Q1. Search

(a) Rubik’s Search

Note: You do not need to know what a Rubik’s cube is in order to solve this problem.

A Rubik’s cube has about $4.3 \times 10^{19}$ possible configurations, but any configuration can be solved in 20 moves or less. We pose the problem of solving a Rubik’s cube as a search problem, where the states are the possible configurations, and there is an edge between two states if we can get from one state to another in a single move. Thus, we have $4.3 \times 10^{19}$ states. Each edge has cost 1. Note that the state space graph does contain cycles. Since we can make 27 moves from each state, the branching factor is 27. Since any configuration can be solved in 20 moves or less, we have $h^*(n) \leq 20$.

For each of the following searches, estimate the approximate number of states expanded. Mark the option that is closest to the number of states expanded by the search. Assume that the shortest solution for our start state takes exactly 20 moves. Note that $27^{20}$ is much larger than $4.3 \times 10^{19}$.

(i) DFS Tree Search

| Best Case: | 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |
| Worst Case: | 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |

(ii) DFS graph search

| Best Case: | 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |
| Worst Case: | 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |

(iii) BFS graph search

| Best Case: | 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |
| Worst Case: | 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |

(iv) A* tree search with a perfect heuristic, $h^*(n)$, Best Case

| 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |

(v) A* tree search with a bad heuristic, $h(n) = 20 - h^*(n)$, Worst Case

| 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |

(vi) A* graph search with a perfect heuristic, $h^*(n)$, Best Case

| 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |

(vii) A* graph search with a bad heuristic, $h(n) = 20 - h^*(n)$, Worst Case

| 20 | $4.3 \times 10^{19}$ | $27^{20}$ | $\infty$ (never finishes) |
Q2. Searching with Heuristics

Consider the A* searching process on a connected undirected graph, with starting node \( S \) and the goal node \( G \). Suppose the cost for each connection edge is always positive. We define \( h^*(X) \) as the shortest (optimal) distance to \( G \) from a node \( X \).

Note: You may want to solve Questions (a) and (b) at the same time.

(a) Suppose \( h \) is an admissible heuristic, and we conduct A* tree search using heuristic \( h' \) and finally find a solution. Let \( C \) be the cost of the found path (directed by \( h' \), defined in part (a)) from \( S \) to \( G \).

(i) Choose one best answer for each condition below.

1. If \( h'(X) = \frac{1}{2} h(X) \) for all Node \( X \), then
   \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

2. If \( h'(X) = \frac{h(X) + h^*(X)}{2} \) for all Node \( X \), then
   \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

3. If \( h'(X) = h(X) + h^*(X) \) for all Node \( X \), then
   \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

4. If we define the set \( K(X) \) for a node \( X \) as all its neighbor nodes \( Y \) satisfying \( h^*(X) > h^*(Y) \), and the following always holds

   \[ h'(X) \leq \begin{cases} 
   \min_{Y \in K(X)} h(Y) - h(X) + h(Y) & \text{if } K(X) \neq \emptyset \\
   h(X) & \text{if } K(X) = \emptyset 
   \end{cases} \]

   then,
   \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

5. If \( K \) is the same as above, we have

   \[ h'(X) = \begin{cases} 
   \min_{Y \in K(X)} h(Y) + \text{cost}(X, Y) & \text{if } K(X) \neq \emptyset \\
   h(X) & \text{if } K(X) = \emptyset 
   \end{cases} \]

   where \( \text{cost}(X, Y) \) is the cost of the edge connecting \( X \) and \( Y \),

   then,
   \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

6. If \( h'(X) = \min_{Y \in K(X)+\{X\}} h(Y) \) \( (K \) is the same as above),

   \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

(ii) In which of the conditions above, \( h' \) is still admissible and for sure to dominate \( h \)? Check all that apply. Remember we say \( h_1 \) dominates \( h_2 \) when \( h_1(X) \geq h_2(X) \) holds for all \( X \).

(b) Suppose \( h \) is a consistent heuristic, and we conduct A* graph search using heuristic \( h' \) and finally find a solution.

(i) Answer exactly the same questions for each conditions in Question (a)(i).

1. \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

2. \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

3. \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

4. \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

5. \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

6. \( C = h^*(S) \) \( \bigcirc \) \( C > h^*(S) \) \( \bigcirc \) \( C \geq h^*(S) \)

(ii) In which of the conditions above, \( h' \) is still consistent and for sure to dominate \( h \)? Check all that apply.

\[ \bigcirc \] \[ 1 \] \[ 2 \] \[ 3 \] \[ 4 \] \[ 5 \] \[ 6 \]