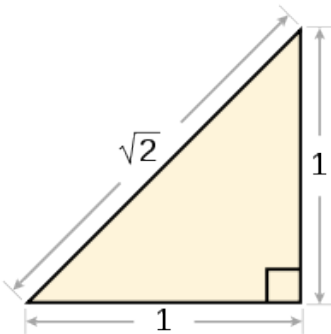


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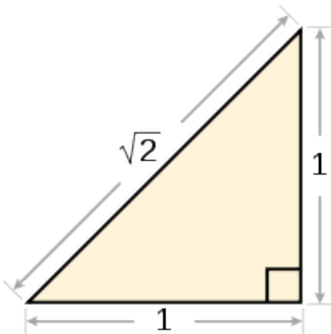


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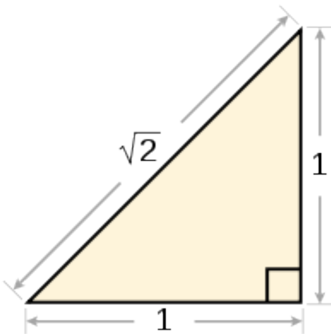


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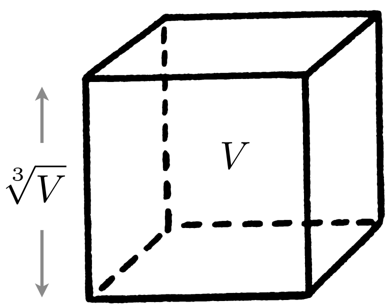
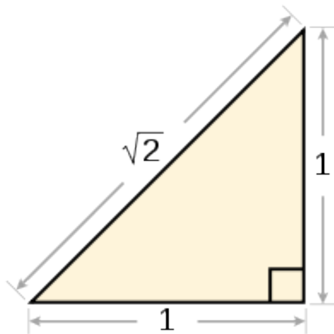


Using Newton's Method

How to find the **square root** of 2?

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>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
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$f(x) = x^2 - 2$
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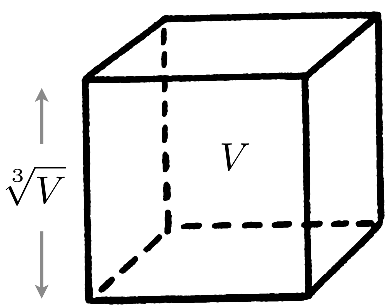


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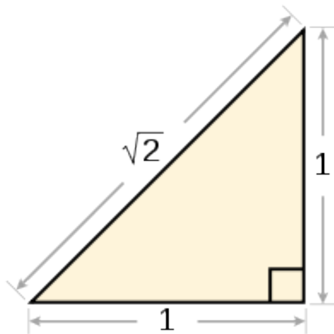


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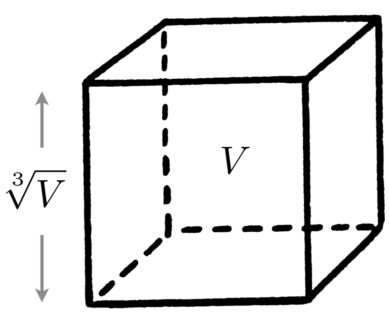


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Iterative Improvement

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Implementing Newton's Method

(Demo)