Q1. Local Search

(a) Hill Climbing
   (i) Hill-climbing is complete. □ True ■ False
   Consider hill-climbing for 8-queen.
   (ii) Hill-climbing is optimal. □ True ■ False
   no completeness indicates no optimality.

(b) Simulated Annealing
   (i) The higher the temperature T is, the more likely the randomly chosen state will be expanded. ■ True □ False
   The higher T is, the larger $e^{\Delta E/T}$ is given $\Delta E$ is negative.
   (ii) In one round of simulated annealing, the temperature is 2 and the current state S has energy 1. It has 3 successors:
   A with energy 2; B with energy 1; C with energy 1-ln 4. If we assume the temperature does not change, What’s the probability that these states will be chosen to expand after S eventually?
   A, B will be expanded with probability $\frac{2}{5}$, C will be expanded with probability $\frac{1}{5}$.
   Proof. First, the problem is asking which node will be expanded next, not in this round. A, B and C are randomly selected for expansion. If A or B is selected, they will surely be expanded. If C is selected, it has probability of 1/2 to be expanded and 1/2 to restart the random selection. Thus the probability ratio between A, B and C is 2:2:1.
   (iii) On a undirected graph. If T decreases slowly enough, simulated annealing is guaranteed to converge to the optimal state. ■ True □ False

(c) Local Beam Search
   The following state graph is being explored with 2-beam graph search. A state’s score is its accumulated distance to the start state and lower scores are considered better. Which of the following statements are true?

   States A and B will be expanded before C and D. ■
   States A and D will be expanded before B and C. □
   States B and D will be expanded before A and C. □
   None of above. □

(d) Genetic Algorithm
   (i) In genetic algorithm, cross-over combine the genetic information of two parents to generate new offspring. ■ True □ False
   (ii) In genetic algorithm, mutation involves a probability that some arbitrary bits in a genetic sequence will be flipped from its original state.
(e) Gradient Descent

(i) Gradient descent is optimal. ☐ True ☐ False
   False. Gradient descent can become trapped in a local minimum.

(ii) For a function $f(x)$ with derivative $f'(x)$, write down the gradient descent update to go from $x_t$ to $x_{t+1}$. Learning rate is $\alpha$.

$$x_{t+1} = x_t - \alpha f'(x_t),$$ where $\alpha$ is the learning rate.
Q2. MedianMiniMax

You’re living in utopia! Despite living in utopia, you still believe that you need to maximize your utility in life, other people want to minimize your utility, and the world is a 0 sum game. But because you live in utopia, a benevolent social planner occasionally steps in and chooses an option that is a compromise. Essentially, the social planner (represented as the pentagon) is a median node that chooses the successor with median utility. Your struggle with your fellow citizens can be modelled as follows:

There are some nodes that we are sometimes able to prune. In each part, mark all of the terminal nodes such that there exists a possible situation for which the node can be pruned. In other words, you must consider all possible pruning situations. Assume that evaluation order is left to right and all $V_i$’s are distinct.

Note that as long as there exists ANY pruning situation (does not have to be the same situation for every node), you should mark the node as prunable. Also, alpha-beta pruning does not apply here, simply prune a sub-tree when you can reason that its value will not affect your final utility.

(a) \[ V_1 \quad V_2 \quad V_3 \quad V_4 \quad \] (b) \[ V_5 \quad V_6 \quad V_7 \quad V_8 \quad \] (c) \[ V_9 \quad V_{10} \quad V_{11} \quad V_{12} \quad \] (d) \[ V_{13} \quad V_{14} \quad V_{15} \quad V_{16} \quad \]

- (a) $V_1$, $V_2$, $V_3$, $V_4$, None
- (b) $V_5$, $V_6$, $V_7$, $V_8$, None
- (c) $V_9$, $V_{10}$, $V_{11}$, $V_{12}$, None
- (d) $V_{13}$, $V_{14}$, $V_{15}$, $V_{16}$, None
**Part a:**
For the left median node with three children, at least two of the children’s values must be known since one of them will be guaranteed to be the value of the median node passed up to the final maximizer. For this reason, none of the nodes in part a can be pruned.

**Part b (pruning \( V_7, V_8 \)):**
Let \( min_1, min_2, min_3 \) be the values of the three minimizer nodes in this subtree.

In this case, we may not need to know the final value \( min_3 \). The reason for this is that we may be able to put a bound on its value after exploring only partially, and determine the value of the median node as either \( min_1 \) or \( min_2 \) if \( min_3 \leq \min(min_1, min_2) \) or \( min_3 \geq \max(min_1, min_2) \).

We can put an upper bound on \( min_3 \) by exploring the left subtree \( V_5, V_6 \) and if \( \max(V_5, V_6) \) is lower than both \( min_1 \) and \( min_2 \), the median node’s value is set as the smaller of \( min_1, min_2 \) and we don’t have to explore \( V_7, V_8 \) in Figure 1.

**Part b (pruning \( V_6 \)):**
It’s possible for us to put a lower bound on \( min_3 \). If \( V_5 \) is larger than both \( min_1 \) and \( min_2 \), we do not need to explore \( V_6 \).

The reason for this is subtle, but if the minimizer chooses the left subtree, we know that \( min_3 \geq V_5 \geq \max(min_1, min_2) \) and we don’t need \( V_6 \) to get the correct value for the median node which will be the larger of \( min_1, min_2 \).

If the minimizer chooses the value of the right subtree, the value at \( V_6 \) is unnecessary again since the minimizer never chose its subtree.
Part c (pruning $V_{11}, V_{12}$):
Assume the highest maximizer node has a current value $\max_1 \geq Z$ set by the left subtree and the three minimizers on this right subtree have value $\min_1, \min_2, \min_3$.

In this part, if $\min_1 \leq \max(V_9, V_{10}) \leq Z$, we do not have to explore $V_{11}, V_{12}$. Once again, the reasoning is subtle, but we can now realize if either $\min_2 \leq Z$ or $\min_3 \leq Z$ then the value of the right median node is for sure $\leq Z$ and is useless.

Only if both $\min_2, \min_3 \geq Z$ will the whole right subtree have an effect on the highest maximizer, but in this case the exact value of $\min_1$ is not needed, just the information that it is $\leq Z$. Clearly in both cases, $V_{11}, V_{12}$ are not needed since an exact value of $\min_1$ is not needed.

We will also take the time to note that if $V_9 \geq Z$ we do have to continue the exploring as $V_{10}$ could be even greater and the final value of the top maximizer, so $V_{10}$ can’t really be pruned.

Part d (pruning $V_{14}, V_{15}, V_{16}$):
Continuing from part c, if we find that $\min_1 \leq Z$ and $\min_2 \leq Z$ we can stop.

We can realize this as soon we explore $V_{13}$. Once we figure this out, we know that our median node’s value must be one of these two values, and neither will replace $Z$ so we can stop.